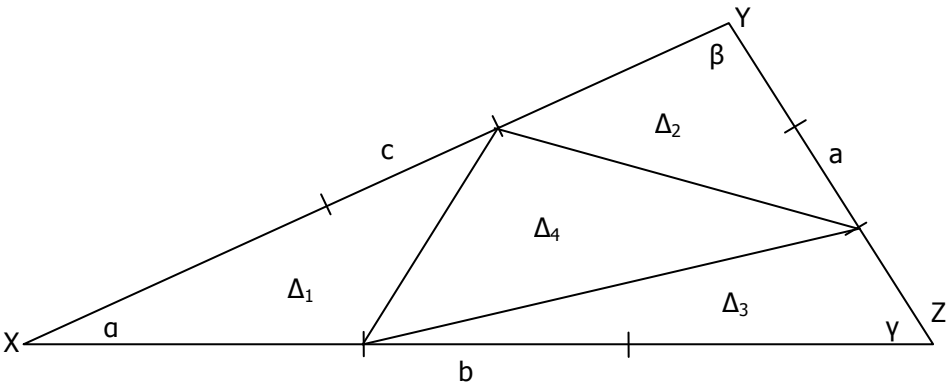
Triangle in a Triangle



Let the area of triangle XYZ be Δ, and the smaller triangles be Δ_1 through Δ_4, as above.

The area of the large triangle can be found as $\frac{1}{2}bc\sin\alpha = \Delta$.

The proportions into which each side of the large triangle has been split means that, the base of Δ_1 is $\frac{1}{3}$ the length of the base of the large triangle, and the sloping side $\frac{2}{3}$ that of the large triangle. So the area of Δ_1 is $\frac{1}{2}\frac{b}{3}\frac{2c}{3}\sin\alpha = \frac{2}{9}\left(\frac{1}{2}bc\sin\alpha\right) = \frac{2}{9}\Delta$. The same holds for triangles 2 and 3. So $\Delta_1 = \Delta_2 = \Delta_3 = \frac{2}{9}\Delta$

The area of the inner triangle, triangle 4, is therefore $\Delta - 3\times\frac{2}{9}\Delta = \frac{1}{3}\Delta$.