Embedding Problem Solving Day 4 – Thursday 2 March

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Aims of the Programme

- To explore ways of integrating problem solving into the primary mathematics curriculum.

- To support teachers in nurturing confident, resourceful and enthusiastic learners of mathematics in their schools.

Overview of the Six Days

Autumn term:	Problem solving
Spring term:	Reasoning
Summer term:	Fluency

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Day 4 of 6

- Progression in reasoning, in particular focus on proof
- Communicating reasoning
- Chance to share classroom experiences and any dissemination to colleagues
- Opportunity to reflect on Chapter 9 of Mathematical Mindsets

Where are you on the reasoning journey?

Reminder of tasks we explored on day 3:

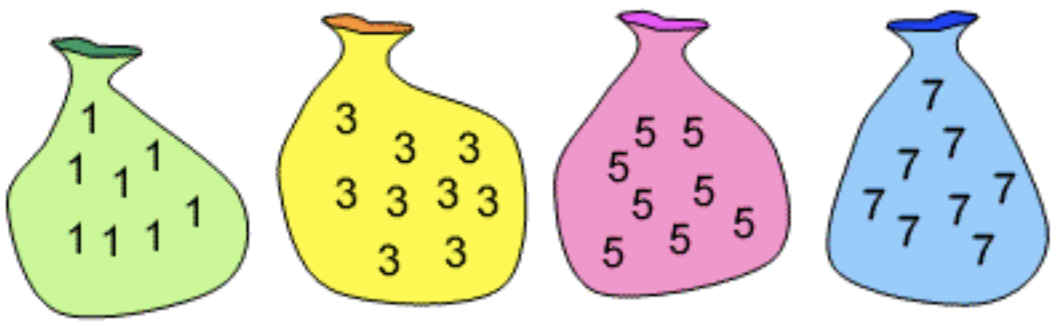
Dicey Addition, Dicey Operations, Three Neighbours, Strike It Out, Sizing Them Up, Forgot the Numbers, Heads and Feet, Ken Kens

Progression in reasoning:

- Describing
- Explaining
- Convincing
- Justifying
- Proving

Make 37 (1885)

Four bags contain a large number of 1s, 3s, 5s and 7s.



Pick any ten numbers from the bags above so that their total is 37.

Proof by Contradiction: Make 37

- Our conjecture is that is is **impossible** to make a total of 37 by adding ten numbers from the bags of 1s, 3s, 5s and 7s.
- So, using proof by contradiction, let's assume it **is** possible.
- The numbers 1, 3, 5 and 7 are all odd numbers, so we can only choose odd numbers.
- Two odd numbers added together always make an even number.
- Adding ten odd numbers together is the same as adding five pairs of odd numbers together.
- Adding five pairs of odd numbers together is the same as adding five even numbers together.
- Five even numbers added together will always make an even total.
- This creates a contradiction: 37, the total we are aiming for, is odd not even.
- Therefore, 37 cannot be made by adding ten numbers from the bags of 1s, 3s, 5s and 7s. We have proven our conjecture.

Article: An Introduction to Proof by Contradiction (4717)

- Proof is key to mathematics; proof by contradiction comes up again and again.
- To prove something by contradiction, we assume that what we want to prove is not true, and then show that the consequences of this are impossible.
- Parking ticket example.
- Pure mathematics example $\sqrt{2}$ is irrational.

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Methods for Proving

- Proof by counter-example
- Proof by exhaustion
- Proof by contradiction
- Proof by logical reasoning
- Generic proof

Communicating Proof

- Visually
- Algebraically
- Through a series of statements (written or spoken)

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Carroll Diagram

	Proof by exhaustion	Proof by counter-example	Proof by logical reasoning	Proof by contradiction	Generic proof
Algebraic communication					
Visual communication					
Communication by logical statements					

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List of tasks

Dicey Addition

Dicey Operations

Three Neighbours

Strike It Out

Sizing Them Up

Forgot the Numbers

Heads and Feet

Ken Kens

Make 37

Carroll Diagram cont.

	Proof by exhaustion	Proof by counter-example	Proof by logical reasoning	Proof by contradiction	Generic proof
Algebraic communication					
Visual communication					
Communication by logical statements	Dicey Addition Dicey Operations		Dicey Addition Dicey Operations		

Always, Sometimes or Never?

Year 5-6: Number

When you multiply two numbers you will always get a bigger number	If you add a number to 5 your answer will be bigger than 5
A square number has an even number of factors	The sum ... numb...
Dividing a whole number by a half makes it twice as big	

Year 1-2: Number

When you add two numbers you can change the order and the answer will be the same	If you add 10 and take away 1 it is the same as adding 9
When you add 10 to a number the answer is a multiple of 10	When you subtract one number from another number you can change the order and the answer will be the same.

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Always, Sometimes or Never? KS1 (12671)

"Four-sided shapes are called squares"

Always, Sometimes or Never? Number (12672)

"Multiples of 5 end in a 5"

Carroll Diagram

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Visual communication					
Communication by logical statements					

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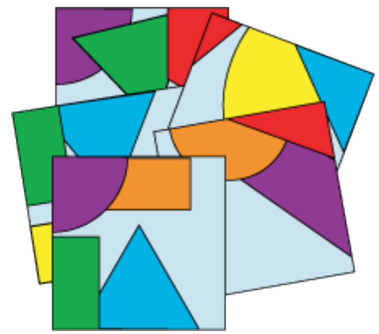
Jig Shapes (6886)



Share all the cards out amongst the group.

Can you each work out what shape or shapes you have part of on your card?

Can you describe the shapes without showing it to anyone else?

What will the rest of the shape or shapes look like do you think?

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Strategies That May Help me to Communicate my Reasoning

- Modelling
- Group work
- Understanding how others work
- Personal notes and recording

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Confidence of staff and children (including subject knowledge)	Questioning	Enjoyment/Engagement
Vocabulary	"Greater depth" /Differentiation/ Challenge	Collaboration
Resources - linked to curriculum/SoW	Independence/Resilience /Learning from mistakes	Assessment/Evidence /Recording

Dissemination

On your table, discuss ways in which you shared ideas from this project with colleagues. In particular:

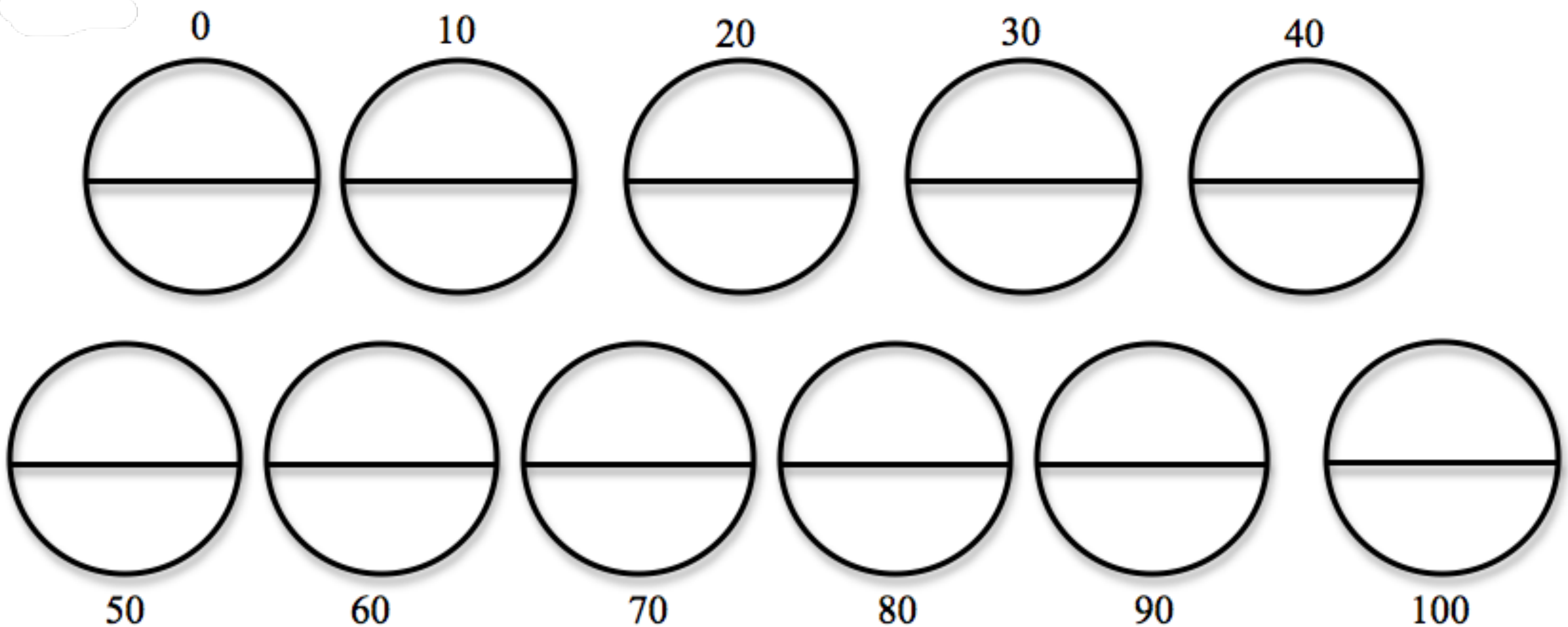
- Did you use the ‘Celebrating Solutions’ Feature? If so, how?
- What went well?
- Were there any surprises?
- What might you do differently next time?

Mathematical Mindsets - Chapter 9

Encourage students to Be Mathematicians

This video shows a third-grade American class and their teacher Deborah Ball. She encourages her students to be inquirers and to make conjectures about mathematics.

Reasoned Rounding (10945)



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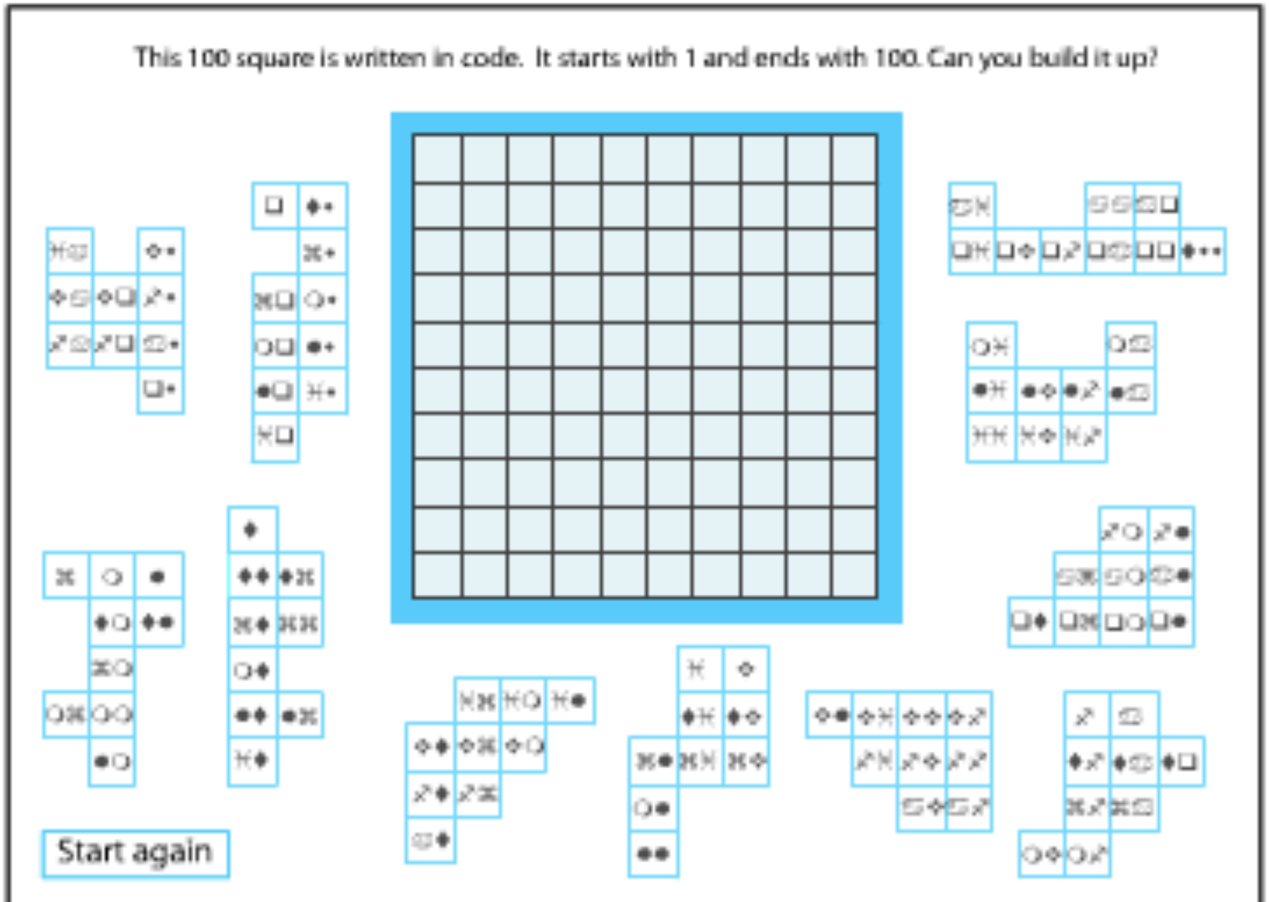
Coded Hundred Square (6554)



- I think this because…
- If this is true then……
- I know that the next one is …..because…
- This can't work because ….
- When I tried xxx I noticed that …..
- The pattern looks like……
- All the numbers begin with…..
- Because xxxx then I think xxxxx
- It will never work because…..

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Teacher Takeaway

- Read chapter 4 from Mathematical Mindsets
- Try a task from today with your class
- Ask a colleague to do likewise and then discuss the outcomes
- Share the nrich.maths.org/towerhamlets page with someone who hasn't seen it before and have a conversation about it

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Mathematical Mindsets

Chapter 9 – Teaching Mathematics for a Growth Mindset

It begins by summarising previous chapters, so that the reader has a guide to setting up a growth mindset class:

- Encourage all students
- Believe in all your students
- Value struggle and failure
- Give growth praise and help

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Teaching Mathematics for a Growth Mindset cont.

- Encourage students to be mathematicians
- Encourage intuition and freedom of thought
- Teach Mathematics as:
  - An open subject
  - A subject of patterns and connections
  - A visual and creative subject
- Value depth over speed
- Encourage students to pose questions, reason, justify and be skeptical
- Teach with technology, manipulatives and models

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School Fair Necklaces (9692)

Rob and Jennie were making necklaces to sell at the school fair. They decided to make them very mathematical.
Each necklace was to have eight beads, four of one colour and four of another.
And each had to be symmetrical, like this:

How many different necklaces could they make?

Carroll Diagram

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