Task 2 **S-I Model - task 2**

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(a) $s+i=1$ because the entire population is divided into only two compartments: those who are susceptible and those who are infected. Together they make up 100% of the population

(b) $\frac{ds}{dt}$ is negative because the number of susceptible people in the population decreases as they become infected

$\frac{di}{dt}$ is positive because the number of infectives increases as susceptible people become infected.

(c) The total rate of change is zero because the population remains constant in this model.

(d) $\frac{di}{dt} = 0.6si$

since $s+i=1$ then

$$\frac{di}{dt} = 0.6i(1-i)$$

(e) $\frac{di}{dt} = 0.6i(1-i)$

$$\frac{1}{0.6i(1-i)} \frac{di}{dt} = 1$$

$$\int \frac{1}{0.6i(1-i)} \frac{di}{\cancel{dt}} \cancel{dt} = \int 1 \, dt$$

$$\int \frac{1}{0.6i(1-i)} \, di = \int 1 \, dt$$

$$\int \frac{5}{3i(1-i)} \, di = t$$

Partial fractions

$$\frac{5}{3i(1-i)} = \frac{A}{3i} + \frac{B}{1-i}$$

$$5 = A(1-i) + B3i$$

$i=1$, $5=3B$, $B=\frac{5}{3}$

$i=0$, $A=5$

$$\int \frac{5}{3i(1-i)}\, di = t$$

$$\int \frac{5}{3i} + \frac{5}{3(1-i)}\, di = t$$

$$\frac{5}{3}\int \frac{1}{i} + \frac{1}{1-i}\, di = t$$

$$t = \frac{5}{3}\left[\ln i + \frac{\ln|1-i|}{-1} + \ln A\right]$$

$$t = \frac{5}{3}\ln\left|\frac{Ai}{1-i}\right|$$

Initially $t = 0$, $i = 0.05$

$$0 = \frac{5}{3}\ln\left|\frac{A.0.05}{0.95}\right|$$

$$\frac{A}{19} = 1$$

$$A = 19$$

$$t = \frac{5}{3}\ln\left|\frac{19i}{1-i}\right|$$

$$\frac{3t}{5} = \ln\left|\frac{19i}{1-i}\right|$$

$$e^{\frac{3t}{5}} = \frac{19i}{1-i}$$

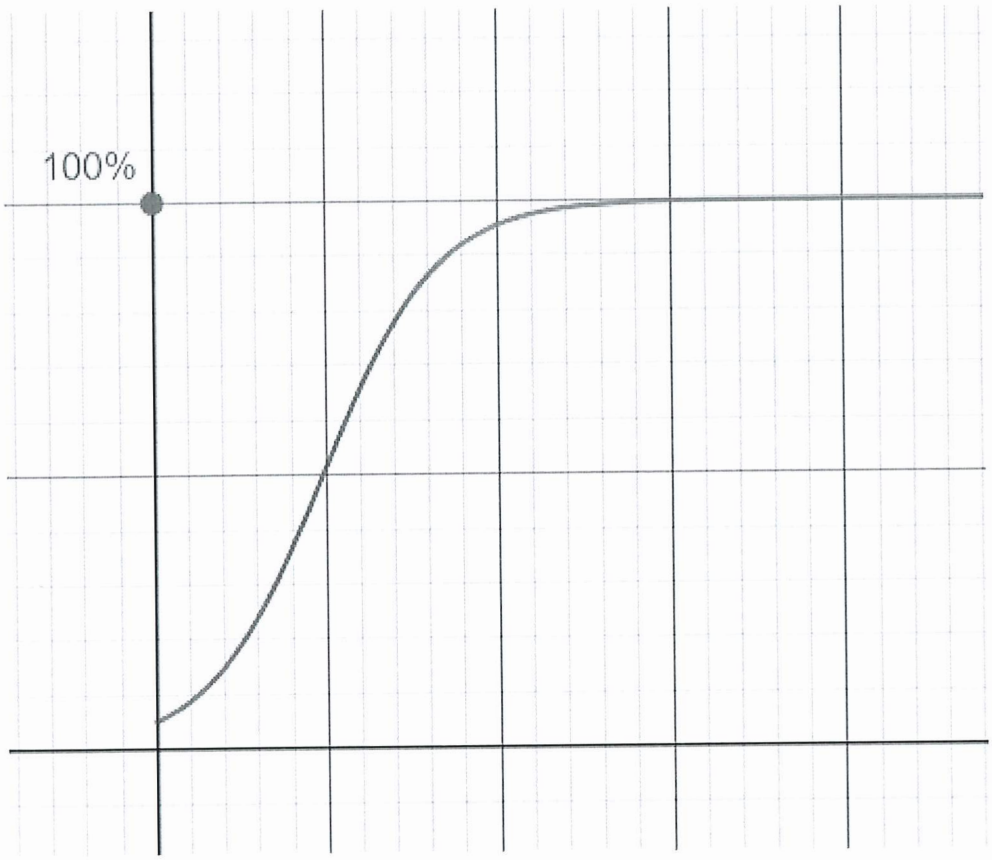
$$e^{\frac{3t}{5}} - ie^{\frac{3t}{5}} = 19i$$

$$e^{\frac{3t}{5}} = 19i + ie^{\frac{3t}{5}}$$

$$e^{\frac{3t}{5}} = i(19 + e^{\frac{3t}{5}})$$

$$i = \frac{e^{\frac{3t}{5}}}{19 + e^{\frac{3t}{5}}}$$

(f)



The graph shows that the infectives group increases from 5% of the population until 100% of the population is infected.