

A Little Light Thinking

Imagine a machine with two lights. When you enter a number, a light turns on if your number belongs to one of the computer's chosen linear sequences. If your number belongs to both sequences, both lights will switch on.

Here are some examples of possible sequence rules that the computer might choose:

$$\begin{array}{ccc}
 8n + 3 & 4n + 1 & 11n + 10 \\
 6n + 5 & 3n + 2 & 5n + 1 \\
 3n + 1 & & \\
 12n + 4 & 10n + 1 & 9n + 3
 \end{array}$$

Identify some pairs of sequences for which it is possible to turn both lights on.

What do you notice about their rules?

For each pair of sequences, find a number that would turn both lights on.

Can you find a way of generating lots of numbers, once you have found one that works?

Identify some pairs of sequences for which it is **not** possible to turn both lights on.

What can you say about their rules that convinces you that it is not possible?

If the two sequences are described by the rules $an+b$ and $cn+d$, can you explain the conditions for determining whether the lights will ever switch on together?