The Trouble with Quizzles
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UNIVERSITY OF CAMBRIDGE

https://nrich.maths.org/cf2022

$y_n \rightarrow$ Number of Quizzles in year n

$y_0 \rightarrow$ Starting number of Quizzles

$$y_{n+1} = k y_n$$

$$y_0 = 1000$$

$$y_{n+1} = ky_n$$

What happens to the number of Quizzles if:

- $k = 1$
- $k > 1$
- $k < 1$

$x_n \rightarrow$ The proportion of the maximum possible number of Quizzles that there are in year n

(For example, $x_3 = 0.5$ means that in year 3 the population of Quizzles is half of the maximum possible population)

$$x_{n+1} = kx_n(1 - x_n)$$

$$x_{n+1} = 2x_n(1 - x_n)$$

If $x_0 = 0.3$ what is x_1, x_2 and x_3?
What happens as the years increase?
What if you started with a different x_0?

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1. Can you find a parameter (*k* value) where the population dies out?

2. Can you find a parameter so that the population settles to a non-zero constant value (which is not 0.5)?

3. Can you find a parameter so that the population eventually oscillates between two values? Or eventually cycles between three or four values?

4. Why have we chosen 0 and 4 as limits for the *k* slider?

$$x_{n+1} = 1.5x_n(1 - x_n)$$

$$0.6 \leq x_0 \leq 0.8$$

$$x_{n+1} = 3.2x_n(1 - x_n)$$

$$0.6 \leq x_0 \leq 0.8$$

$$x_{n+1} = 3.5x_n(1 - x_n)$$

$$0.6 \leq x_0 \leq 0.8$$

$$x_{n+1} = 3.7x_n(1 - x_n)$$

$$0.6 \leq x_0 \leq 0.8$$

$$x_{n+1} = 3.7x_n(1 - x_n)$$

$$0.69 \leq x_0 \leq 0.71$$

$$x_{n+1} = 3.7x_n(1 - x_n)$$

$$0.699 \leq x_0 \leq 0.701$$

Logistic Map

$$x_{n+1} = kx_n(1 - x_n)$$

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