STEP Support Programme

Statistics Questions

2010 S1 Q12

1 A discrete random variable X takes only positive integer values. Define $\mathrm{E}(X)$ for this case, and show that

$$\mathrm{E}(X) = \sum_{n=1}^{\infty} \mathrm{P}\left(X \geqslant n\right).$$

I am collecting toy penguins from cereal boxes. Each box contains either one daddy penguin or one mummy penguin. The probability that a given box contains a daddy penguin is p and the probability that a given box contains a mummy penguin is q, where $p \neq 0$, $q \neq 0$ and $p + q = 1$.

Let X be the number of boxes that I need to open to get at least one of each kind of penguin. Show that $\mathrm{P}(X \geqslant 4) = p^3 + q^3$, and that

$$\mathrm{E}(X) = \frac{1}{pq} - 1.$$

Hence show that $\mathrm{E}(X) \geqslant 3$.

2013 S2 Q12

2 The random variable U has a Poisson distribution with parameter λ. The random variables X and Y are defined as follows.

$$X = \begin{cases} U & \text{if } U \text{ is } 1,\ 3,\ 5,\ 7,\ \ldots \\ 0 & \text{otherwise} \end{cases}$$

$$Y = \begin{cases} U & \text{if } U \text{ is } 2,\ 4,\ 6,\ 8,\ \ldots \\ 0 & \text{otherwise} \end{cases}$$

(i) Find $\mathrm{E}(X)$ and $\mathrm{E}(Y)$ in terms of λ, α and β, where

$$\alpha = 1 + \frac{\lambda^2}{2!} + \frac{\lambda^4}{4!} + \cdots \quad \text{and} \quad \beta = \frac{\lambda}{1!} + \frac{\lambda^3}{3!} + \frac{\lambda^5}{5!} + \cdots.$$

(ii) Show that

$$\mathrm{Var}\,(X) = \frac{\lambda\alpha + \lambda^2\beta}{\alpha + \beta} - \frac{\lambda^2\alpha^2}{(\alpha+\beta)^2}$$

and obtain the corresponding expression for $\mathrm{Var}\,(Y)$. Are there any non-zero values of λ for which $\mathrm{Var}\,(X) + \mathrm{Var}\,(Y) = \mathrm{Var}\,(X+Y)$?