**Cuboid Challenge**

First, I cut out 5 square sheets of paper with a length of 20cm. Then, I took each sheet and cut corners of different size squares, measuring: 1cm², 2cm², 3cm², 4cm² and 5cm² respectively. So, for example, on the first sheet, I cut corners of 1cm². On the second sheet, I cut corners of 2cm² etc. until I had used up all the sheets. I folded what was left to make a box and recorded the volumes. I found that cutting corners of different lengths affected the volume.

Here is a table to show my results:

|  |
| --- |
| Length of corner cut out (cm) Volume of box (cm³) |
| 1 length=18, width=18, height=1. Volume= 324 |
| 2 l=16, w=16, h=2, V=512 |
| 3 l=14, w=14, h=3, V=588 |
| 4 l=12, w=14, h=4, V=576 |
| 5 l=10, w=10, h=5, V=500 |

As you can see from my table, with corners of a 3cm length, the volume was 588cm³. With corners measuring 4cm in length, the volume was 576cm³. Therefore, the highest volume may have been achieved with corners measuring between 3cm and 4cm in length. (I confirmed this by having plotted a graph of volume against length of cut.)

I decided to work out a formula for finding the volume of a 20cm square with a cut out of length ‘x’. This is my formula: (20-2x)(20-2x)x . I used it to clarify my answer:

|  |  |
| --- | --- |
| Length of corner cut out (cm) | Volume of box (cm³) |
| 3.1 | 590.364 |
| 3.2 | 591.872 |
| 3.3 | 592.548 |
| 3.4 | 592.416 |

I found that the largest volume of the box had a corner cut out measuring 3.3cm. I used the formula again to improve my answer:

|  |  |
| --- | --- |
| Length of corner cut out (cm) | Volume of box (cm³) |
| 3.31 | 592.570764 |
| 3.32 | 592.585472 |
| 3.33 | 592.592148 |
| 3.34 | 592.590816 |

I found here that the largest volume had a corner cut measuring 3.33cm. I used the formula once more to improve my answer:

|  |  |
| --- | --- |
| Length of corner cut out (cm) | Volume of box (cm³) |
| 3.331 | 592.5923748 |
| 3.332 | 592.5925215 |
| 3.333 | 592.5925881 |
| 3.334 | 592.5925748 |

I concluded that the highest volume for a 20cm² square is a cut out of 3.333 (592. 5925881cm³)

I repeated the same procedure for squares of length of 18cm and 24cm. To find out the largest volume, I again used the formula, substituting the 20 for 18 and 24. (I also plotted graphs to confirm the answers to the largest volume of those measures.)

The Largest Volume of 18cm

|  |  |
| --- | --- |
| Length of corner cut out (cm) | Volume of box (cm³) |
| 1 | 256 |
| 2 | 392 |
| 3 | 432 |
| 4 | 400 |

I found that the largest volume was between the cut out of 3cm and 4cm (392-432cm³). I used the formula once more to improve my answer:

|  |  |
| --- | --- |
| Length of square cut out (cm) | Volume of box (cm³) |
| 3.1 | 431.644 |
| 3.2 | 430.592 |
|  |  |

I saw that the largest volume cut out was lower than 3.1cm, so I investigated the numbers between 2 and 3cm to find out their volumes. I predicted that the largest volume for 20cm would be nearest the cut out of 3cm:

|  |  |
| --- | --- |
| Length of corner cut out (cm) | Volume of box (cm³) |
| 2.6 | 425.984 |
| 2.7 | 428.652 |
| 2.8 | 430.528 |
| 2.9 | 431.636 |

I then concluded that the largest volume for an 18cm box is a cut out of 3cm (432cm³), as the numbers between 2 and 3cm were smaller than the volume of 3cm.

The Largest Volume of 24cm

I repeated the same procedure for a square of length 24cm. To find out the largest volume, I used the formula, substituting the 18 for the 24.Here is my first table to find the largest volume:

|  |  |
| --- | --- |
| Length of square cut out (cm) | Volume of box (cm³) |
| 1 | 484 |
| 2 | 800 |
| 3 | 972 |
| 4 | 1024 |
| 5 | 980 |

I found that the largest volume was between the cut out of 4 and 5cm (972-1024cm³) I used the formula again to improve my answer:

|  |  |
| --- | --- |
| Length of square cut out (cm) | Volume of box (cm³) |
| 4.1 | 1023.524 |
| 4.2 | 1022.112 |

I saw that the volumes above were smaller than the volume with the 4cm cut out, so I investigated numbers between 3cm and 4cm. I predicted that the highest volume of 18cm would be nearest the cut out of 4cm, so I started with a cut out of 3.9:

|  |  |
| --- | --- |
| Length of square cut out (cm) | Volume of box (cm³) |
| 3.9 | 1023.516 |

I saw that this was also lower than the volume of the 4cm cut out, and concluded that the highest volume of an 18cm box has a cut out of 4cm and a volume of 1024cm³.

I was then ready to answer the question of: “Can you find a relationship between the size of paper and the size of cut that produces the maximum volume?” To answer this, I set my results of the investigation in a table:

|  |  |
| --- | --- |
| Size of paper(cm) | Size of cut that produces maximum volume (cm³) |
| 18 | 3 |
| 20 | 3.33 |
| 24 | 4 |
|  |  |

I found that there is a relationship between the size of cut that produced the maximum volume and the size of the paper. The maximum volume occurred when the cut out square was a sixth of the original square. I decided to put this to the test using two predictions:

|  |  |
| --- | --- |
| Size of paper(cm) | Size of cut that produces maximum volume (cm³) |
| 36 | 6 |
| 50 | 8.333 |

Therefore, it is clear that the maximum possible volume for a square sheet of paper measuring 20cm in length is 592.5925881cm³ with a size cut out of 3.333cm because this size of cut measures one sixth of the paper’s length.