Please use the post-it notes on your table to jot down any reflections/questions resulting from days 1 and 2 (which focused on problem solving)

Embedding Problem Solving Day 3 – Thursday 19 January

Fran Watson fw279@cam.ac.uk
Liz Woodham emp1001@cam.ac.uk
NRICH Primary Team

Aims of the Programme

- To explore ways of integrating problem solving into the primary mathematics curriculum.
- To support teachers in nurturing confident, resourceful and enthusiastic learners of mathematics in their schools.

Overview of the Six Days

Autumn term: Problem solving
Spring term: Reasoning
Summer term: Fluency

Day 3 of 6

- Progression in reasoning
- Chance to share classroom experiences and any dissemination to colleagues
- Opportunity to reflect on chapter 5 of Mathematical Mindsets
- Exploration of 'Celebrating Solutions' Feature

Sessions 1 and 2

Exploring Progression in Reasoning

Dicey Addition (11863)

Find a partner and a 0-9 dice.
Each of you draw an addition grid like this:

☐☐ + ☐☐ = ______

Take turns to throw the dice. After each throw of the dice, you each decide which of your cells to put that number in. Throw the dice four times each until all the cells are full.
Whoever has the sum closer to 100 wins.

What can you articulate about your reasoning now that you have had more than one go at Dicey Addition?

Dicey Operations (6606)

Find a partner and a 0-9 dice.
Each of you should draw an addition grid like this:

	☐	☐	☐
	☐	☐	☐
+	☐	☐	☐
	___	___	___

Take turns to throw the dice and decide which of your cells to fill. You must fill a cell before throwing the dice again.
Each time the dice is thrown, you *both* use that number in one of your cells.
When you have filled all nine cells, ***whoever has the sum closer to 1000 wins.***

How would you structure these games to help children develop their reasoning skills?

Progression in Reasoning

- Describing
- Explaining
- Convincing
- Justifying
- Proving

Delving into Proof with Dicey Operations

This time, throw the dice nine times before placing any of the numbers in the cells.
How would you place the numbers so that the total is as **close as possible** to 1000?
Could you convince another pair that your way did indeed produce the closest possible sum to 1000?
Could you prove it?

Three Neighbours (8108)

Take three numbers that are 'next door neighbours' when you count. These are called consecutive numbers.

Add them together.

What do you notice?

Take another three consecutive numbers and add them together.

What do you notice?

Can you prove that this is always true by looking carefully at one of your examples?

nrich.maths.org

UNIVERSITY OF CAMBRIDGE

Communicating Proof

- Visually
- Algebraically
- Through a series of statements (written or spoken)

nrich.maths.org

UNIVERSITY OF CAMBRIDGE

Methods for Proving

- Proof by counter-example
- Proof by exhaustion
- Proof by contradiction
- Proof by logical reasoning
- Generic proof

nrich.maths.org

UNIVERSITY OF CAMBRIDGE

Mathematical Mindsets: Chapter 5

Five Cs of mathematical excitement:

Curiosity, connection making, challenge, creativity, collaboration

Ways to increase the power of a mathematical task:

- Open it up so there are multiple methods and representations
- Include inquiry opportunities
- Ask the problem before teaching the method
- Add a visual component and ask learners how they see the mathematics
- Extend the task to make it lower threshold, higher ceiling
- Ask learners to convince and reason

nrich.maths.org

UNIVERSITY OF CAMBRIDGE

Sharing Classroom Experiences

Chat to colleagues on your table about what you have tried out in your classroom since last time.

- What went well?
- Were there any surprises?
- What might you do differently next time?

Please pull out three key points on your table and record them on flipchart paper.

nrich.maths.org

UNIVERSITY OF CAMBRIDGE

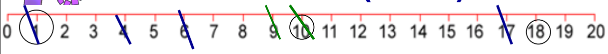
Reflective Journals

Take five minutes to order your thoughts and note down points that you would like to contribute to the discussion

nrich.maths.org

UNIVERSITY OF CAMBRIDGE

Strike it Out (6589)



6 + 4 = 10

10 take away 9 makes 1

1 add 17 is 18

18……

Competitive aim – stop your partner from going

Collaborative aim – cross off as many as possible

Exploring Proof

Is it possible to create a string of number sentences that uses all the numbers on the:

- 0-20 number line?
- 1-20 number line?
- Any number line with a set of consecutive whole numbers?

0-20 number line

- If zero is included in the number sentence then all three numbers are not distinct.
- This is true whether you use addition or subtraction in your number sentence.
- This does not obey the rules of the game.
- Therefore, on 0-20 number line we cannot ever use up all the numbers.

1-20 number line

- The first number sentence uses up three distinct nos.
- The second number sentence uses up two new nos.
- The third number sentence uses up another two new nos.
- The total number of numbers used so far is 3+2+2=7.
- Each subsequent number sentence will use 2 more new nos.
- Therefore, the total number of numbers used will go up in twos from seven.
- The 1-20 number line has 20 numbers on it.
- Going up in 2s from 7 we will never get to 20 exactly – only 19 or 21.
- Therefore, it is impossible to use exactly all the numbers on the 1-20 number line.

Any number line

- The first number sentence uses an odd number of distinct nos.
- Each subsequent number sentence uses an even number of distinct new nos.
- Odd + any number of even numbers is always odd.
- Therefore the total number of numbers used is always odd.
- Therefore we will never be able to use all the numbers on any number line that has an even number of consecutive distinct numbers on it.

- However, this does not prove that we can always use all the numbers on a number line with an odd number of consecutive distinct numbers.
- We only know that we have the correct number of numbers to make it a possibility but not a certainty.
- This does not give us any insight into the individual number sentences and the order in which the numbers need to be used.

Progression in Reasoning

- Describing
- Explaining
- Convincing
- Justifying
- Proving

Session 3

Celebrating Solutions Feature

Jo Boaler's Research youcubed.org

- 'Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School'
  - three schools
  - longitudinal study
- Book: 'Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning'
  - Amber Hill and Phoenix Park schools
  - over three years
- Follow-up study of Amber Hill and Phoenix Park students into adulthood

nrich.maths.org
© University of Cambridge
UNIVERSITY OF CAMBRIDGE

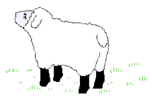
Analysing Others' Reasoning (12990)

- There are two tasks on your table – 'Sizing Them Up' and 'Forgot the Numbers'
- Choose which to have a go at first
- Take a few minutes to get into the task
- On your table, you will find some children's starting points. Each pair, please take a different starting point and work it up into a full solution
- Share your solution with others on your table

nrich.maths.org
© University of Cambridge
UNIVERSITY OF CAMBRIDGE

Using NRICH Solutions with Colleagues (12988)

Heads and Feet (924)



On a farm there were some hens and sheep.

Altogether there were 8 heads and 22 feet.

How many hens were there?

nrich.maths.org
© University of Cambridge
UNIVERSITY OF CAMBRIDGE

Using NRICH Solutions with Colleagues cont. (12988)

- Have a go at the Heads and Feet task
- Take a look at the children's responses
- Discuss:

  What does each solution show that the learner knows?

  What understandings have been demonstrated?

  What might each child need to work on?

nrich.maths.org
© University of Cambridge
UNIVERSITY OF CAMBRIDGE

Strategies That May Help me to Communicate my Reasoning

- Modelling
- Group work
- Understanding how others work
- Personal notes and recording

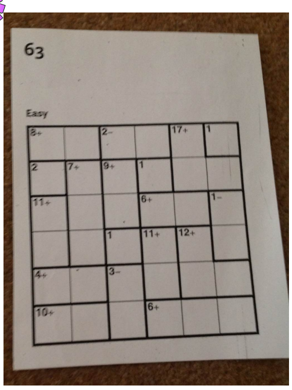
nrich.maths.org
© University of Cambridge
UNIVERSITY OF CAMBRIDGE

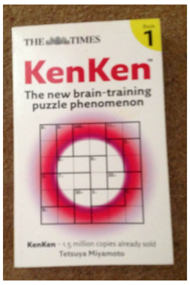
http://www.kenkenpuzzle.com

nrich.maths.org
© University of Cambridge
UNIVERSITY OF CAMBRIDGE

- How would you describe reasoning?
- How do you know when a learner is reasoning?
- How can you tell the difference between novice, proficient and expert reasoners?

nrich.maths.org
© University of Cambridge
UNIVERSITY OF CAMBRIDGE





nrich.maths.org
© University of Cambridge
UNIVERSITY OF CAMBRIDGE

Further NRICH Support

- Reasoning Feature (includes articles and tasks to help identify reasoning opportunities and support children to get better at reasoning) http://nrich.maths.org/11018
- Mastering Mathematics: Developing Generalising and Proof Feature http://nrich.maths.org/11458
- Celebrating Solutions Feature (includes an article outlining ways in which solutions to NRICH tasks can be used as a teaching resource in their own right) http://nrich.maths.org/12940

nrich.maths.org
© University of Cambridge
UNIVERSITY OF CAMBRIDGE

Reflective Journals

Jot down any reflections, reminders and/or ideas in your journal.

nrich.maths.org
© University of Cambridge
UNIVERSITY OF CAMBRIDGE

Teacher Takeaway

- Read chapter 9 from Mathematical Mindsets
- Put planning into action in your classroom – try one of the live tasks with your class
- Try out an activity with colleagues and analyse children's solutions together (either those published on NRICH or your own children's work)
- Refer to nrich.maths.org/towerhamlets

nrich.maths.org
© University of Cambridge
UNIVERSITY OF CAMBRIDGE

Feed Forward Planning

- Talk to the colleague from your school about ways in which you will:
  - -each implement some of today's content in your own classrooms
  - -work together with one or more colleagues, making use of the Celebrating Solutions Feature (12940)
- Explore the Teachers' Resources on the NRICH site for each task you plan to use

nrich.maths.org

UNIVERSITY OF CAMBRIDGE

References

Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning. Revised and Expanded Edition.* Mahwah, NJ: Lawrence Erlbaum Association.

Boaler, J. (2012). 'From psychological imprisonment to intellectual freedow – the different roles that school mathematics can take in students' lives.' Proceedings from the 12th International Congress on Mathematical Education

Boaler, J & Staples, M. (2008). Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School. *Teachers' College Record.* 110 (3), 608-645.

nrich.maths.org

UNIVERSITY OF CAMBRIDGE