More Twisting and Turning

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All simplified fractions can be split into the following categories:

1. 1/n
2. m/n
3. –n/m
4. n/m
5. -1/n
6. –m/n

where 1< m<n

Case 1: 1/n

These fractions can be disentangled by turning (→*-n*) then twisting *n* times, taking the value back to 0.

Case 2: m/n

We can reduce *m/n* to the form *1/n*. To do this, we turn the fraction (→*-n/m*) then continue twisting until the first positive fraction is reached.

Let *n= km+r* (where *r* is the remainder). With one turn *m/n* →*-n/m= -k- r/m*. Therefore *k+1* twists are required to get the first positive fraction:

-n/m+ (k+1) = ***(m-r)/m***

As *m-r<m* and *m<n*, both numerator and denominator of *m/n* have been reduced.

Similarly, we keep repeating the above procedure to *(m-r)/m* until the fraction becomes of the form *1/a* (*a* is a positive integer). This is Case 1 and we can disentangle this fraction.

Case 3: -n/m

With *k+1* twists, *-n/m* becomes *(m-r)/m*, which is Case 2.

Case 4: n/m

With one turn and one twist, *n/m* becomes *1-m/n = (n-m)/n*, which is Case 2.

Case 5: -1/n

With one twist, *-1/n* becomes *(n-1)/n* which is Case 2.

Case 6: -m/n

With one twist, *-m/n* becomes *(n-m)/n*, which is Case 2

Thus, we can disentangle any fraction.

In addition, we can disentangle any negative integer *n* by twisting *n* times.
We can disentangle any positive integer *n* by turning to give *-1/n*, which is Case 5.

In summary, we can disentangle any rational number using the following simple rule:

<u>General Rule:</u>

If we reach a positive number/fraction, we turn; if it is a negative number/fraction, twist.

Q.E.D