The following proofs are all dodgy. Why? Make sure that you give a good, clear reason.

1. **A pound equals a penny**

   Proof: $1 = 100p = (10p)^2 = (0.1)^2 = 0.01 = 1p$

2. **2 = 3**

   Proof: $2 \times 0 = 0$, so $2 = 0/0$. Also, $3 \times 0 = 0$, so $3 = 0/0$. Since $0/0$, however it is defined, is clearly the same as $0/0$ we must have $2 = 3$

3. **All numbers are equal**

   Proof: Choose any two numbers a and b and let $s = a + b$. Thus

   $a + b = s$

   $(a + b)(a - b) = s(a - b)$

   $a^2 - b^2 = sa - sb$

   $a^2 - sa = b^2 - sb$

   $a^2 - sa + s^2/4 = b^2 - sb + s^2/4$

   $(a - s/2)^2 = (b - s/2)^2$

   $a - s/2 = b - s/2$

   $a = b$

4. **There is a living man with at least four heads**

   Proof: No man has three heads. Any living man has at least one head more than than no man. Therefore, there is a living man with at least four heads.

5. $\infty = -1$

   Proof: Let

   $$x = 1 + 2 + 4 + 8 + \dots$$

   This is obviously infinite. Thus,

   $$1 + 2x = 1 + 2(1 + 2 + 4 + \cdots) = 1 + 2 + 4 + 8 + \cdots = x$$

   Thus, $1 + 2x = x$. Rearranging this gives $x = -1$. Thus, $\infty = -1$.

6. **All numbers are the same**

Proof: Suppose that all numbers were not the same. Choose two numbers a and b which are not the same. Therefore one is bigger; we can suppose that $a > b$. Therefore, there is a positive number c such that $a = b + c$. Therefore, multiplying by $(a - b)$ gives

$$a(a - b) = (b - +)(a - b)$$

Expanding gives

$$a^2 - ab = ab - b^2 + ca - bc$$

Rearranging gives

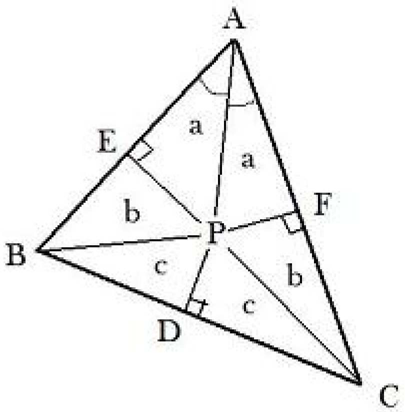
$$a^2 - ab - ac = ab - b^2 - bc$$

Taking out a common factor gives

$$a(a - b - c) = b(a - b - c)$$

Dividing through gives $a = b$, therefore a and b could not have been different after all, hence all numbers are the same.

7. **All triangles are isosceles**



Proof: Consider a triangle ABC (see the diagram).

1. Draw a line bisecting the angle A.

2. Draw a line bisecting the segment BC that is perpendicular to BC. If these two lines are parallel, then we know that we have an isosceles triangle.

3. Assume now that they are not parallel. Then they must intersect at a point P. We may now draw lines from P to E and F that are perpendicular to AB and AC, respectively.

4. The two triangles (a) are equal since they have two equal angles and share one side. Thus, PE = PF.

5. Since DP is perpendicular to BC, the two triangles (c) must be right angle triangles. Since DP bisects BC, D is the midpoint of BC. Thus, the two triangles gamma share two sides and one angle and are therefore equal triangles. Thus, PB = PC.

6. With two equal sides (PB = PC and PF = PE) and an equal angle each, the two triangles (b) must therefore be equal triangles. Hence, BE + EA = CF + FA and the triangle must be isosceles.

8. $\mathbf{1 = -1.}$

   Proof: To see why, follow this chain of reasoning:

   Clearly, $-1 = -1$

   Therefore, $\frac{-1}{1} = \frac{-1}{1}$

   Therefore, $\frac{-1}{1} = \frac{1}{-1}$

   Therefore, $\sqrt{\frac{-1}{1}} = \sqrt{\frac{1}{-1}}$

   Therefore, $\frac{\sqrt{-1}}{1} = \frac{1}{\sqrt{-1}}$

   Therefore, $\sqrt{-1}^2 = 1^2$

   Therefore, $-1 = 1$

9. **Any two real numbers are the same**

   Proof: Pick any two real numbers b and b. Then, if $a^b = a^c$, then $b = c$. Therefore, since $1^x = 1^y$, we may deduce $x = y$ for any two real numbers x and y.

10. $\mathbf{0 = 1}$

    Proof: $0 = 0 + 0 + 0 + \cdots$. But $0 = 1 - 1$, so

    $$0 = (1-1) + (1-1) + (1-1) + \cdots$$

    So, by rearranging the brackets we have

    $$0 = 1 + (-1+1) + (-1+1) + (-1+1) + \cdots = 1 + 0 + 0 + 0 + \cdots = 1$$

11. **The smallest positive number is 1**

    Proof: Suppose that x is the smallest positive number. Clearly $x \leq 1$ and also $x^2 > 0$. Since x is the smallest positive number, x^2 can't be smaller then x, so we must have $x^2 \geq x$. We can divide both sides of this by the positive number x to get $x^2 \geq x$. Since x is both less than or equal to 1 and greater than or equal to 1, x must equal 1. Thus the smallest positive number is 1.

12. **1 = 2**

    Proof: If you expand the nth power of a bracket $(a+b)$ then you get

    $$(a+b)^n = a^n + \frac{n}{1}a^{n-1}b + \frac{n(n-1)}{2\times 1}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3\times 2\times 1}a^{n-3}b^3 + \cdots$$
    $$\cdots + \frac{n(n-1)(n-2)}{3\times 2\times 1}a^3b^{n-3} + \frac{n(n-1)}{2\times 1}a^2b^{n-2} + \frac{n}{1}ab^{n-1} + b^n$$

    (This is called the binomial theorem)

    Put $n = 0$. Then $(a+b)^0 = 1, a^0 = 1, b^0 = 1$ and all other terms vanish because they have n as a factor. Thus,

    $$1 = 1 + 0 + \cdots + 0 + 1 = 2$$

13. **3 = 0**

    Proof: Consider the quadratic equation

    $$x^2 + x + 1 = 0$$

    Then, we can see that

    $$x^2 = -x - 1$$

    Assuming that x is not zero (which it clearly isn't, from the equation) we can divide by x to give

    $$x = -1 - \frac{1}{x}$$

    Substitute this back into the x term in the middle of the original equation, so

    $$x^2 + \left(-1 - \frac{1}{x}\right) + 1 = 0$$

    This reduces to

    $$x^2 = \frac{1}{x}$$

    So, $x^3 = 1$, so $x = 1$ is the solution. Substituting back into the equation for x gives

    $$1^2 + 1 + 1 = 0$$

    Therefore, $3 = 0$.