Impossible Sums - Proof 3

Cut out the statements and put them in order, to prove that powers of 2 cannot be written as the sum of two or more consecutive numbers.

Factorising gives $\frac{1}{2}[n(n+1)-m(m+1)]$	A
The sum of the consecutive numbers $m+1, m+2, \ldots, n$ is the same as the difference between the n^{th} and m^{th} triangular numbers	B
If n and m are both odd, or both even, then $n+m+1$ is odd. Since $n, m \geq 1$ we know that $n+m+1>1$.	C
Factorising gives $\frac{1}{2}(n-m)(n+m+1)$	D
The n^{th} triangular number is equal to $\frac{1}{2}n(n+1)$	E
This is equal to $\frac{1}{2}[n^2+n-m^2-m]$	F
Therefore the expression $\frac{1}{2}(n-m)(n+m+1)$ always has an odd factor which is greater than 1	G
We can write the difference between the n^{th} and m^{th} triangular numbers as $\frac{1}{2}n(n+1)-\frac{1}{2}m(m+1)$, where $n \geq 1$, $m \geq 1$ and $n > m+1$	H
If one of n, m is odd and the other is even then $n-m$ is odd. We know that $n > m+1$, so we have $n-m>1$.	I
Substituting $n=m$ into $\frac{1}{2}[n^2+n-m^2-m]$ gives 0, so $(n-m)$ is a factor of $\frac{1}{2}[n^2+n-m^2-m]$	J
Numbers of the form 2^n have no odd factors other than 1	K
Since any sum of consecutive numbers must have an odd factor greater than 1, it cannot be of the form 2^n	L