UNIVERSITY OF CAMBRIDGE

The Language of Mathematical Problem Solving, Reasoning and Fluency

25 June 2019
Tower Hamlets CPD Centre

Fran Watson fw279@cam.ac.uk
Liz Woodham emp1001@cam.ac.uk
@nrichmaths

2018-19 Project Overview

Developing mathematical language through the three aims.

13 Nov and 13 Dec – Problem Solving
29 Jan, 26 Feb and 2 Apr – Reasoning
25 June – Fluency

nrich.maths.org/towerhamlets2018

Neighbourly Addition (14222)

As I walked down the street this morning, I noticed that all of my neighbours' house numbers were odd!

I added three house numbers together as I walked past: 7+9+11=27

Further down the road, I passed some bigger numbers. I added another set of three neighbouring house numbers: 15+17+19=51

Can you find some other totals I could make, by adding together the house numbers of three (odd) next-door-neighbours?

Is there anything special about all the totals?
Is there a quick way to work out the total?
Can you predict what would happen if I walked down the other side of the street instead (where all the houses have even numbers)?

Are there any patterns if I add together four house numbers instead of just three?
Or five house numbers?
Or...

Can you explain and justify the patterns you have noticed?

Staff meeting reflection

What were the positives?
Were there any pitfalls?

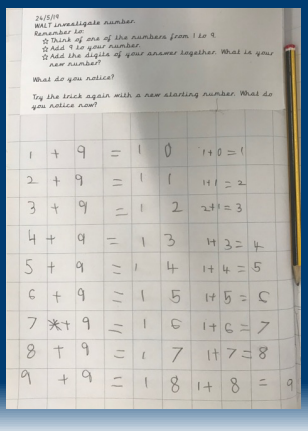
Which points from the staff meeting advice document we circulated did you find most helpful?
Please select your top three and give your reasons.

Submitted solutions

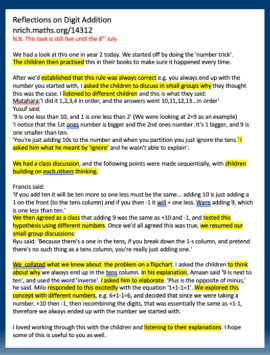
I'm Eight

Age 5 to 11 ★
Published November 1998,February 2011.

We had over 70 solutions sent in for this task!
Tayla from Marion Primary School in Australia and Luna from Marner Primary School in England, sent in what they called "all the possible additions equalling 8":
5+3 =8 3+5 =8 6+2=8 2+6=8 1+7=8 7+1=8 4+4=8



Teacher submission



Reflections on Digit Addition
nrich.maths.org/14312
N.B. This task is still live until the 8th July

We had a look at this one in year 2 today. We started off by doing the 'number trick'. The children then practised this in their books to make sure it happened every time.

After we'd established that this rule was always correct e.g. you always end up with the number you started with, I asked the children to discuss in small groups why they thought this was the case. I listened to different children and this is what they said:
Mutahara:'I did it 1,2,3,4 in order, and the answers went 10,11,12,13...in order'
Yusuf said:
'9 is one less than 10, and 1 is one less than 2' (We were looking at 2+9 as an example)
'I notice that the 1st ones number is bigger and the 2nd ones number. It's 1 bigger, and 9 is one smaller than ten.
'You're just adding 10s to the number and when you partition you just ignore the tens.' I asked him what he meant by 'ignore' and he wasn't able to explain'.

We had a class discussion, and the following points were made sequentially, with children building on each others thinking.

Francis said:
'If you add ten it will be ten more so one less must be the same... adding 10 is just adding a 1 on the front (to the tens column) and if you then -1 it will = one less. Were adding 9, which is one less than ten.'
We then agreed as a class that adding 9 was the same as +10 and -1, and tested this hypothesis using different numbers. Once we'd all agreed this was true, we resumed our small group discussions.
Ryu said: 'Because there's a one in the tens, if you break down the 1+ column, and pretend there's no such thing as a tens column, you're really just adding one.'

We collated what we knew about the problem on a flipchart. I asked the children to think about why we always end up in the tens column. In his explanation, Amaan said '9 is next to ten', and used the word 'inverse'. I asked him to elaborate. 'Plus is the opposite of minus,' he said. Milo responded to this excitedly with the equation '1+1-1='. We explored this concept with different numbers, e.g. 6+1-1=6, and decided that since we were taking a number, +10 then -1, then recombining the digits, that was essentially the same as +1-1, therefore we always ended up with the number we started with.

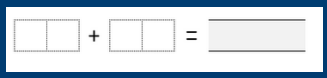
I loved working through this with the children and listening to their explanations. I hope some of this is useful to you as well.

What do you understand by the term 'mathematical fluency'?

Dicey Addition (11863)

Find a partner and a 0-9 dice.

Each of you draw an addition grid like this:



Take turns to throw the dice.

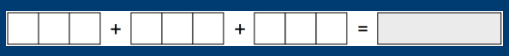
After each throw of the dice, you each decide which of your cells to put that number in.

Throw the dice four times xuntil all the cells are full.

Whoever has the sum closer to 100 wins.

Dicey Operations in Line (13261)

Each of you draw an addition layout like this:



Throw the dice nine times each until all the cells are full.

Whoever has the sum closest to 1000 wins.

Being fluent

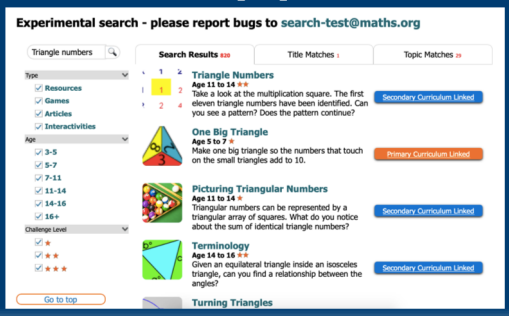
At NRICH, we believe that 'being fluent' means being fluent with:

- Facts
- Calculation strategies
- Concepts
- Representations
- Using mathematical content in new contexts
- Making connections across mathematical content
- Problem-solving strategies
- If I know this, then I know that…
- Explaining and reasoning

Key aspects of fluency

Accuracy
Efficiency
Flexibility
Understanding
Reasonableness

Exclusive – new NRICH search facility (beta-search)



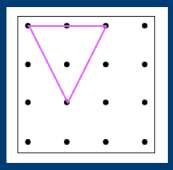
Tasks to talk about

School Fair Necklaces (**9692**)
Strike It Out (**6589**)
Three Neighbours (**8108**)
Always, Sometimes or Never? (**14023**)
Dart Target (**12261**)

Inside Triangles (5648)

I have joined three dots to make a triangle which has one dot inside it.

How many different triangles with one dot in the middle can you draw?



How do you know you've found them all?

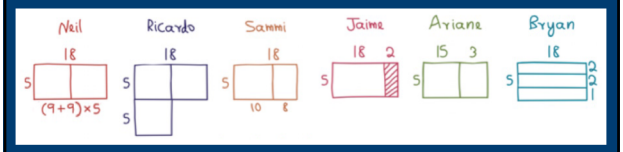
121 – 78
121 -59
121 – 4
121 - 20

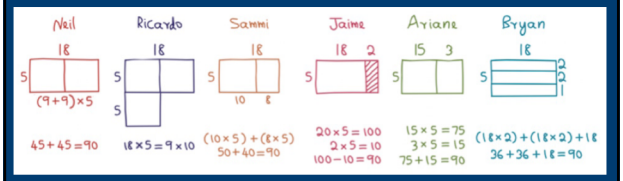
How would *you* do it?

5 x 18 =

From youcubed.org

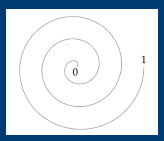
Can you spot yours?





Who's who?

Spiralling Decimals (10326)

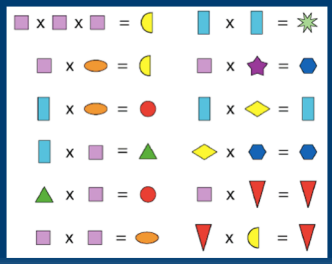


0.5	0.25	0.75	0.3
0.35	0.9	0.99	0.999
0.1	0.01	0.05	0.79
0.64	0.32	0.54	0.865

Shape Times Shape (5714)

The coloured shapes stand for eleven of the numbers from 0 to 12. Each shape is a different number.

Can you work out what they are from these multiplications?



1) Students struggle to get started. Think about ways to facilitate this process, as it's the most important step. Many of the students I was working with wanted to give up immediately because they didn't know where to begin, but once they got started they were off to the races.

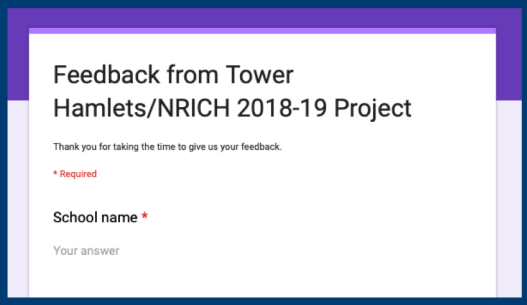
2) In trying to start with the first clue, they often confused repeated addition and repeated multiplication and thus went down a rabbit-hole of incorrect thinking. This was a great opportunity to discuss the difference.

3) The students needed a concrete way of to keep track of which numbers they had already used. Older students may not need this, but it is also a constant reminder of the constraints of the puzzle. In my next iteration, I will add a number tracker to the bottom of the sheet.

4) Don't forget the cultural connection. After they solved the problem, I discussed with them where the symbols came from and why they are important. The next step is a full #STEAM project on Adinkra symbols for them to complete. This was just the warm-up 😜

~~Yes, but...~~

Yes, and...



nrich.maths.org/towerhamlets2018

Further support

- NRICH fluency feature (10821)
- NRICH games (14095)
- NRICH interactives (13881)
- YouCubed (https://www.youcubed.org)
- More to Maths than Counting (https://www.towerhamlets.gov.uk/Documents/Children-and-families-services/Early-Years/More_to_Maths_the_counting_booklet.pdf)

References

Boaler, J. (2015) Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts (see https://www.youcubed.org/evidence/fluency-without-fear/)

Kilpatrick J. , Swafford J. and Findell B. eds (2011) Adding It Up: Helping Children Learn Mathematics. Washington DC: National Academy Press (see https://www.nap.edu/catalog/9822/adding-it-up-helping-children-learn-mathematics).

National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. Reston, VA: NCTM.

Russell, Susan Jo. (May, 2000). *Developing Computational Fluency with Whole Numbers in the Elementary Grades.* In Ferrucci, Beverly J. and Heid, M. Kathleen (eds). Millenium Focus Issue: Perspectives on Principles and Standards. The New England Math Journal. Volume XXXII, Number 2. Keene, NH: Association of Teachers of Mathematics in New England. Pages 40-54.