

Developing fluency and understanding of fractions through rich mathematical tasks

By Keith Ellis of Oakthorpe Primary School

Abstract/Summary

This collaborative project involving six primary schools explored the concrete models and visual representations of fractions and how the use of a rich mathematical task supports arithmetic proficiency and fluency of understanding. Through reflective meetings and gap tasks all schools were involved in using the Fair Feast rich task (<http://nrich.maths.org/2361>) to explore how three main visual representations of fractions could be taught across the Primary age range. According to Nunes et al (2006) fractions can be represented using part-whole, linear and quotient, and there is an imbalance in the priority that is given to these in the classroom. This was found in the questions pupils answered and how they responded to the rich task. The use of the rich task has shown the importance of teaching 'tricky' maths through a real and practical context to increase motivation, confidence with reasoning, and conceptual understanding when solving other problems. Through planned intervention with groups of year 5 pupils and case studies of some pupils: pupils were exposed to a range of representations including the quotient, had opportunities to solve problems practically with clear reasoning opportunities, and were expected to draw their own visual representations to make sense of problems. With the implementation of the new National Curriculum in 2014, this project also gives recommendations for teaching the fraction objectives.

Background

The group of schools has worked together in the past as part of the NCETM Host Schools project in February 2013. The collaborative project came about as a need to extend the work of this group and the use of rich mathematical tasks to develop a better understanding of fractions. This was also timed to help engage subject leaders with the new National Curriculum and support its implementation.

Aims of the Collaborative Teacher Project

- Pupils will have an increased fluency and proficiency in fractions including other areas of maths linked to this (proportional reasoning, multiplication, division, fractions of quantities)
- Pupils will show an increased motivation and engagement in tackling problems through the use of rich mathematical tasks
- Teachers from across all schools will have the opportunity to try rich mathematical tasks and have opportunities to reflect on the progression within the arithmetic proficiency that pupils' responses to these tasks illustrate.
- Teachers will have an increased knowledge of a range of visual representations and concrete models to use when teaching fractions.
- Representatives from the schools (subject leaders) will develop their reflections about progression within rich tasks and their pedagogical approach to teaching fractions. They will also increase their confidence and capacity to lead whole school initiatives to develop pupil progress in arithmetic proficiency.
- Schools will have a model on which to base further action research to explore proficiency in other areas of arithmetic. It will also aid them in their whole school discussion around the

implementation of the new National Curriculum and how this could be approached through the use of rich mathematical tasks.

Details of those involved in the Collaborative Teacher Project

Oakthorpe Primary School, Leicestershire (2 teachers – subject leader and Y5/6 teacher)

Donisthorpe Primary School, Leicestershire (2 teachers – subject leader and Y5 teacher)

Albert Village Primary School, Leicestershire (2 teachers – subject leader and Y5 teacher)

Viscount Beaumont CE Primary School, Leicestershire (1 teacher – subject leader and Y5/6 teacher)

St Mary's CE Primary School, Leicestershire (1 teacher – subject leader and Y6 teacher)

Riverside Primary School, Leicestershire (2 teachers - subject leader and Y5 teacher)

Supported by Dr Jenni Back

A description of the Collaborative Teacher Project

As part of the project, the group met on three occasions across the Autumn term in 2013. The project was organised in such a way that there were gap tasks between each of the sessions and time was given within the session for reflection on these tasks. Before the first session began pre reading was sent out for the participants to engage with the visual representations of fractions. Also participants were expected to carry out a pre-test with year 5 pupils focusing on the use of different representations. This ensured that all participants had an understanding of how fractions can be represented and how the pupils responded to these.

The first meeting explored visual representations in the teaching of fractions and what is already used by schools. Current research was shared surrounding the use of visual representations and concrete models in the teaching and learning of fractions. Nunes et al (2006) gave clear examples showing how there is an imbalance in the use of representations in school when looking at the type of test questions pupils receive. According to the research, there are three main representations of fractions that should be used with pupils – part-whole, linear and quotient. Before the session, each school was asked to bring along completed pre-tests (courtesy of P. Barmby). Examples of questions from this test for each of the 3 visual representations can be seen below:

Part whole questions:

Write the fraction that is shaded grey in the shape below:

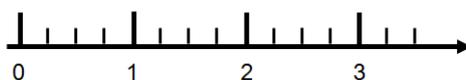


A cake was cut into 8 pieces of equal size. John ate 3 pieces of the cake. What fraction of the cake did John eat? (Tick one box)

 $\frac{1}{8}$ $\frac{3}{8}$ $\frac{3}{5}$ $\frac{8}{3}$

Linear questions:

Draw an arrow to show where $\frac{1}{2}$ is on the number line below:



Quotient questions:

A bar of chocolate is shared equally between 4 pupils. What fraction of the chocolate bar will each child get?

Three pizzas are evenly divided among four pupils. How much pizza will each child get?

Twelve biscuits were evenly shared among some friends. If each of them gets $1\frac{1}{2}$ biscuits, how many friends are there altogether?

These test papers were then analysed to see which questions were answered correctly. There was an obvious difference between the number of correct answers in the part-whole questions compared to the linear and quotient questions. This was discussed with the group and it was concluded that this was because there was an imbalance in the way that we expose pupils to the different representations.

As a group we then discussed how manipulatives can be used with pupils to help support these three visual representations of fractions. We looked at the pros and cons of using different resources including fractions cards, fraction walls, Cuisenaire rods, Dienes apparatus and fractions shapes. It was decided that many of the resources helped to support the part-whole representation of fractions and it was more difficult for teachers to help pupils use these resources to support with their understanding of linear and quotient. The best resource to do this was the use of Cuisenaire rods as they were more flexible in the values they could represent.



The afternoon of the first session was then dedicated to working through a rich mathematical task that would be used by all classes across the schools to help develop an understanding in the progression of visual representation of fractions from EYFS to Lower KS3. We picked Fair Feast as the rich task taken from the nrich website (<http://nrich.maths.org/2361>). Time was given to discuss how this could be adapted for all classes in the school but also time was given for subject leaders to think about how they will get the key messages across to staff about the use of different representations. The first session ended with clear expectations for what the participants needed to bring to the next meeting in order to explore progression further.

The second session gave the participants an opportunity to find out from the rest of the group how they had approached the dissemination of subject knowledge to the rest of the school and how the different schools had tackled the rich task. All classes within every school had used the task and each school came with evidence of how the pupils had responded to the task from EYFS to year 6. This evidence was displayed on a full wall of the classroom we were working in so we could see a clear progression of visual representations. Time was then given for reflection and analysis of when different visual representations were used and how the rich task was adapted by the pupils. The progression was also compared to the fraction objectives in the 2014 National Curriculum. At this point we found that a lot of the objectives were not covered but there was time dedicated to pupils getting a good understanding of how to represent fractions securely in a real-life context.

The rest of the second session was used to develop intervention for a group of year 5 pupils who had been identified from the pre-test as needing support with their understanding and would benefit from intervention to help them make more significant progress. The work from the progression wall was used to help support how the teachers would best represent fractions for them. The teachers planned for the use of visual representations as well as how they could secure pupils' understanding of equivalence. Participants were given time between the second

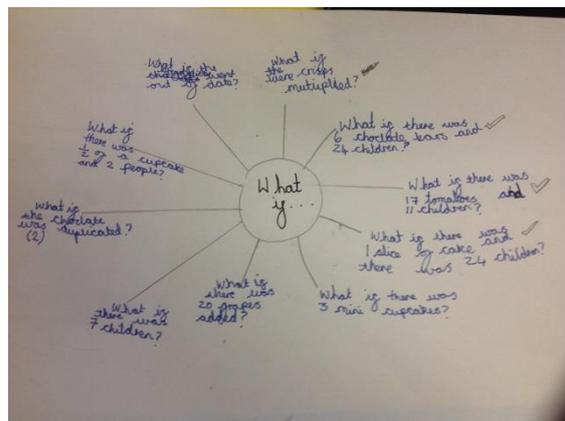
and third session to work with this group to support their understanding of fractions and complete the same test to measure the impact.

The third meeting was then used to analyse the results from the intervention and measure the impact on each school including pupils, focus pupils, staff and subject leader. A case study was developed for a pupil from each school including progress they had made and their attitudes to fractions (see appendix 2).

What has been learned from the project?

For more information about the progression of fractions using the rich task (A Fair Feast) and linked to the new National Curriculum please see appendix 1. This progression gives evidence of how and when to use different representations of fractions (e.g. when do pupils begin to understand the use of quotient relating to fractions and when to use a linear representation of fractions). Evidence of what has been learned from the project can also be found in the pupil case studies.

School A



All pupils were involved in a fractions week, which was introduced by using the basic Fair Feast activity. Reception and KS1 explored this using different visual and practical representations and a range of vocabulary relating to sharing (see progression document in appendix 1). The older pupils were invited to offer their own 'what if' questions based on the original problem. These were then explored in more depth and the pupils spent time drafting and redrafting their explanations for how to solve the problem. A fractions week display was created to show the impact of the week and give an opportunity for pupils to share their learning. Pupils were also given the opportunity to respond to questions on a whole school blog and this shows excellent evidence of pupils' confidence and willingness to have a go. ((In) fractions week I wasn't confident about fractions because some fractions are hard especially improper fractions – they're the hardest – Year 6 girl). The focus pupils became more confident to try out problems and they were happy to try out different fractions problems as they were able to relate back to their understanding of fractions through the fair feast. The pupils are now less likely to groan at the mention of fractions. There is evidence in the intervention group of pupils making links between multiplication and division facts and how this supports their fluency of fractions.

School B

All pupils were engaged in the task across this large school. There was time dedicated in a staff meeting to discuss the vocabulary that should be used and how best to approach the task with the pupils. There was an expectation that pupils work in groups and record their learning on big sheets of paper. Pupils have a better understanding of the fundamentals of fractions, what fractions are and how they can be represented. Further up the school there is more confidence with calculating with fractions. The focus pupils have an increased motivation and enjoyment when working with fractions and more resilience when working on rich tasks. All focus pupils have a better understanding of different representations of fractions and all improved their score in the fractions test.

School C

All classes were involved in the rich task. Initially the teachers were nervous about dedicating a whole week to the rich task as it would be difficult to plan for what the pupils were going to want to investigate. Time was spent in deep immersion of what to expect the pupils to achieve by the end of week according to year group expectations. Pupils have shown a change of attitude towards fractions. Pupils now view this more as a 'fun' aspect of maths rather than something to be feared. Pupils are using a range of practical resources in different ways, independently, to support their learning. Pupils are talking more about maths / fractions between each other. Pupils are able to attempt problems in different ways and are developing their language of communication to explain their ideas more clearly. Many of the pupils did not even attempt some questions in the pre-test but in the second test many of the focus pupils attempted the questions that they found tricky even if they did not get it correct. Focus pupils have shown a deeper understanding of some elements of fractions – part whole / linear fractions. Some more able pupils were able to make wider links between fractions, decimals and percentages and use this fluently. Pupils were attempting problems more widely and using visual representations to support their learning and the learning of their peers.

School D

All pupils were involved in the rich task across the school. The Fair Feast was used as a stimulus and teachers decided how they could adapt it into a week of maths learning linked with their class topic. For example, year 5 had an Egyptian feast with pizza and year 1/2 focused on a teddy bear's picnic with concrete materials. Pupils were given the opportunity to ask their own 'what if' questions. Pupils across the school have a more positive attitude towards fractions and evidence from the work gathered after the rich tasks demonstrates the different approaches the pupils used. All focus pupils improved their score in the test and the pupil questionnaire showed that they have a more positive attitude towards fractions.

School E

All pupils from across the school were involved in the rich task and there was a range of different ways in which the pupils presented their findings. The younger pupils shared the feast fairly using simple unit fractions and the older pupils explored different fractions of the feast and discussed how the food could be shared fairly. Younger pupils have a greater understanding of fractions and the vocabulary relating to fractions that will hopefully give them more confidence in the future. The focus group of pupils have an increased confidence in fractions and enjoyed the practical aspects of the task. All focus pupils had improved test scores.

School F

All pupils were involved in the rich task across the school and evidence of response to the task was seen from EYFS to year 6. Younger pupils were teacher led in halving the food in the fair feast. Older pupils found different fractions of the feast and also explored proportion and ratio using recipes for pizza. Pupils are more positive towards fractions and enjoyed having a hands-on activity related to real-life fractions. Evidence for this is the variety of work that had been produced using many different styles of presentation. The focus group had quality time to talk about fractions, their test scores improved particularly with the quotient problems. It was an opportunity to intervene with pupils who showed a lack of confidence and understanding.

Impact on teachers' practice

Teachers' reflections after the rich task activity:

'It has given me a good starting point when covering fractions over the year.' (Year 5/6 teacher)

'Ensure that all representations are taught alongside each other.' (Reception teacher)

Further evidence of impact on teachers' practice can be found in the pupil case studies (appendix 2) which outlines what was successful in helping pupils to understand fractions further.

School A

Evidence in planning shows that teachers have a greater purpose and confidence in using rich tasks in other aspects of maths. Greater understanding of progression in fractions and the use of different representations allows for planning for progression to take place. The teacher involved in the project has gained a lot from using rich tasks over a week to judge the pupils' starting points and give opportunities for the pupils to explore misconceptions further. She has improved the way in which she teaches fractions and presents fractions in a range of ways (i.e. as a number line and in the context of division).

School B

All teachers have a clearer understanding of the different representations of fractions, confidence in using a rich task to support and develop pupils' understanding and an insight into the new curriculum and progression throughout the key stages. The teacher involved in the project has an increased confidence in teaching and supporting pupils with misconceptions, an increased awareness of the different representations and methods that can be used, a better understanding of how rich tasks can be used to extend pupils understanding and uncover misconceptions.

School C

Teachers have been able to spend more time teaching fractions which has given them a deeper understanding of the most efficient teaching strategies as well as being able to tailor their teaching towards the pupils. Teachers now have a wider view of progression in fractions across the school and have used a range of resources in a variety of ways, this has also raised awareness of the new National Curriculum expectations. They are also attempting more open ended / rich tasks in maths and using this as a start for planning. Their understanding of fractions has been deepened in terms of how different areas relate to each other. The use of the rich task across the school showed how valuable it is to engage in open ended tasks and actually provided good CPD for staff in terms of using this in planning as well as showing progress. It has showed the importance and significance of fractions in all areas of maths and helped to develop the subject knowledge of the maths leader further.

School D

The reflective aspects of the meetings gave the opportunity for a teacher to reflect on how activities were approached and how it made them think about their own misconceptions before they worked with the pupils (i.e. when 3 pizzas are divided into quarters each piece is $\frac{1}{4}$ not $\frac{1}{22}$ as it needs to be clear what you are finding a fraction of). Staff have a greater understanding of how using a rich task can have a greater impact on pupils' learning and how they could be used in other aspects of maths that are 'tricky' to teach. The time dedicated to planning a fractions week shows how a whole school approach allows for teachers to work together to make these aspects of maths less 'tricky'. As one of the participants is about to take on leadership of maths in the school, it has given her an insight of how to plan for accountability and ensure that impact on progress is measured across the school.

School E

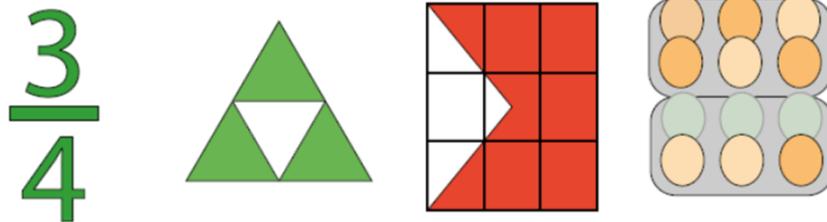
Using the rich task was a good introduction to fractions and the teachers are more aware of the National Curriculum expectations as well as knowing how this can be introduced to pupils in a real life context. The year 5/6 teacher has a greater understanding of 'gaps' in individual needs and how these misconceptions can be unpicked by going back to visual representations and practical contexts. As the maths subject leader, the participant has recognised the importance of leading progression throughout the school to ensure continuity of approach and language – *'We could use this approach for other areas of the new maths curriculum.'*

School F

Staff have a greater awareness of rich tasks and have a greater awareness of misconceptions and ways to represent fractions using a range of contexts (i.e. part-whole and quotient). Resources have been ordered by the teaching involved to be used in future fractions teaching. Greater use of visual representations have been used in planning. As a leader, the participant has had the opportunity to raise the profile of fractions with the maths subject leader and the year 5 teacher. Fractions has been identified as a key area for development across the school and how it is linked to multiplicative reasoning and ratio and proportion.

A particular activity that had an impact on pupils' understanding of fractions was a card sort activity found on the rich website (<http://rich.maths.org/8283/note>). When looking at the impact between the two tests, a lot of the participants noticed the impact that different representations of fractions as part-whole had on the pupils' understanding. If a fraction of a shape is represented in an 'unconventional' or 'unusual' way, many pupils answered the question incorrectly in the pre-test. After completing this activity and giving opportunities for pupils to reason about what they see, pupils were more confident in answering this question on the second test and had a deeper understanding of how fractions can be represented.

For example:



From the case studies (appendix 2) there are three main points that come out from the project which demonstrate how the focus pupils made progress in their understanding and fluency with fractions:

- ***Drawing their own visualisations***
In the first test, the focus pupils on the whole were chosen to receive intervention because of their lack of attempts at questions by drawing what they see. Many of the teachers involved in the project cited the use of the rich task which expected pupils to draw visualisations of what they could see in the practical context. This was then followed up during the intervention and there was a noticeable improvement in the way that the focus pupils tackled these same questions in the second test.
- ***Reasoning about fractions***
Case Study Pupil D is a good example of how opportunities for reasoning helped this pupil to develop her understanding of fractions. Due to the practical aspect of the rich task and subsequent tasks in the intervention group, this pupil was able to make sense of what she could see and communicate her thinking based around the practical context. This subsequently helped her to make sense of 'unusual' and 'unconventional' images of part-whole fractions.
- ***Focus on quotient***
As Nunes et al (2006) suggest in their research, many of the questions that pupils are asked to work out using fractions use the part-whole representation of fractions. Very few questions expect pupils to use fractions in the context of division (quotient). This was something that was found in the first test as many pupils were unable to answer questions in a division context and many were confused that the answer should be expressed as a fraction (e.g. 3 pizzas shared between 4 people). With a focus on this during the intervention, it resulted in the second test having a greater percentage of pupils answering the quotient question correctly (Case study Pupil E is a good example). This demonstrates that if pupils are exposed to this representation of fractions earlier, they are able to apply

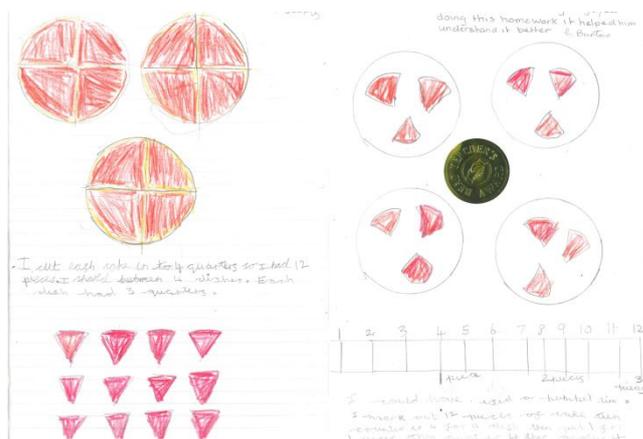
their understanding of fractions in a division context. Often this way of representing fractions is left until the pupils are older but there is no reason why pupils should not be expected to solve problems like this in a practical context.

Impact on others

As explained above, there was a noticeable difference in the pupils' ability to answer and confidence in attempting quotient problems using their understanding of fractions. All questions show an improvement in the percentage answered correctly and there is also a decrease in the percentage of questions not attempted in the second test. This therefore indicates an increased confidence as more of the questions were at least attempted by the focus pupils. There is less of an impact on the questions that asked pupils interpret fractions using a linear representation. On the whole, there was an increase in the percentage of questions answered correctly for the part-whole questions. This supports the findings from the pupil case studies in that the pupils were expected to reason about what they could see and used the practical context from the rich task to help them make sense of what they see.

One school used a blog to help the pupils to express their thinking and discuss their own ways to interpret the problem of the Fair Feast. Pupils were invited to ask their own 'what if' questions for other pupils to respond to. This had great impact on pupils' confidence and understanding (see quotes above) and also allowed parents to be involved in the project too. A parents meeting was set up during the fractions week where parents would be given a brief overview of how to represent fractions before they were invited to look around the classrooms to see how this looked in practice.

There was a great deal of positive feedback from the parents about this meeting – *'Thank you for inviting the parents in this morning – I've learnt lots of useful things to put into practice at home.'* It also had an impact on the homework that pupils did as a result of this meeting and there was evidence of the pupils using different representations practically. One example is from a pupil in year 2 who developed his own jam tart problem (3 tarts shared between 4 people) with support from his Mum.



Advice to teachers who may want to try something similar

Recommendations for teaching of fractions within the new National Curriculum

- Begin with a rich task – links to real life situation, plan for preconceptions and break down into steps.
- Visual representations of fractions need to be taught alongside each other in a blended approach (number line, part whole, quotient)

- Don't be afraid to ask pupils to work out problems using division where the answer is a fraction. If worked out practically, young pupils can explain their thinking.
- Different resources need to be used to help pupils reason about fractions – don't just rely on one resource.
- Use a whole school approach for staff to see progression to help remove the default pedagogical approach.
- Consider the vocabulary and language relating to fractions and how pupils make sense of it.
- All pupils to 'say what they see' and 'show what they say'
- Ensure pupils have a secure knowledge of times tables and number bonds to help with fluency
- Don't just tackle the list of objectives from the National Curriculum – build on prior knowledge and secure foundations
- Give it time and richness to explore fractions
- Make connections with percentages, decimals, ratio proportion.

References and resources produced or used

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