$\int_a^5 10x + 3 \, dx = 114$

$\frac{10x^2}{2} + \frac{3x^1}{1} = 114$ Integrate $10x + 3$

$[5x^2 + 3x]_a^5 = 114$ Substitute a and 5

$[5(5)^2 + 3(5)] - [5(a)^2 + 3(a)] = 114$

$[5(25) + 15] - [5(a^2) + 3a] = 114$

$[125 + 15] - [5a^2 + 3a] = 114$

$140 - 5a^2 - 3a = 114$ Rearrange to equal 0

$5a^2 + 3a + 114 - 140 = 0$

$5a^2 + 3a - 26 = 0$ Factorise the quadratic

	26	1	13	2
5	26	1	13	2
1	1	26	2	13
	5	130	10	65

	-13	13
5	-13	13
1	2	-2
	10	-10

$(5a + 13)(a - 2) = 0$ Find values for a

$a = -\frac{13}{5}$ or 2

$a = 2$ Must be 2 as answer must be positive

$$\int_{2a}^{9} b\sqrt{x} + \frac{a}{\sqrt{x}} \, dx = 42$$

$$\int_{2\times2}^{9} b\sqrt{x} + \frac{2}{\sqrt{x}} \, dx = 42$$ Substitute 2 into a

$$\int_{4}^{9} bx^{1/2} + 2x^{-\frac{1}{2}} \, dx = 42$$ Put into index form

$$\frac{bx^{3/2}}{3/2} + \frac{2x^{1/2}}{1/2} = 42$$ Integrate $bx^{1/2} + 2x^{-\frac{1}{2}}$

$$\left[\tfrac{2}{3}bx^{3/2} + 4x^{1/2}\right]_4^9 = 42$$ Substitute 9 and 4

$$\left[\tfrac{2}{3}b(9)^{3/2} + 4(9)^{1/2}\right] - \left[\tfrac{2}{3}b(4)^{3/2} + 4(4)^{1/2}\right] = 42$$

$$\left[\tfrac{2}{3}b(27) + 4(3)\right] - \left[\tfrac{2}{3}b(8) + 4(2)\right] = 42$$

$$\left[18b + 12\right] - \left[\tfrac{16}{3}b + 8\right] = 42$$

$$18b + 12 - \tfrac{16}{3}b - 8 = 42$$ Rearrange to find b

$$18b - \tfrac{16}{3}b = 42 - 12 + 8$$

$$\tfrac{38}{3}b = 38$$

$$b = 3$$

$$\int_{1/2}^{1} \frac{1}{x^5} - \frac{1}{x^2} \, dx = \frac{c+1}{4}$$

$$\int_{1/2}^{1} x^{-5} - x^{-2} \, dx = \frac{c+1}{4}$$ Put into index form

$$\frac{x^{-4}}{-4} - \frac{x^{-1}}{-1} = \frac{c+1}{4}$$ Integrate $x^{-5} - x^{-2}$

$$-\frac{1}{4}x^{-4} - -x^{-1} = \frac{c+1}{4}$$

$$\left[x^{-1} - \frac{1}{4}x^{-4}\right]_{1/2}^{1} = \frac{c+1}{4}$$ Substitute 1 and ½

$$\left[(1)^{-1} - \frac{1}{4}(1)^{-4}\right] - \left[(1/2)^{-1} - \frac{1}{4}(1/2)^{-4}\right] = \frac{c+1}{4}$$

$$\left[1 - \frac{1}{4}\right] - \left[2 - 4\right] = \frac{c+1}{4}$$

$$3/4 \quad - \quad -2 = \frac{c+1}{4}$$ Rearrange to find c

$$11/4 = \frac{c+1}{4}$$

$$11 = c+1$$

$$c = 10$$

$$\int_6^{c+2} x^{\frac{b}{a}}\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx = ab^a d^a$$ Substitute values into a, b and c and put into index form

$$\int_6^{10+2} x^{\frac{3}{2}}\left(x^{1/2} - x^{-\frac{1}{2}}\right) dx = 2 \times 3^2 \times d^2$$

$$\int_6^{12} x^2 - x^1 \; dx = 18d^2$$ Multiply out brackets

$$\frac{x^3}{3} - \frac{x^2}{2} = 18d^2$$ Integrate $x^2 - x$

$$\left[\frac{1}{3}x^3 - \frac{1}{2}x^2\right]_6^{12} = 18d^2$$ Substitute 12 and 6

$$\left[\frac{1}{3}(12)^3 - \frac{1}{2}(12)^2\right] - \left[\frac{1}{3}(6)^3 - \frac{1}{2}(6)^2\right] = 18d^2$$

$$\left[\frac{1}{3}(1728) - \frac{1}{2}(144)\right] - \left[\frac{1}{3}(216) - \frac{1}{2}(36)\right] = 18d^2$$

$$[576 - 72] - [72 - 18] = 18d^2$$

$$504 - 54 = 18d^2$$ Rearrange to find d

$$450 = 18d^2$$

$$25 = d^2$$

$$d = 5$$

$$\boxed{\begin{gathered} a = 2 \\ b = 3 \\ c = 10 \\ d = 5 \end{gathered}}$$