Problems to develop mathematical processes and applications

Introduction

There are 15 problems in total and each problem is linked to a strand of mathematical processes and applications and to the sub-strands of content. Click next or select the title of a mathematical problem above to open it.

Incorporating these problems into unit plans which are spread throughout Key Stage 3 would enhance most schemes of work and help to broaden the learning experience of many pupils. Developing a scheme of work in this way would be an excellent step towards implementing the new Key Stage 3 programme of study.

Overview

Each problem is described in an overview paper and associated guidance papers.

Each problem comprises the following sections:

- Problem outline: This gives a general idea of what the problem is about.
- Why do this problem?: This explains how pupils will benefit from working together on this mathematical activity.
- For the problem itself and some associated teachers' notes: This contains a link to the problem on the NRICH website which contains all the resources needed to use the problem with pupils.
- Curriculum references: This shows the specific process and content strands the problem relates to.
- Other useful links, including pages from the Framework supplement of examples, and problems and articles from the NRICH website can be found in the 'Related links'.

Guidance

For every problem there are two documents that offer alternative formats of the same guidance:

- *How the problem might unfold* describes what the teacher and pupils might do based on experience of how this problem develops in the classroom. This format might be useful to give a feel for the sequence of the learning.
- *How the problem might be focused* also describes what the teacher and the pupils might do, this time set against each of the mathematical processes. This format is useful to help teachers take specific actions to help pupils focus on a particular sub-strand of the mathematical processes.

Cross-referencing tables

PROBLEM–MATHEMATICAL PROCESSES AND APPLICATIONS CROSS-REFERENCE TABLE

Problem	Representing	Analysing – use mathematical reasoning	Analysing – appropriate mathematical procedures	Interpreting and evaluating	Communicating and reflecting
All in a jumble		✔			

Problem	Representing	Analysing – use mathematical reasoning	Analysing – appropriate mathematical procedures	Interpreting and evaluating	Communicating and reflecting
Consecutive sums	✔				
Got it!		✔			
Harmonic triangle				✔	
Isosceles triangle				✔	
More number pyramids	✔				
Odds and evens					✔
Reaction timer					✔
Route to infinity					✔
Seven squares					✔
Speeding up, slowing down		✔			
Square it				✔	
Squares in rectangles	✔				
Tilted squares		✔			
Triangles in circles				✔	

PROBLEM–RANGE AND CONTENT CROSS-REFERENCE TABLE

Problem	Number	Algebra	Geometry and measures	Statistics
All in a jumble			Measures and mensuration	
Consecutive sums	Number operations Integers, powers and roots	Equations, formulae, expressions and identities		

Problem	Number	Algebra	Geometry and measures	Statistics
Got it!	Mental calculation methods Integers, powers and roots			
Harmonic triangle	Fractions, decimals, percentages, ratio and proportion	Equations, formulae, expressions and identities		
Isosceles triangle			Transformations and coordinates Measures and mensuration	
More number pyramids		Equations, formulae, expressions and identities		
Odds and evens				Interpreting and discussing results Probability
Reaction timer				Specifying a problem, planning and collecting data Processing and representing data Interpreting and discussing results
Route to infinity		Sequences, functions and graphs	Transformations and coordinates	
Seven squares		Sequences, functions and graphs Equations, formulae, expressions & identities		
Speeding up, slowing down		Sequences, functions and graphs	Construction and loci Transformations and coordinates	
Square it			Geometrical reasoning	
Squares in rectangles		Sequences, functions and graphs		
Tilted squares		Equations, formulae, expressions and identities	Geometrical reasoning Measures and mensuration	
Triangles in circles			Geometrical reasoning	

Mathematics problem: All in a Jumble

Problem outline

This problem gives a set of 'jumbled up' measurements that need to be sorted. The interactivity changes each time you use it, so it is possible to use the same problem several times with a group. Cards are also available to support group work away from the computer. For some items there are a few numbers which could be valid and the aim is to juggle them to find a combination where every measurement makes sense.

Why do this problem?

This problem gives opportunities for pupils to focus on effective ways of working with data that needs ordering. The relatively closed nature of the problem provides a secure context in which pupils can work together and put forward the cases for their views. The openness comes from the routes to a solution, not the solution itself. Time can also be spent helping pupils reflect on ways of working together by asking about which collaboration had the most positive impact on their thinking.

For the problem itself and some associated teachers' notes

All in a Jumble (link opens in new window) from NRICH (link opens in new window).

Curriculum references: process

The guidance sections 'What teachers might do' offer suggested actions that can help to draw out pupils' skills in how to use mathematical reasoning. There is, however, a breadth of opportunities to develop a range of process skills including:

- Make and justify conjectures, considering special cases and counter-examples.
- Work systematically and logically towards results and solutions.
- Use estimation and approximation to support decisions and check reasonableness of answers.
- Form convincing arguments.
- Engage in mathematical discussion of the appropriateness and accuracy of data matches, making connections with the context.

Curriculum references: content

Geometry and measures: Measures and mensuration
Select and use appropriate units, estimating and using the equivalence of units.

Useful links

Other useful links including problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: Consecutive sums

Problem outline

This problem investigates the sums of consecutive numbers. The aim is to identify patterns and relationships and explain them, for example, why all odd numbers can be written as the sum of two consecutive numbers, or identifying a rule for determining the number of ways a particular number can be represented as the sum of consecutive numbers.

Why do this problem?

This problem can provide high quality activity for all pupils by offering an accessible context in which to explore the structure of numbers at a wide range of levels. First, through experimentation, learners might discover relationships, then they might begin to pose their own problems and finally, produce convincing arguments or proofs for what they have discovered.

For the problem itself and some associated teachers' notes

Consecutive Sums (link opens in new window) from NRICH (link opens in new window).

Curriculum references: process

The guidance sections ‘What teachers might do' offer suggested actions that can help to draw out pupils' skills in representing. There is, however, a breadth of opportunities to develop a range of process skills including:

- Simplify a situation or problem to help identify and classify patterns.
- Pose problems, making and beginning to justify conjectures and generalisations.
- Appreciate that there are a number of different techniques and approaches that can be used to analyse a situation.
- Form convincing arguments based on findings, and make general statements.

Curriculum references: content

Number: Number operations; Integers, powers and roots

Understand addition, subtraction, multiplication and division as they apply to whole numbers and decimals; know how to use the laws of arithmetic and inverse operations. Construct and simplify linear expressions.

Useful links

Other useful links including problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: Got It!

Problem outline

This is an adding game that can be played against the computer or by pairs of pupils. Starting with the Got It! target of 23, players take turns to add a whole number from 1 to 4 to the running total.

The player who hits the target of 23 wins the game. The aim is to find a winning strategy and then change the game, choose a new target or a new range of numbers to add on, and generalise the winning strategy.

Why do this problem?

This is a motivating context in which learners can use their mental addition and subtraction skills. However, the real challenge is not the calculation but the search for a winning strategy that will always work. This involves conjecturing, refining ideas, generalising and using knowledge of factors and multiples to aid efficiency.

For the problem itself and some associated teachers' notes

Got It! (link opens in new window) from NRICH (link opens in new window).

Curriculum references: process

The guidance sections ‘What teachers might do' offer suggested actions that can help to draw out pupils' skills in how to use mathematical reasoning. There is, however, a breadth of opportunities to develop a range of process skills including:

- See the mathematics in the structure of the game and make connections with other similar contexts.
- Use effective ways of recording that help reveal patterns and relationships.
- Make and begin to justify conjectures and generalisations, considering special cases and counter-examples.
- Explore the effects of varying values.
- Form convincing arguments and communicate them effectively.
- Engage with someone else's reasoning.

Curriculum references: content

Number: Mental calculation methods; integers, powers and roots
Using addition and subtraction facts and utilising knowledge of factors and multiples.

Useful links

Other useful links including problems and articles from the NRICH website can be found in the 'Related links'.

Useful links

Other useful links including problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: Harmonic triangle

Problem outline

In this problem you are given the start of a triangle of terms. Each fraction in the triangle is equal to the sum of the two fractions below it. You are asked to extend the triangle, notice patterns and justify their continuation.

Why do this problem?

This problem provides a fraction-based challenge for pupils who already have a good understanding of fraction addition and subtraction. Most benefit can be gained if discussion goes beyond 'pattern spotting' to making the connection between the values in the triangle and the underpinning structure (a sort of 'unifying principle'). It has the potential to lead to algebraic manipulation of fractions as learners generalise and justify their findings.

For the problem itself and some associated teachers' notes

Harmonic triangle (link opens in new window) from NRICH (link opens in new window).

Curriculum references: process

The guidance sections 'What the teacher might do' offer suggested actions that can help to draw out pupils' skills in interpreting and evaluating. There is, however, a breadth of opportunities to develop a range of process skills including:

- Select mathematical methods and tools, such as inverses, to use.
- Use knowledge of related problems.
- Identify and classify patterns and make conjectures and generalisations from them.
- Calculate accurately, recording methods, solutions and conclusions.
- Form convincing arguments and engage with those of someone else, discussing strengths and limitations of approaches.

Curriculum references: content

Number: Fractions, decimals, percentages, ratio and proportion

Add and subtract fractions

Algebra: Equations, formulae, expressions and identities

Use symbols to represent general terms and potentially extend to manipulating algebraic fractions.

Useful links

Other useful links including problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: Isosceles triangles

Problem outline

In this problem you are asked to draw some isosceles triangles with an area of 9 cm^2 and a vertex at the point (20, 20). If all the vertices must have whole-number coordinates, how many triangles is it possible to draw? There is interactivity to support discussion and the organisation of solutions if required.

Why do this problem?

This problem calls on pupils to connect a wide range of mathematical knowledge including coordinates, properties of isosceles triangles, area of a triangle, factors and symmetry. The solution, which is numerically quite small, is attainable without being trivial and requires an organised and systematic approach. An emphasis should be placed on being able to justify, through the rigour of the approach, that all the solutions could be, or have been found. This is more important than finding some or even all of the solutions.

For the problem itself and some associated teachers' notes

Isosceles triangles (link opens in new window) from NRICH (link opens in new window).

Curriculum references: process

The guidance sections 'What teachers might do' offer suggested actions that can help to draw out pupils' skills in interpreting and evaluating. There is, however, a breadth of opportunities to develop a range of process skills including:

- Select mathematical information and tools to use, making connections within mathematics.
- Work logically towards results and solutions, recognising the impact of constraints and assumptions, and identifying and classifying patterns.
- Make and begin to justify conjectures and generalisations, forming convincing arguments.
- Engage with someone else's approaches, arguments and results.
- Consider the elegance and efficiency of alternative solutions.

Curriculum references: content

Geometry and measures: Transformations and coordinates; Measures and mensuration.
Apply knowledge of reflection and rotation to investigate isosceles triangles within a coordinate system using ICT.

Useful links

Other useful links including problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: More number pyramids

Problem outline

This problem involves looking for, describing and explaining patterns and relationships in a pyramid of numbers where each of the four cells at the bottom of the pyramid are related by a simple rule to numbers in subsequent rows. What is the number at the vertex of pyramids built in this way and how does it relate to the 'starter' number? Can you explain why you only get multiples of 4 at the top when you start with an integer in the bottom left-hand corner?

Why do this problem?

This problem offers a context for pupils to discover things that must be true, and asks them to find out why this is the case. Patterns are revealed very quickly but the aim is to generalise them and represent them in a meaningful way. At higher levels this will involve manipulating algebraic expressions, but it is the structure that explains the result not the algebra. One challenge is to make that connection.

For the problem itself and some associated teachers' notes

More number pyramids (link opens in new window) from NRICH (link opens in new window).

Curriculum references: process

The guidance sections 'What teachers might do' offer suggested actions that can help to draw out pupils' skills in representing. There is, however, a breadth of opportunities to develop a range of process skills including:

- Select representations that enable the structure to be seen.
- Identify patterns and make and defend conjectures, using a range of arguments.
- Consider the effectiveness of different forms of representation and strategies used, including the use of algebra and of counter-examples.
- Engage in mathematical discussion of findings, looking for the equivalence in both the different approaches to the problem and different representations.

Curriculum references: content

Algebra: Equations, formulae, expressions and identities

Construct and simplify algebraic expressions that represent a practical relationship. Use the relationship to explore and interpret the underpinning mathematical structure of a situation.

Useful links

Other useful links including problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: Odds and evens

Problem outline

In this problem you are asked to draw randomly from the four sets of balls marked with odd and even numbers; you win if, having randomly drawn two balls from any set, their sum is even. The interactivity enables you to test out ideas but

the aim is to justify experimental results compared with those of a theoretical approach to finding the likelihood of winning.

Why do this problem?

The starting point can be making and justifying conjectures about the set of balls that gives the best chance of winning by getting an even total. Conjectures and justifications may initially hinge on the relative number of even-numbered balls in a set. When the experiment is run, conjectures may be confounded and further ideas may emerge. This can lead in to calculating theoretical probabilities. The important point is to establish convincing arguments.

For the problem itself and some associated teachers' notes

Odds and Evens (link opens in new window) from NRICH (link opens in new window).

Curriculum references: process

The guidance sections 'What teachers might do' offer suggested actions that can help to draw out pupils' skills in communicating and reflecting. There is, however, a breadth of opportunities to develop a range of process skills including:

- Choose between representations and methods, working logically towards results.
- Recognise that a number of different techniques can be used and use these to form and share convincing arguments.
- Discuss the elegance and efficiency of different approaches.
- Look for equivalence in the different approaches to the problem.
- Make connections between the theoretical results and the outcomes from the experimental context.

Curriculum references: content

Statistics: Interpreting and discussing results, probability.

Interpret data collected by the interactivity and relate it to the original context, discussing why results will vary. Compare distributions from the different contexts. Calculate probabilities, using systematic recording of all possibilities or tree diagrams.

Useful links

Other useful links including problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: Reaction timer

Problem outline

This problem is created within a reaction timer environment and offers opportunities for pupils to pose their own questions or test given or self-generated hypotheses. The data can then be analysed by applying current content

knowledge of statistical techniques or to extend knowledge of statistical techniques, thus offering considerable flexibility. At the highest level, discussions can arise about possible sources of bias and how these can be minimised.

Why do this problem?

Learners need lots of practice at formulating, as well as testing hypotheses and this problem is an ideal starting point for developing the skills of making and testing hypotheses of their own. The aim is to spend time refining initial hypotheses by asking questions such as 'Is this always true? When might it not be true? Do we want to add some conditions, such as …?' The next step is to make decisions about the information that is required to answer the refined hypothesis, how to analyse the data that are collected and what that analysis says about the original question. Does it support or refute the hypothesis?

For the problem itself and some associated teachers' notes

Reaction Timer (link opens in new window) from NRICH (link opens in new window).

Curriculum references: process

The guidance sections 'What teachers might do' offer suggested actions that can help to draw out pupils' skills in communicating and reflecting. There is, however, a breadth of opportunities to develop a range of process skills including:

- State hypotheses.
- Select mathematical information, methods and tools to use.
- Make diagrams and present statistics that represent findings.
- Use data and statistics to form convincing arguments, looking for evidence that supports or refutes the hypothesis.
- Engage with someone else's discussion of results, and consider the effectiveness of their own and other people's approaches.

Curriculum references: content

Statistics: Specifying a problem, planning and collecting data; Processing and representing data; Interpreting and discussing results.

Use reaction times as the focus for defining hypotheses. Make decisions on data and statistics that will inform the testing of the hypothesis. Report findings and reflect on the findings and statistical arguments of others.

Useful links

Other useful links, including pages from the Framework supplement of examples, and problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: Route to infinity

Problem outline

In this problem pupils are asked to take some time looking at the route the arrows follow on a coordinate grid. By identifying and applying the system of arrows beyond what is visible, the aim is to work out:

- How many points the route passes through before reaching a given point, for example (9, 4).
- What direction the arrows will be pointing in when they get there.

Why do this problem?

This problem offers a good opportunity for pupils to internalise patterns and share what they see. In doing this they will begin to identify and talk about relationships they are all able to connect with. Through this they begin to make convincing cases for their solutions to the problem. On the way, they will be working with coordinates and seeing how different areas of mathematics can come together in surprising places (for example, sequences, coordinates, triangular numbers).

For the problem itself and some associated teachers' notes

Route to Infinity (link opens in new window) from NRICH (link opens in new window).

Curriculum references: process

The guidance sections 'What teachers might do' offer suggested actions that can help to draw out pupils' skills in communicating and reflecting. There is, however, a breadth of opportunities to develop a range of process skills including:

- Select mathematical information, methods and tools to use.
- Make connections within mathematics.
- Appreciate that there are a number of different techniques that can be used to analyse a situation and check working.
- Look and account for patterns and use them to extend the mathematics.
- Consider the elegance and efficiency of alternative strategies and solutions.

Curriculum references: content

Geometry and measures: Transformations and coordinates

Use 2-D coordinates and find relationships between points and a geometric relationship.

Algebra: Sequences functions and graphs

Generate terms in a sequence and determine a general rule.

Useful links

Other useful links including problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: Seven squares

Problem outline

This problem looks at patterns, and challenges pupils to describe them clearly – verbally, numerically and, where appropriate, algebraically. It starts with an image of seven squares that can be generalised to 100 squares, 1000 squares and so on. It asks pupils to look at the different ways that the number of matchsticks can be calculated.

Why do this problem?

This problem does not assume prior knowledge of algebra and could be a good way to introduce, practise or assess algebraic fluency. It is often the case that pupils are guided to take a situation, create a table of results and translate it into algebra. Only then do they 'look back' to make sense of what has been found in terms of the original structure. Here the emphasis is on giving learners the opportunity to see the structure first and to use their insights to generate the algebra. Different pupils will see the structure differently and yet still end up in the same place, and this is at the heart of the problem. There is no single correct way to see the structure and there are multiple routes to a solution, all of which, surprisingly to some, lead to the same algebraic generalisation.

For the problem itself and some associated teachers' notes

Seven Squares (link opens in new window) (link opens in new window) from NRICH (link opens in new window) (link opens in new window).

Curriculum references: process

The guidance sections 'What teachers might do' offer suggested actions that can help to draw out pupils' skills in communicating and reflecting. There is, however, a breadth of opportunities to develop a range of process skills including:

- Make sense of a situation in order to represent it mathematically, using appropriate variables and symbols.
- Form convincing arguments based on the context and its mathematical structure; relate findings to the original context.
- Engage in someone else's reasoning and look for equivalences.

Curriculum references: content

Algebra: Equations, formulae, expressions and identities; Sequences, functions and graphs

Create expressions that represent a pattern emerging from a practical context, justifying it in terms of the structure. Simplify expressions to demonstrate their equivalence, including the use of brackets.

Useful links

Other useful links, including pages from the Framework supplement of examples, and problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: Speeding up, slowing down

Problem outline

This problem involves experimenting with an interactivity that shows regular polygons 'rolling' along a horizontal surface and plots graphs related to the motion of a red dot. The aim is to explore and explain how the speed varies when a dot is placed in different positions on the regular polygon:

- at the centre
- on the edge
- on a vertex.

A card-sorting activity is also available and the idea of using cards and working away from the computer could be extended further.

Why do this problem?

This problem provides a visual context in which to consider how speed–time graphs represent movement over time. It is designed to provide opportunities to discuss and refine ideas by allowing time for pupils to justify predictions and then modify their views in light of what others say, and what they see. This can offer opportunities to address misconceptions and improve understanding.

For the problem itself and some associated teachers' notes

Speeding Up, Slowing Down (link opens in new window) (link opens in new window) from NRICH (link opens in new window).

Curriculum references: process

The guidance sections 'What teachers might do' offer suggested actions that can help to draw out pupils' skills in how to use mathematical reasoning. There is, however, a breadth of opportunities to develop a range of process skills including:

- Visualise and work with dynamic images.
- Explore the effects of varying values, taking account of feedback and learning from mistakes.
- Make connections between particular situations and outcomes within the context.
- Make and justify conjectures using convincing arguments.
- Engage with someone else’s mathematical reasoning.

Curriculum references: content

Algebra: Sequences functions and graphs, Geometry and measures: Construction and loci

Plot and interpret graphs arising from practical situations. Using and interpreting loci and the effects of transformations.

Useful links

The Traffic program and teacher guidance from Improving Learning in Mathematics (2005) DFES 1599-2005DOC-EN. This is part of the Standards Unit, Success for All resources distributed to secondary schools in 2006. Downloaded

from: teachingandlearning.qia.org.uk (link opens in new window) or order from: www.ncetm.org.uk (link opens in new window)

Other useful links including problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: Square it

Problem outline

This game involves two players choosing points on a square dotted grid with the aim of being the first one to 'make a square'. The aim is to begin to analyse the game to identify some winning strategies, for example, 'Where are the best places to start?'

Why do this problem?

This game offers an excellent opportunity to practise visualising squares and angles on grids and also encourages pupils to look at strategies using systematic approaches. Describing strategies to others helps them to focus and clarify mathematical reasoning. Working with tilted squares also provides an opportunity to examine the properties of gradients of parallel and perpendicular lines.

For the problem itself and some associated teachers' notes

Square It (link opens in new window) from NRICH (link opens in new window).

Curriculum references: process

The guidance sections 'What teachers might do' offer suggested actions that can help to draw out pupils' skills in interpreting and evaluating. There is, however, a breadth of opportunities to develop a range of process skills including:

- Identify the mathematical aspects of the game and make connections with other mathematics they have learned.
- Visualise and plan ahead.
- Develop strategies and consider the effectiveness of the strategies of others, learning from feedback.
- Work systematically and refine approaches.

Curriculum references: content

Geometry and measures: geometrical reasoning; transformation and coordinates.

Pupils will identify parallel and perpendicular lines, utilise properties of squares and recognise their rotations.

Useful links

Other useful links including problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: Squares in rectangles

Problem outline

This problem is about the number of squares in a given size of rectangle. For example, a 2 by 3 rectangle contains eight squares and a 3 by 4 rectangle contains 20 squares. What size rectangles, or rectangle, contain exactly 100 squares? Is there more than one answer? Can you find them all?

Why do this problem?

This problem is a context for systematic number work, geometrical thinking and problem solving. It contains great opportunities for discussing a general idea resulting from changing one variable and how this can be extended when other changes are made. This means that something you notice in one situation can be used in a slightly different situation. For example, one way to investigate this problem is to consider how the number of squares increases as you add another column to a rectangle with a fixed number of rows. The method can be extended to rectangles with more rows.

For the problem itself and some associated teachers' notes

Square in Rectangles (link opens in new window) (link opens in new window) from NRICH (link opens in new window) (link opens in new window).

Curriculum references: process

The guidance sections 'What teachers might do' offer suggested actions that can help to draw out pupils' skills in representing. There is, however, a breadth of opportunities to develop a range of process skills including:

- Select information and ways of representing it.
- Simplify the situation or problem in order to represent it mathematically and identify things to vary systematically.
- Identify and classify patterns, make and begin to justify conjectures and generalisations.
- Reason inductively and use this to extend patterns and check working.
- Relate generalisations to the original context.
- Consider the effectiveness of alternative strategies and look for equivalence in relation to the different approaches to the problem.

Curriculum references: content

Algebra: Sequences, functions and graphs

Generate sequences from a practical context, identifying generalisations and use them to see overarching relationships.

Useful links

Other useful links including problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: Tilted squares

Problem outline

This problem hinges around finding areas of squares drawn on a square-dotted grid. It involves identifying patterns in the areas of squares with different tilts in order to be able to predict the area of a square with any tilt.

Why do this problem?

This is a rich task for sharing ideas and ways of working. It combines the need to adopt systematic approaches to collecting data with extension of knowledge of areas and a potential link to Pythagoras' Theorem. It also encourages identification of and managing variables and considering the constraints imposed by the environment. There is plenty of opportunity for learners to take different routes, depending on the variables they introduce, and to structure their findings in different ways.

For the problem itself and some associated teachers' notes

Tilted Squares (link opens in new window) (link opens in new window) from NRICH (link opens in new window) (link opens in new window).

Curriculum references: process

The guidance sections 'What teachers might do' offer suggested actions that can help to draw out pupils' skills in how to use mathematical reasoning. There is, however, a breadth of opportunities to develop a range of process skills including:

- Simplify the situation or problem in order to represent it mathematically, using appropriate variables, symbols, diagrams and models.
- Explore the effects of varying values, working logically towards results and solutions and recognising the impact of constraints.
- Calculate areas accurately and find effective ways of recording methods, solutions and conclusions to share.
- Make connections within mathematics, using knowledge of related problems and leading into new mathematics.
- Form convincing arguments, communicate findings effectively and engage with someone else’s mathematics.

Curriculum references: content

Algebra: Equations, formulae, expressions and identities.

Geometry: Geometrical reasoning; Measures and mensuration.

Working from a geometric context, reasoning about the properties of squares and calculating areas means that the problem can lead to the use of algebraic representation of generalisations.

Useful links

Other useful links including problems and articles from the NRICH website can be found in the 'Related links'.

Mathematics problem: Triangles in circles

Problem outline

This problem uses the nine-pin circular geoboard. Pupils are asked to find all the 'unique' isosceles triangles with a vertex at the centre of the circle and calculate their angles.

Can they visualise all the other isosceles triangles on geoboards with different numbers of pins? Can they extend this knowledge of how to find angles of isosceles triangles to finding the angles of any triangle inscribed in a nine-pin circle? What about circular geoboards with different numbers of pins?

Why do this problem?

This problem challenges learners to apply what they know about angles in triangles. By using the nine-pin circle learners can concentrate on the geometrical structure without having to worry about the arithmetic. It makes an excellent lead into work on angles in regular polygons and later, circle theorems. There is plenty of room for the generalisation of results relating to sizes of angles but also on the number of unique triangles.

For the problem itself and some associated teachers' notes

Triangles in Circles (link opens in new window) (link opens in new window) from NRICH (link opens in new window) (link opens in new window).

Curriculum references: process

The 'What the teacher might do' sections in the problem guidance offer suggested actions that can help to draw out pupils' skills in interpreting and evaluating. There is, however, a breadth of opportunities to develop a range of process skills including:

- Make connections within mathematics using knowledge of related problems.
- Make and begin to justify conjectures and generalisations, form arguments into a logical sequence.
- Appreciate the difference between evidence and proof.
- Engage with someone else’s mathematical reasoning.
- Engage in mathematical discussion of ideas and what constitutes a logical step.

Curriculum references: content

Geometry and measures: Geometrical reasoning

Know and use angle properties to solve geometrical problems. Explain reasoning with diagrams and text and justify inferences.

Useful links

Other useful links, including pages from the Framework supplement of examples, and problems and articles from the NRICH website can be found in the 'Related links'.
