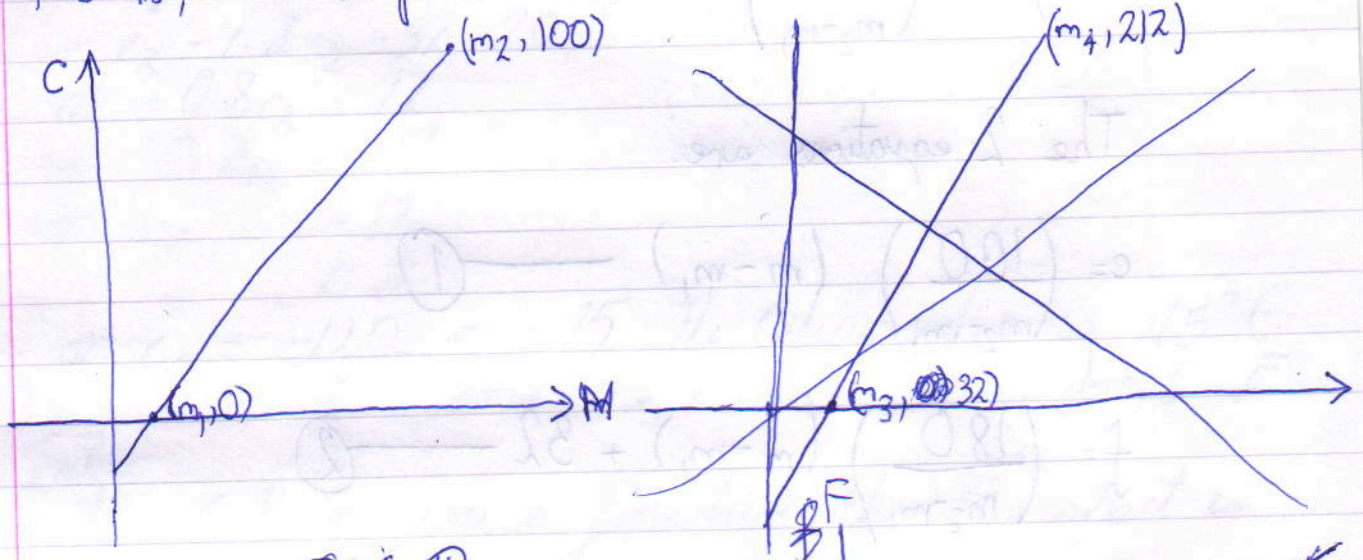
15.5.11. Temperature Nikoriska Paul / Leicester High School For Girls / Y4

Here, I am using a mercury thermometer

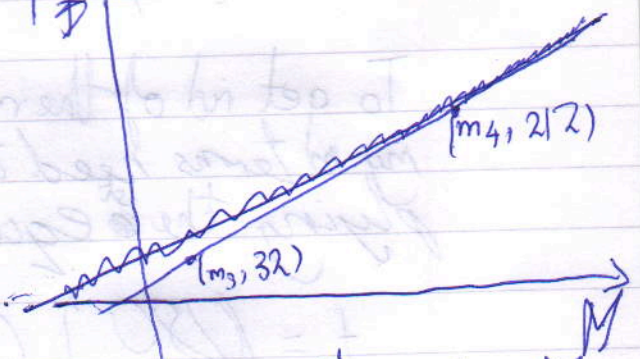
(Here I define my variables)

$M \in \mathbb{R}$, $M > 0$, M is length of mercury column
$C \in \mathbb{R}$, C is temprature in °Celsius
$F \in \mathbb{R}$, F is temprature in °Farenheit



(Here I define my Constants)

$m_1, m_2, m_3, m_4 \in \mathbb{R}$
m_1 is length of mercury at freezing point of water, $m_1 = m_3$
m_2 is length of mercury at boiling point of water, $m_2 = m_4$



Using slope of linear function we find C as a function of M.
I use my points $(m_1, 0)$, $(m_2, 100)$ and a general point (m, c)

$$\therefore \frac{c-0}{m-m_1} = \frac{100-0}{m_2-m_1}$$

$$\text{or } \frac{c}{m-m_1} = \frac{100}{m_2-m_1}$$

$$\text{or } c = \left(\frac{100}{m_2-m_1}\right)(m-m_1)$$

Similarly:

$$\frac{f-32}{m-m_1} = \frac{212-32}{m_2-m_1}$$

$$\text{or } \frac{f-32}{m-m_1} = \frac{180}{m_2-m_1}$$

$$\text{or } f-32 = \left(\frac{180}{m_2-m_1}\right)(m-m_1)$$

$$\text{or } f = \left(\frac{180}{m_2-m_1}\right)(m-m_1)+32$$

The 2 equations are:

$$c = \left(\frac{100}{m_2-m_1}\right)(m-m_1) \quad \text{——} (1)$$

and

$$f = \left(\frac{180}{m_2-m_1}\right)(m-m_1)+32 \quad \text{——} (2)$$

To get rid of the m terms by doing simple subtraction; my terms need to be equal. We can do this by multiplying the 1st equation by 1.8.

$$f = \left(\frac{180}{m_2-m_1}\right)(m-m_1)+32$$

$$-\left[1.8c = \left(\frac{1.8\times 100}{m_2-m_1}\right)(m-m_1)\right]$$

$$\text{or } f-1.8c = 32$$

$$\text{or } f = 32+1.8c \quad \text{——} (3)$$

Let the temperature, in Celsius and Fahrenheit that is equal

~~The value which is the same as~~ scales be c

$\therefore c_1 = 32+1.8c_1$ (from (3))

$$\text{or } c_1-1.8c_1 = 32$$

$$\text{or } -0.8c_1 = 32$$

$$\text{or } 0.8c_1 = -32$$

$$\text{or } c_1 = \frac{-32}{0.8} = -40 \quad \text{Ans}$$

Let,
the Celsius value for which the Farenheit is 20° more is c_2

$\therefore c_2 + 20 = 32 + 1.8c_2$ (from ③)
or $c_2 + 20 - 1.8c_2 = 32$
or $c_2 - 1.8c_2 = 32 - 20$
or $-0.8c_2 = 12$
or $0.8c_2 = -12$
or $c_2 = \frac{-12}{0.8}$

Ans or $c_2 = \frac{-120}{8} = -15$. The Celsius value is $-15°C$ and the Farenheit value is $+5°F$

Let,
the Celsius value for which the Farenheit is 20° lower is c_3

$\therefore c_3 - 20 = 32 + 1.8c_3$
or $c_3 - 20 - 1.8c_3 = 32$
or $c_3 - 1.8c_3 = 32 + 20$
or $-0.8c_3 = 52$
or $0.8c_3 = -52$
or $c_3 = \frac{-52}{0.8}$

Ans or $c_3 = \frac{-520}{8} = -65$ The Celsius value is $-65°C$ and the Farenheit value is $-85°F$

As the gradations for Celsius and Kelvin are the same you simply add or translate

$\boxed{K = 273.15 + C}$ - ④

We can find the relation between Kelvin and Farenheit by using the relation between Farenheit and Celsius

~~f = 32 + 1.8 (C + 273.15)~~

~~K = 273.15 + K~~

~~or K = 273.15 + K~~

~~or K + K = 273.15~~

~~This is not possible~~

$$f = 32 + 1.8(K - 273.15) \quad \text{——⑤}$$

Is there a common temprature on the °C and K scales?

$K = 273.15 + K$

or $K - K = 273.15$.

This is not possible (∵ the relation is translation) Ans

$f = 32 + 1.8(f - 273.15)$

or $f = 32 + 1.8f - 491.67$

or $f - 1.8f = 32 - 491.67$

or $-0.8f = -459.67$

or $0.8f = 459.67$

or $f = \frac{459.67}{0.8} \approx \frac{4600}{8} = \frac{1150}{2} = 575\ °F$ Ans

Is there a common temperature on the °F and K scales?