



## Impossible Sums - Proof of Converse

Can you prove that any number which is not a power of 2 can be written as a sum of consecutive positive numbers?

Below is a proof that has been scrambled up. Can you cut up the statements and rearrange them into their original order?

Consider the $k$ numbers before $m$ and the $k$ numbers after $m$ , as well as $m$ itself	A
If $m \geq k$ then none of the numbers are negative and so we can write $n$ as a sum of consecutive positive numbers	B
Therefore if a number is not a power of 2 it can be written as a sum of consecutive positive numbers	C
If a number is not a power of 2, then it must have an odd factor which is greater than 1	D
Assume that the number $n$ has a factor equal to $2k + 1$ , where $2k + 1 > 1$	E
These numbers in ascending order are $m - k, m - k + 1, \dots, m - 1, m, m + 1, \dots, m + k - 1, m + k$	F
If $m < k$ then there will be some negative numbers in our list, but these will cancel out with their positive equivalents, leaving $n$ as a sum of consecutive positive numbers	G
We can write $n = (2k + 1)m$ , where $m$ is a whole number	H
The sum of these $2k + 1$ numbers is equal to $(2k + 1)m$ , which is equal to $n$	I