

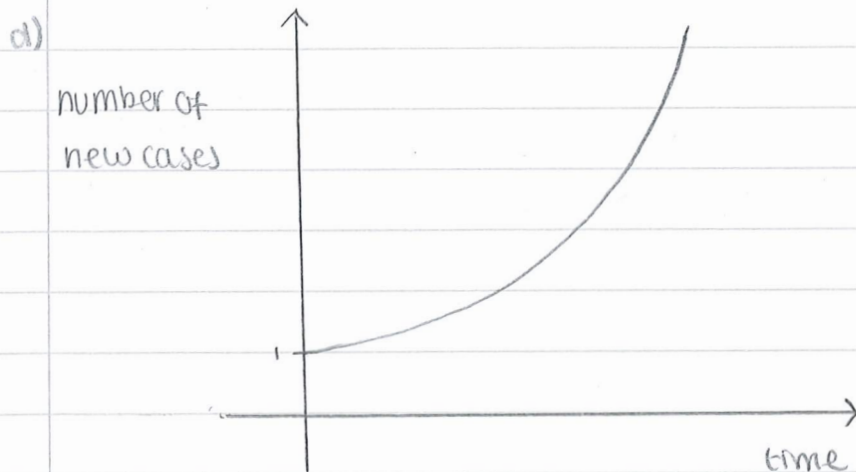
Geometric Model - task 1

- What would happen to a population if the basic reproduction rate of a new infectious disease is 3?

a) First term: $a = 1$
 common ratio: $r = 3$

b) $u_n = 1 \times 3^{n-1}$
 $u_7 = 1 \times 3^{7-1} = 729$

c) $u_n = 3^{n-1}$



e) $\frac{du}{dn} = 3^{n-1} \ln 3 > 0$ for all values of n \therefore The number of cases will continue to rise.

- f) The graph of $u_n = 3^{n-1}$ is always increasing, since the population is fixed then the number of cases cannot continue to rise beyond the population size.

a. 560 people

$$R_0 = 2.5$$

$$1, 2.5, 6.25$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad \begin{array}{l} a=1 \\ r=2.5 \end{array}$$

$$\text{people infected} = \frac{1-2.5^n}{1-2.5}$$

$$\text{After 3 days} \quad \frac{1-2.5^3}{1-2.5} = 9.75 \text{ people are infected.}$$

b.

$$560 = \frac{1-2.5^n}{1-2.5}$$

$$560(1-2.5) = 1-2.5^n$$

$$2.5^n = 1-560(1-2.5)$$

$$2.5^n = 841$$

$$n = \log_{2.5} 841$$

$$= 7.3498$$

≈ 8 days for entire population to become infected.

c.

containment measures in place from day 3 so consider GP: $a=9.75$,
 $r=0.8$

$$S_\infty = \frac{9.75}{1-0.8} = 48.75$$

so 49 people in total will be infected.

d. When $R_0 < 1$, the disease is modelled as a GP with $r < 1$. This means the series is convergent and the disease will die out by itself.

- a) The basic reproduction number for chicken pox is about three times higher than for covid-19. This means that we would expect it to spread much more rapidly through a community. However, when a person recovers from chicken pox they usually develop an immunity preventing them from being infected again. In a community with a range of ages, many people should be immune to chicken pox and so the spread would be slow. In a pre-school community, the spread would be more significant, as very few would be immune. Covid-19 has a smaller R value but could spread more quickly in community where it is a new disease.
- b) An exponential curve will continue to increase without limit. In the case of an infectious outbreak, like covid-19, there is a limit to the number of cases. The first limit is maximum population; once this maximum is reached, the number of cases cannot continue to rise. Another limit is deaths and recoveries; if a person with the disease dies or recovers, they are no longer in the number of cases. For an improved model, we need to take account of the population size and those removed and that the rate of infections will slow down.