STEP Support Programme

STEP II Pure Questions: Hints

1 **(i)** Using the given substitution and careful manipulation should lead you to a differential equation where the variables are separable. Don't forget to substitute back for y again at the end.

(ii) Try to use a similar idea as for part **(i)**. Look at how this equation is different to help you decide which substitution to use.

(iii) This part is asking you to look at your previous two results and the generalise. Simplifying your answers to the previous two parts may help you to spot the general pattern.

2 **(i)** When you differentiate you get a quadratic, and you are told that the roots of this are $x = p$ and $x = q$. This means that the derivative can be written as $k(x - p)(x - q)$.

(ii) Use the facts $p > 0$ and $n > 0$ to approximately place the turning points.

(iii) m is the y coordinate when $x = p$, so $m = 2p^3 - bp^2 + cp$. You will need to use factorisation and your results from part **(i)**.

(iv) Make sure you have drawn $x = p$ and $y = n$ on your sketch. A couple more lines might be useful.

3 **(i)** Find the coordinates of the turning points. Then show whether they are above or below the x-axis. Remember that something squared is always greater than or equal to zero.

(ii) You can use the quadratic formula to find the two possible values of u^3. These both give the same value of x (which is to be expected as there is only one real root of x).

(iii) Use $t^2 - pt + q \equiv (t-\alpha)(t-\beta)$ and it will be helpful to consider $(\alpha+\beta)^3$. The condition on the roots means that either $\alpha^2 = \beta$ or $\beta^2 = \alpha$. There is a connection back to the first two parts of the question.

4 Remember that $\arctan x = \tan^{-1} x$. This is an integration by substitution question, and you will need to use a relationship between $\tan\theta$ and $\sec\theta$.

(i) **(a)** Try to find a substitution which will convert I into something of the same form as the "stem" result. You will find that $a = 1$.

(b) Quite a lot of manipulation of trigonometric functions is needed here, but you do know what you are aiming for. Completing the square might be a useful technique to consider. Remember that $\sin 2A = 2\sin A\cos A$.

(ii) This is very similar to part **(i)(b)**. Start with the same substitution as in the previous part and then use a second one to write the integral in the same form as in the stem.