

*This is how Mrs George's Year 7 Set 1 class from Swavesey Village College, Cambridgeshire, tackled the problem.*

This lesson is one of a series of lessons covering sequences in the summer term.

The students were given the lesson objective:  
Understand another way to analyse sequences.  
Apply this to find the nth term rule.

On the board was the following starter:

I can create a sequence by taking a times-table and shifting it. For example...

7,11,15,19... is the 4 times table shifted by 3.

What is the  
nth term rule?

The students were given 2 minutes to discuss the answer in groups. During open class discussion, three answers were given:  $n+4$ ,  $4n+3$ ,  $7n+4$ . The teacher asked students to justify their answers so eventually the students agreed it was  $4n+3$  because substituting for  $n$  works with this rule.

T: 'Can you think of another reason why this is the answer?'

S: 'Because it is in the question?'

The students were shown another sequence and asked to fill in the gaps and work out the nth term rule.

3,5,7,9... is the \_\_\_\_\_ times table shifted by \_\_\_\_\_ .

When answers were given the teacher wrote the 2 times table underneath so students could check their answer.

The teacher then introduced the students to the interactivity from the problem Shifting Times Tables (<http://nrich.maths.org/6713>). The students wrote down each sequence generated and had some thinking time to work out the rules by themselves before checking as a whole class using the checking button on the interactivity. The students did 4 sequences at levels 1 and 2. Here is an example of some of the dialogue.

The sequence was 5,9,13,17,21.

T: 'Does anyone know the answer?'

S: 'It's going up in 4s.'

T; 'So what's it shifted by?'

S: 'I don't really know what that means.'

T: 'So let's see if anyone else knows.'

S: 'It's 1 because I just worked out  $4 \times 1$ , which is 4, and 5 is one more than 4.'

The teacher then wrote the 4 times table under the sequence to help demonstrate to the class.

The students asked if they could do a level 3 sequence. The teacher explained that these are harder because they are muddled but they could try one. The sequence generated was 86, 50, 62, 38, 116. The students were given 5 minutes thinking time. It then took 5 minutes to get to the right answer as a whole class. Here is how they reached the answer:

T: 'Does anyone know the answer?'

S: 'Is it 12 times table shifted by 2?'

T: 'Let's check.' *Teacher plugs in numbers and is told that one answer is right but one is wrong.* 'Ok, does anyone else know?'

S: '12 by 38?' 'It can't be that because the shift is always less than the times table.' 'I think it's shifted by 4.' 'No, it's 6.'

T: 'Ok we're getting lots of different answers so let's think of a strategy. What did you do first?'

S: 'Put them in order then work out the difference.' 'The difference is 12.' 'No sometimes it's 24.' 'So is the times table 12 or 24?' 'What about when it's 30?'

*Teacher writes it on the board:*

38,	50,	62,	86,	116.
→	→	→	→	
12	12	24	30	

T: 'Do you think this sequence has all the numbers in it? What if some numbers are missing? What could go between 62 and 84 to carry on the rule that it's 12 times table?'

S: '74'

T: 'And what about between 86 and 116?'

S: '86 + 12 is 98 and adding 12 again is 110 but 116 is only 6 away from 110.'

T: 'Good so what do you think this means?'

S: 'Maybe it's the 6 times table and there are lots of numbers missing!'

T: 'Well what's the number in the 6 times table closest to 38?'

S: '36 so the shift could be 2.' 'The shift is 2 because when we said it's 12 and 2 before, only one number was wrong.'

The students were asked to think back to the sequences from levels 1 and 2 and to try to answer some of the following questions in groups:

What do you notice about the difference between the numbers in each sequence?

What about the rule when the numbers in the sequence are all odd?

What about if the numbers are all even?

What can you say if the units digits are all the same?

What if there are only two units digits?

What can you say if the difference between two numbers is prime?

For all of the questions, convince me why your method will always work.

After 5 minutes the teacher stopped the class as some students were struggling with the questions. Open class discussion followed.

T: 'Who can answer question 1?'

S: 'If the sequence is 5,7,9,11...then the difference is 2 and this is the times table.'

T: 'Good. Now for questions 2, if I press go and the sequence was all odd numbers is there something that is always true for the rule?'

S: 'It has to be an even times shifted by odd.'

T: 'How did you figure that out?'

S: 'I noticed it in one of the sequences and then tested it on others.'

T: 'Is it always true?'

S: 'I tried an odd times table like  $3n+1$  and found the sequence alternates between even and odd so it only works for even times tables.'

T: 'What about question 4. What if all the units digits are the same like 16,26,36...'

S: 'It's a 10 times table because only the ten times table always ends with the same digit.'

T: 'What if there are two units digits? 6,11,16,21...'

S: 'We know the 5 times table always ends in 5 or 0 so it's always the 5 times tables.'

After some discussion about primes the lesson ended.

This activity will be continued in the next lesson followed by a simplified version of 'Which Is Bigger?' (<http://nrich.maths.org/7344>) to consolidate understanding.