I have started from the observations appearing in the solution of the problem 'Making tracks', that while the front wheel can point in any direction, the back wheel always points exactly forward towards the front wheel.

This means that the position of the contact point of the front wheel at any moment is placed a distance $d = 1m$ in front of the contact of the rear wheel, on the tangent at the curve traced by the back wheel.

Let $y = f(x)$ be the equation of the curve traced by the back wheel (its trace). The equation of the tangent at this curve in the point (x_b, y_b) is:

$$y - y_b = f'(x_b) \cdot (x - x_b) \qquad (1)$$

The front wheel is situated a distance d in front of the point (x_b, y_b), on this tangent:

$$x_f = x_b + \frac{d}{\sqrt{1 + f'^2(x_b)}} \quad (2) \quad y_f = f(x_b) - \frac{d \cdot f'(x_b)}{\sqrt{1 + f'^2(x_b)}} \quad (3)$$

To find the trajectory of the front wheel, one must find the relation between and y_f and x_f from (2) and (3), preferably without reference to the point (x_b, y_b). One could observe that:

$$(x_f - x_b)^2 + (y_f - f(x_b))^2 = d^2$$

I don't see a solution to eliminate x_b on the general case, so that I'll work the numerical examples.

Numerical examples I have worked numerical examples using the Matlab software: I have defined a vector containing the values of x of interest for each example, I have defined the function $f(x)$ described by the back wheel and calculated it in the points of the domain of definition, I have calculated the derivative of the function in all points x (using its symbolic expression) and finally I have calculated the coordinates of the front wheel, as given by (2) and (3). On the same graph, I represented in blue the trajectory of the back and in red the trajectory of the front wheel respectively.

see the drawings below

From the examination of these curves, it appears that the trajectory of the front wheel is situated on the outside in respect to the centre of curvature. In the case of a circular trajectory of the back wheel, the front one has a circular trajectory, of higher radius. This is understandable easily using geometric arguments: if one considers a circle, a direction on this circle, and the tangents in successive points of the trajectory, measuring a distance d forward on the tangents, a circle of higher radius is found. Its radius is:

$$r_f = \sqrt{r_b^2 + d^2}$$

In the case of a sine trajectory of the back wheel, the trajectory of the front wheel seems also a sine one, but of higher amplitude and with the phase in front of the phase of the back wheel.

Now, I shall work some examples from the problem, calculating the derivative of each function in x_b, replacing it in (2) and (3) and trying to find a relation between x_f, y_f which is the equation of the trajectory followed by the front wheel. I'll replace d directly with 1. Let the coordinates of the back wheel be (x_0, y_0), and of the front one be (x_0, y_0)

1) $y_0 = 1/x_0$

After some calculations, I found from (2) and (3):

$$x_0 = \frac{x-1}{1-y}$$

and replacing into (4):

$$y = \frac{3}{4x}$$

Representing on the same graph $y = 1/x$ and $= 3/(4x)$ shows exactly what I found from numerical calculations previously.

see the pictures below

2) $y_0 = \sin(x_0)$

I found from (2) and (3):

$$x = x_0 + \frac{1}{\sqrt{1+\cos^2 x_0}} \qquad y = \sin(x_0) + \frac{\cos x_0}{\sqrt{1+\cos^2 x_0}} \tag{5}$$

Here, $x - x_0 \leq 1$ From the second eq. in (5), replacing x_0 as a function of x and using the notation $\sqrt{1+\cos^2 x_0} = R$, I found:

$$y = \sin\left(x - \frac{1}{R}\right) + \frac{1}{R} \cdot \cos\left(x - \frac{1}{R}\right) \tag{6}$$

From the study of oscillations, I know that (6) could be written as:

$$y = A\sin(x + \alpha) \tag{7}$$

where A is the amplitude and α is the phase. I have determined by identification these coefficients, finally obtaining:

$$y = \frac{\sqrt{1+R^2}}{R} \sin\left(x + \arccos\left(\frac{R\cos\frac{1}{R} + \sin\frac{1}{R}}{\sqrt{1+R^2}}\right)\right) \tag{8}$$

This proves the 'guess' that the front wheel describes also a sine trajectory, with higher amplitude (the back wheel had amplitude 1, and here $A > 1$ because $R \geq 1$, from its definition), and with phase α (the trajectory of the front wheel is phased in front of the trajectory of the back wheel). Here I have only proved the 'sine' character of the trajectory of the front wheel, because in (8) both A and α depend on x_0.