Digital Roots

By Bernard Bagnall

Digital roots usually first appear - though not by name - when children discover the fascinating things about the results in the 9 times table. They often notice that when the digits of each multiple (9, 18, 27, 36, 45, 54 etc.) are added together they come to 9. Pupils can be encouraged to extend the 9 times table further and so they might look at 135 558 etc. Some discussion is usually needed when the digits add up to 18 or another multiple of 9 rather than just 9 itself - as is the case for 558, 8883. In these cases the sum is considered as a number in itself and its digits added to make 9. Some pupils really enjoy checking big numbers in this way to see if they are multiples of 9, like the year in which they are born (1998 for example).

The general use of digital roots just extends that idea to any number - but does not necessarily imply anything special about multiples. So to obtain the digital root of a number we simply add the digits, and continue to do so until we are left with a single digit. For example:
1 244 > 11 > 2 so the digital root of 1 244 is 2
24 675 > 24 > 6 so the digital root of 24 675 is 6

Pupils therefore often discover that, when they have to obtain the digital root of a large number, they only need to count one out of all the 9's it contains. For example:
If we take the number 4 569 512 597 853, losing one of the two 9s gives you 456 951 257 853.
Then you can do the same with numbers that add to 9 [as we know that when added to the 9 we have kept, the resulting sum will be a multiple of 9 and will therefore have a digital root of 9]. So in the number we have now, we can also lose 4&5, 6&3, 1&8, 2&7 which leaves 9 555.
Now we can find the digital root of 9 555 quite easily: 9+5+5+5=24, then 2+4=6.
That's nice - no big additions to do to get the digital root of 4 569 512 597 853 to be 6!

When you have a sequence of numbers that occur in any investigation, challenge or exploration then finding their digital roots nearly always gives some excitement. The Big Cheese and Sending and Receiving Cards are good examples of problems where finding digital roots might be productive, as well as some other investigations on the website. The notes of these problems give further details.

[We might introduce some pupils at a higher level to modular arithmetic (sometimes known as 'clock' arithmetic) and modulo 9 is equivalent to digital roots. You might find the article Modular Arithmetic useful.]

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