\[ \frac{dV_1}{dr} = -k A_1 \] given in question

\[ V_1 = \frac{4}{3} \pi r_1^3 \]

\[ \frac{dV_1}{dr} = 4\pi r_1^2 \]

chain rule

\[ \frac{dA}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} = -k A_1 \times \frac{4\pi r_1^2}{4\pi r_1^2} = -k \]

\[ \frac{dr}{dt} = -k \]

Solving diff. equation

\[ r_1 = -kt + C \]

\[ 2r = C \rightarrow r_1 = 2r - kt \]

Total height

\[ r_2 = 3r - kt \]

using same logic as we used to find \( r_1 \).

\[ h = 2r_1 + 2r_2 \]

Chain rule

\[ \frac{dV}{dh} = \frac{dV}{dr} \times \frac{dr}{dh} \]

\[ \frac{dt}{dV} = \frac{1}{dV/dr} \]

Total volume

\[ \frac{dV}{dt} = -k \left( A_1 + A_2 \right) \]

\[ \text{the overall rate of change of volume} = \frac{dV}{dt} \]

\[ \frac{dv}{dt} = -k \left( 4\pi r_1^2 + 4\pi r_2^2 \right) \]

Using expressions for \( r_1 \) and \( r_2 \) derived earlier

\[ = -4\pi k \left( \left( 2r - kt \right)^2 + \left( 3r - kt \right)^2 \right) \]

\[ = -4\pi k \left( 4r^2 - 4rkt + k^2 t^2 + 9r^2 - 6rkt + k^2 t^2 \right) \]
\[-4\pi k (13R^2 - 10Rkt + 2k^2t^2)\]

\[\frac{dV}{dh} = -4\pi k (13R^2 - 10Rkt + 2k^2t^2)\]

\[\frac{dh}{dt} = -4k, \quad dt = \frac{1}{4k}\]

\[h = 2(5R - 2kt) = 10R - 4kt\]

\[\Rightarrow \frac{dh}{dt} = -4k, \quad dt = \frac{1}{4k}\]

\[\frac{dV}{dh} = \pi (13R^2 - 10Rkt + 2k^2t^2)\]

\[h^2 = 100R^2 - 80Rkt + 16k^2t^2\]

\[\frac{h^2}{8} = 12.5R^2 - 10Rkt + 2k^2t^2\]

With replacing original expression using \[\frac{h^2}{8}\].

\[\frac{dV}{dh} = \pi \left(\frac{h^2}{8} + \frac{1}{2}R^2\right)\]

\[\frac{dV}{dh} = \pi \left(h^2 + 4R^2\right)\]

at \(0 \leq h < 2R\), top snowball is gone.

\[h = 2\sqrt{2}, \quad V = \frac{4}{3}\pi R^2, \quad V = \frac{4}{3}\pi \left(\frac{h}{2}\right)^3\]

\[V = \frac{1}{6}\pi h^3; \quad dV/dh = \frac{1}{2}\pi h^2, \quad 0 \leq h \leq 2R\]
discontinuity is due to change in function at $h = 2R$ when top snowball has melted.

as $r_1 = 2R - h t$
\[ h t = 2R \text{ when } r_1 = 0 \]
\[ r_2 = 3R - 2R = R \]
\[ h = 2R \text{ when top snowball has melted.} \]