Generating Triples

Let x, y and z present the triples so that

$x^2 + y^2 = z^2$.

When z = y+1, this equation becomes

$x^2 + y^2 = (y+1)^2$.

Then, expanding the brackets and a little manipulation gives

$x^2 + y^2 = y^2 + 2y + 1 \therefore x^2 = 2y + 1 \therefore x^2 - 2y = 1$

x must therefore be odd if the difference of x^2 and the even number 2y is to be the odd number 1. Therefore we see that as long as x is an odd number, an infinite amount of possible solutions can be generated. For example, if x = 7, y = (49-1)/2 = 24 $\therefore$ x = 7, y = 24 and z = 25. This can be verified, and we see that indeed 49 + 576 = 625.

Similarly, when y = z + 2, the equation becomes

$x^2 + y^2 = (y+2)^2 \therefore x^2 + y^2 = y^2 + 4y + 4 \therefore x^2 = 4y + 4 \therefore x^2 = 4(y + 1)$.

We see that x^2 must be a multiple of 4 in this case; therefore x must be divisible by 2. Since y is also to be a positive integer, $x^2 > 4$ therefore $x \geq 6$. From here, again infinite possibilities can be generated as long as x is an even number of 6 or more.

For example, when x = 12, y = 144/4 – 1 = 35 $\therefore$ x = 12, y = 35 and z = 37. And indeed, for verification, 144 + 1225 = 1369

This will follow in a similar fashion if we were to find triples with one length 3 units longer than the hypotenuse, or 4 units longer, or indeed any value.