UNIVERSITY OF CAMBRIDGE

The Language of Mathematical Problem Solving, Reasoning and Fluency

2 April 2019
Tower Hamlets CPD Centre

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2018-19 Project Overview

Developing mathematical language through the three aims.

13 Nov and 13 Dec – Problem Solving
29 Jan, 26 Feb and 2 Apr – Reasoning
25 June – Fluency

nrich.maths.org/towerhamlets2018

Reflecting on staff meeting

What observations do you have in relation to the meeting:

- Beforehand
- During
- Afterwards

Tasks to talk about

Amy's Dominoes (**1044**)
The Remainders Game (**6402**)
Hundred Square (**2397**)
Cumbria 100 square ideas
100 square 'people maths'
Baravelle (**6522**)

Progression in reasoning

Describing
Explaining
Convincing
Justifying
Proving

School Fair Necklaces (9692)

Rob and Jennie were making necklaces to sell at the school fair.

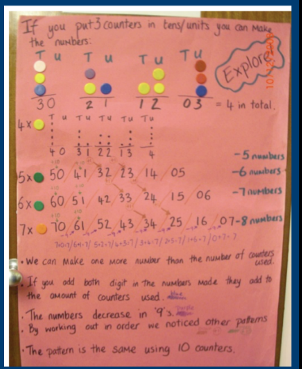
They decided to make them very mathematical.

Each necklace was to have eight beads, four of one colour and four of another.

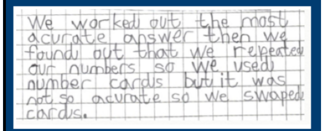
And each had to be symmetrical, like this:

How many different necklaces could they make?

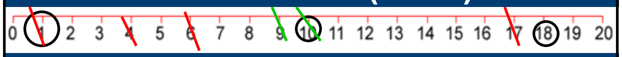
6 Beads →



Two-digit Targets ↓



Strike it Out (6589)



6 + 4 = 10

10 take away 9 makes 1

1 add 17 is 18

18……

Competitive aim – stop your partner from going

Collaborative aim – cross off as many as possible

Exploring Proof

Is it possible to create a string of number sentences that uses all the numbers on the:

- 0-20 number line?
- 1-20 number line?
- Any number line with a set of consecutive whole numbers?

0-20 number line

If zero is included in the number sentence then all three numbers are not distinct.

This is true whether you use addition or subtraction in your number sentence.

This does not obey the rules of the game.

Therefore, on 0-20 number line we cannot ever use up all the numbers.

1-20 number line

The first number sentence uses up three distinct nos.

The second number sentence uses up 2 new nos.

The third number sentence uses up another 2 new nos.

The total number of numbers used so far is 3+2+2 = 7.

Each subsequent number sentence will use 2 more new nos.

Therefore, the total number of numbers used will go up in twos from seven.

The 1-20 number line has 20 numbers on it.

Going up in 2s from 7 we will never get to 20 exactly – only 19 or 21.

Therefore, it is impossible to use exactly all the numbers on the 1-20 number line.

Any number line

The first number sentence uses an odd number of distinct nos.

Each subsequent number sentence uses an even number of distinct new nos.

Odd + any number of even numbers is always odd.

Therefore, the total number of numbers used is always odd.

Therefore, we will never be able to use all the numbers on any number line that has an even number of consecutive distinct numbers on it.

However, this does not prove that we can always use all the numbers on a number line with an odd number of consecutive distinct numbers.

We only know that we have the correct number of numbers to make it a possibility but not a certainty.

This does not give us any insight into the individual number sentences and the order in which the numbers need to be used.

Jot down a number.

Jot down the next two consecutive numbers.

Add your numbers together.

Jot down another number.

Jot down the next two consecutive numbers.

Add your numbers together.

Jot down a number…

Three Neighbours (8108)

Take three numbers that are 'next door neighbours' when you count. These are called consecutive numbers.

Add them together.

What do you notice?

Take another three consecutive numbers and add them together.

What do you notice?

Can you prove that this is always true by looking carefully at one of your examples?

Experience suggests that there is a significant difference between being asked for three examples all at once, and being asked for three one after the other with pauses for the construction at each stage. By the third request, many learners are 'feeling bored' and so challenge themselves by constructing more complex examples. The point of the task is to get learners to become aware of the range of choices open to them and, more specifically, the general class of possibilities from which they can choose, so they do not jump at the first thing that comes to mind.

Working on your own mathematics course

John Mason for The Open University

Methods for proving

- *Proof by exhaustion* - this is about working systematically to find all possibilities

  (School Fair Necklaces)
- *Proof by logical reasoning* - a complete chain of reasoning, with no room for ambiguity and no missing steps

  (Strike It Out)
- *Generic proof* - the structure that underpins a particular example is unpacked to illustrate a generality which will always be true

  (Three Neighbours)

Always, sometimes or never true?

"Teachers love teaching."

Always, Sometimes or Never? (14023)

When you add two numbers you can change the order and the answer will be the same	If you add 10 and take away 1 it is the same as adding 9
When you add 10 to a number the answer is a multiple of 10	When you subtract one number from another number you can change the order and the answer will be the same.

When you cut off a piece from a 2D shape you reduce the area and perimeter	Triangles tessellate
The number of lines of symmetry in a regular shape is equal to the number of sides	Quadrilaterals can be cut into two equal triangles

Wandering to wonder

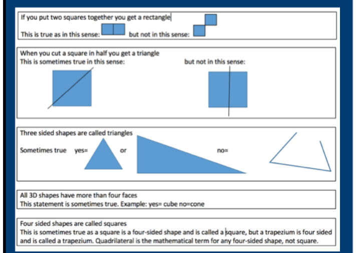
What did the the annotations prompt you to wonder/notice/question?

Reasoning walks

What did you observe?

What will you do next?

Using NRICH solutions



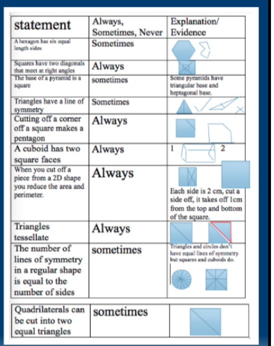
If you put two squares together you get a rectangle
This is true as in this sense: but not in this sense:

When you cut a square in half you get a triangle
This is sometimes true in this sense: but not in this sense:

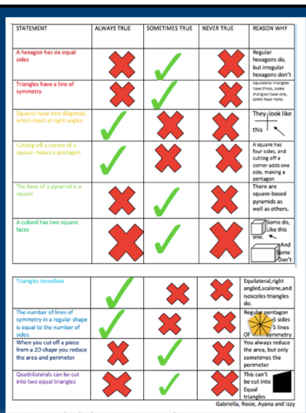
Three sided shapes are called triangles
Sometimes true yes= or no=

All 3D shapes have more than four faces
This statement is sometimes true. Example: yes= cube no=cone

Four sided shapes are called squares
This is sometimes true as a square is a four-sided shape and is called a square, but a trapezium is four sided and is called a trapezium. Quadrilateral is the mathematical term for any four-sided shape, not square.



statement	Always, Sometimes, Never	Explanation/ Evidence
A hexagon has six equal length sides	Sometimes	
Squares have two diagonals that meet at right angles	Always	
The base of a pyramid is a square	sometimes	Some pyramids have triangular base and heptagonal base.
Triangles have a line of symmetry	Sometimes	
Cutting off a corner off a square makes a pentagon	Always	
A cuboid has two square faces	Always	1 2
When you cut off a piece from a 2D shape you reduce the area and perimeter.	Always	Each side is 2 cm, cut a side off, it takes off 1cm from the top and bottom of the square.
Triangles tessellate	Always	
The number of lines of symmetry in a regular shape is equal to the number of sides	sometimes	Triangles and circles don't have equal lines of symmetry but squares and cuboids do.
Quadrilaterals can be cut into two equal triangles	sometimes	

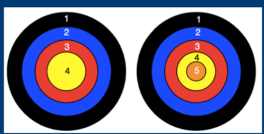


STATEMENT	ALWAYS TRUE	SOMETIMES TRUE	NEVER TRUE	REASON WHY
A hexagon has six equal sides	✗	✓	✗	Regular hexagons do, but irregular hexagons don't
Triangles have a line of symmetry	✗	✓	✗	Equilateral triangles have three, some triangles have one, some have none.
Squares have two diagonals which meet at right angles	✓	✗	✗	They look like this
Cutting off a corner of a square makes a pentagon	✓	✗	✗	A square has four sides, and cutting off a corner adds one side, making a pentagon
The base of a pyramid is a square	✗	✓	✗	There are square-based pyramids as well as others.
A cuboid has two square faces	✗	✓	✗	Some do, Like this one. And Some Don't
Triangles tessellate	✓	✗	✗	Equilateral,right angled,scalene,and isosceles triangles do.
The number of lines of symmetry in a regular shape is equal to the number of sides.	✓	✗	✗	Regular pentagon 5 sides 5 lines Of symmetry
When you cut off a piece from a 2D shape you reduce the area and perimeter	✗	✓	✗	You always reduce the area, but only sometimes the perimeter
Quadrilaterals can be cut into two equal triangles	✗	✓	✗	This can't be cut into Equal triangles

Gabriella, Rosie, Ayana and Izzy

Dart Target (12261)

We have two different targets and a set of three darts.



You use three darts for each game. All the darts always hit the target.

An Important Rule:-
You can only have one dart in the same coloured area on any one target, but you could, for example, have a dart in red 3 on the left target and another dart in red 3 on the right target.

CHALLENGE 1

Each time you have a game, you add up the score of the three darts. Your challenge is to have four games each with the same total score, but made in different ways.

However, note that 1 and 2 on the left target and 3 on the right target is the same as 1 on the left target and 2 and 3 on the right target, so is counted as just one solution.

CHALLENGE 2

Have three games. The three totals must give three consecutive numbers.
In the nine darts used no number must occur more than twice.
So, when Raj chooses:

Game 1: 2 + 3 + 4 = 9
Game 2: 1 + 4 + 5 = 10
Game 3: 1 + 5 + 5 = 11

This is NOT allowed as three 5s have been used and the maximum is two.

CHALLENGE 3

You now move on to having three targets with four darts for each game. As before you can only have one dart in any coloured sector of a target.

The targets are:
1, 2, 3
1, 2, 3, 4
1, 2, 3, 4, 5.

Firstly see how many different totals you can make.

Finally find all the ways of getting the set of three consecutive 11, 12, 13

But in a set of three answers you must not use the same number more than three times.

So, when Sara chooses:

Game 1 1 + 2 + 3 + 5 = 11
Game 2 1 + 2 + 4 + 5 = 12
Game 3 2 + 2 + 4 + 5 = 13

This is not allowed as four 2s have been used and the maximum is three.

Find as many answers as you can.

National Young Mathematicians' Award (11587)

- NRICH has worked with Explore Learning since 2010 to write the questions for the NYMA
- Teams of four children take part in two rounds of local events, then ten teams (five primary and five secondary) come to Cambridge for the final
- See the feature 'Using NYMA Tasks to Develop Problem-solving and Group-working Skills' (11538)

Maths Club Activities (14061)

NRICH webinar (14206)

Date: Tuesday 11th June 2019
Times: 10am and 5pm (repeated)
Place: ANYWHERE! (with internet access)
Who: Students aged 9 - 13
What: Collaborative mathematics led by members of the NRICH team

Live tasks (8802)

Tables Teaser (14242)
Starfish Spotting (182)
I'm Eight (55)
Looking at Lego (14220)
Multiplication Square Jigsaw (5573)
Shape Times Shape (5714)
Mystery Matrix (1070)
Missing Multipliers (7382)

Plan for dissemination

After	**Trialling NRICH tasks with:**
Day 1	Your own class
Day 2	Your own class + 1 other
Day 3 (and before Day 5)	Colleagues (staff meeting input + feedback/reflections)
Day 5	Participate in NRICH Webinar 11th June &/or submit solutions to a live task

Further NRICH support

Past features:

- Working Systematically (9752)
- Mastering Mathematics: The Challenge of Generalising and Proof (11458)

Teacher takeaway

- Try a task from today in your setting, invite a colleague to do so too and then talk about the outcomes
- Join in the NRICH webinar on June 11 and/or submit solutions to one or more live tasks
- Read "Mastering Mathematics: The Challenge of Generalising and Proof" article (11488)

Please come with artefacts and ready to discuss all of the above on Day 6

References

Gardiner, T. (2000). *Maths Challenge Book 1*. Oxford: Oxford University Press.
(Challenge Books 2 and 3 are also available)

Vygotsky, L.S. (1978). *Mind in society: The psychology of higher mental functions.* Cambridge, MA: Harvard University Press

Working on your own mathematics course
John Mason for The Open University

Open University website
Psychology of Mathematics Education 26
Anne Watson and John Mason