

Chapter Nine

Common Practices of AskNRICH

Perspective One:

Two Exemplar Threads

*With many thanks to
NRICH, 'Peter'
and all other AskNRICHers*

Doing Mathematics in Different Places: an Exploration of Young People's Activities as they
make Independent Use of a Web-Based Discussion Board

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Each chapter has been edited to enable it, as far as is feasible, to 'standalone'.

The chapter numbers and numbering of sub-headings has been left unchanged from the original Thesis.

However, each edited chapter has its own page numbering and any cross-references *within* the chapters and *between* chapters on the NRICH website use these (new) page numbers followed by specifying the page number(s) in the original Thesis chapters.

Where appropriate, references may be given to other chapters (not included on the website) within the full Thesis, either by specifying the Section or providing the Thesis page number(s).

If in a chapter reference is made to any appendices, then the relevant appendix is attached at the end of that chapter.

Each chapter has its own list of references.

[The Thesis title, abstract and acknowledgement pages together with a table of contents for these edited chapters and glossary from the Thesis are also included. The table of contents of the full Thesis appears after Chapter Fifteen].

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Perspective One: Two Exemplar Threads

Just keep practising, and only look at the hints when you're really, really stuck - you'll gain more if you struggle with the question a bit before looking at the hint. They will become easier if you keep hammering away at problems ☺.

*[Advice from a school-aged peer in **ExThd2**]*

9.1 Introduction

This chapter is the first of three, each focusing on the interpretive analyses of a selection of message threads from a different Perspective. The work reported in this chapter, and the two following, contributes to addressing: *How is AskNRICH typically used?* and *What are participants' common practices when using the dynamic web-board?*. These chapters also inform *What results from participants' practices when using the dynamic web-board?*

The Perspective for this chapter uses examination of threads that show the general nature, common practices and use made of the web-board by the AskNRICHers, reported here primarily through the analysis of two exemplar threads [**ExThds**].

Thus the purpose of this chapter is to:

- i. briefly describe the two exemplar threads
- ii. provide examples, using annotated extracts, of the outcomes of applying the data analysis processes to the exemplar threads
- iii. convey the general nature, common practices and use of AskNRICH by presenting the themes derived from the features found by the coding process
- iv. discuss the general practices revealed by these features and themes

The intention of this chapter is to report on general practices found to be common amongst the AskNRICHers from actions and activities evident within the **ExThds**. The theoretical underpinnings used in the discussion section within this chapter are the concepts of Socratic Dialogue and Scaffolding [discussed in Thesis Chapter Seven Section 7.5 pp151-154].

The remaining sections of this chapter start with a brief introduction to the **ExThds** each accompanied by their tabulated extracts of posts, commentaries and indices. The chapter continues by explaining the 29 features, grouped under four themes, created from the open coding process. The final section is a discussion on the general practices revealed by these features and themes.

9.2 Exemplar Threads

In the two threads, the participants asking the original question are *Plea1* and *Plea2* respectively. Another participant, who joins in the second thread also seeking help, is labelled *Plea3*. Any information on participants such as, for example, age, is tied to the time of the analysis, May 2008.

9.2.1 Exemplar Thread One – Attempting to Solve Simultaneous Equations

The first thread [**ExThd1**] was selected for analysis as the topic, simultaneous equations, is taught in school as part of the National Curriculum [DfEE/QCA 1999: 62], and the problem could be considered as a classic routine exercise question. The thread, which is typical of all threads posted for the purpose of finding a solution to a ‘straightforward’ question, shows the nature and the type of help and the process by which it is offered. The question involves solving two simultaneous equations, one linear and one quadratic. Circumstantial evidence suggests that *Plea1* was working on two GCSE homework questions.

Plea1, still a fairly new poster in early 2008, was making their 20th post. Since then they have continued sporadically to post new problems that they need help with. *Help1* has veteran poster status and is at school (aged between 16 and 18). *Help2*, making his first post ever, is also at school and continues to use AskNRICH.

Table 9.1 [pp4-6] shows the posts in précised form together with the final, précised, third-iteration, interpretive commentary, produced using the refined method in which that related to the mathematics undertaken and that related to the actions of the posters are separated into two columns. The allocated code, explained in Section 9.3 below, appears in the final column. [For details of methodology see Thesis Chapter Six Section 6.3.4.1 pp121-123].

9.2.2 Exemplar Thread Two – Number Theory Topic

In this thread [ExThd2] *Plea2*, ‘by the way I am in year 10’ thus assumed aged 15 to 16, is preparing for National Competitions by attempting a series of number theory questions based on self-directed reading of the UKMT Number Theory Book. The topic is, at the level it is being worked on, not common in school mathematics and beyond that expected in GCSE examinations. However it is a topic that young, capable, aspiring mathematicians need to study [Houston 2009]. Part-way through, *Plea2* is joined by another school student *Plea3*, one year younger. Both participants attempt to make sense of the more abstract and, at times incorrect, helping posts with only spasmodic interjections by more able participants acting as ‘sages/experts’ collaborating in the sense of hints or nudges to lead towards the solution (and being in a teaching role).

In contrast to *Plea1* seeking help with two homework questions, *Plea2* is seeking to increase his own knowledge both beyond his chronological age and on a topic outside of ‘normal’ school lessons. Thus *Plea2* appears to be pursuing mathematical study ‘at leisure’, as also evidenced by the thread starting five days before Christmas, i.e. in the school holiday, and the seven exchanges on that day and two further on Christmas Eve.

This thread also shows the nature of generic mathematical advice that more-experienced peers give to someone encountering such challenging problems for the first time, a significant, additional reason for its selection. Several posts, especially at the beginning, aim to support the participant seeking help to have confidence in pursuing the type of challenging mathematics problems that are not just routine practice and, as such, more likely to be studied by an individual at home. The thread only starts to focus on the specific mathematical problem¹ after some eight posts discussing problem-solving practices. Table 9.2 [pp7-8] presents post extracts interspersed with an interpretive commentary and relevant codes on the first eight posts of the thread. Table 9.3 [pp9-12] presents the remaining specifically mathematical posts in the thread.

¹ *Plea2* subsequently and periodically returned to the thread to ask about other number theory questions. For the purposes of the exemplar thread only messages up until the first problem is resolved have been included here.

Table 9.1 Exemplar Thread One: Posts, Dual Commentaries and Codes

Post Number Day/time	Post précis	Commentary Mathematics focused	Commentary Actions focused	Code
P1 Friday 7.29pm	Plea1: <i>We are solving simultaneous equations, one linear, one quadratic. I am stuck on two. I know the answers but I can't work out how to get them. Any help is greatly appreciated. [Two questions and answers stated]. I can usually solve them, but these two got me really muddled. Thanks in advance.</i>	Linear equation of form $y=ax+b$, a quadratic is a polynomial of degree two of form $y=ax^2+bx+c$ Solutions values known and by substitution can be seen to be correct. Does not indicate the method being used	Posted at beginning of weekend, out of school time Content to show current inability Politeness	TM LRO SPP
P2 Friday 8.13pm	Help1: <i>The method you want to use here is substitution [Provides a worked solution to an alternative question] ... See if you can do it for yours now. If you can't, post your working and we can see where you've gone wrong ...</i>	Suggests the method to use, and explains it through working through an alternative example	44 minutes before reply Relevant example especially devised Offers encouragement to try with reassurance of further help if required	TB TREG TRSM SPC
P3 Friday 8.46pm & P4 8.54pm	Plea1: <i>I'm sure I've made a really silly mistake [includes workings] ... doesn't factorise ...I'm not sure what happened ... THESE AREN'T THE RIGHT ANSWERS. Thanks in advance</i>	Used method given to find correctly $x = \frac{2}{3}y$, substituting into the quadratic but expands $(y + \frac{2}{3})^2$ not $(\frac{2}{3}y)^2$. Knows error exists as quadratic does not factorise as it must For second problem the values derived are incorrect	Has spent over half an hour (assumedly) trying to get correct solutions Posts mathematical workings Suggests own inabilities Apparently frustrated but is persevering Asking for further help with no explicit 'write down the solution for me' Perseverance	TB LRW LRO LRP LRU LRP

Post Number Day/time	Post précis	Commentary Mathematics focused	Commentary Actions focused	Code
P5 Friday 9.58pm	Help2: <i>For 1 you've just made a mistake in expanding the expression, can you see it? For 2 in a few steps you have divided/multiplied by x, which means that you have to check the extra case x=0. Additionally you've made a silly mistake in expanding Can you solve it now?</i>	See above Error made in multiplication manipulation: multiplying a bracket by 5x would not make the denominator 5x times larger, a typical 'silly mistake' that mathematicians can be prone to even if here it was made through inexperience. The error made the equation more complex and included a special case of division though this will also be true with correct expansion	Second helper [and first time poster] now involved Teaching locates and signals error but leaves Plea1 to attempt to correct for self Teaching aware of special case, anticipates misconception, 'future-proofs' Supportive atmosphere, still asking if explanations are sufficient to complete solution plus 'silly mistake' is a repeat of Plea1 's own turn of phrase	TA TRSE TRAD SPC
P6 Friday 10.07pm	Plea1: [obtains expression that] <i>cancels down to $x^2 = 5x$ is that correct and if so, what is the algebraic way then, to solve it to make 5, and 0. i can see how the numbers go in but not how to solve it algebraically</i>	$x^2 = 5x$ often causes more problems than quadratics which have all three terms. Solution involves the case where $x=0$ alluded to in P5	Plea1 has continued to work on the problems (2.5 hours since first posting) and is explicit as to where he has reached and what he would like to know	TE LRO
P7 Friday 10.12pm	Moderator: <i>For 1, I suspect it's the sort of blunder you become blind to when going back, because you're too busy checking the steps you did do. So try this:</i>	'blind to blunders' is also something a mathematician can be prone to Required to expand $(y + \frac{2}{3})^2$ and $(\frac{2}{3}y)^2$ to highlight Plea1 's original error [see P4]	Third helper now involved. Postings beginning to overlap and offering correct advice but involving a range of perspectives as to what should be done Expansion will make Plea1 's original error clear 'Comfort' offered with 'blunder'	TA TROH TRSE SPC
P8 Friday 10.15pm	Help2: <i>For 2 we consider the following cases:</i>	Provides the solution for q2 involving the case of $x(x-5)=0$	Gives direct instruction (and completes the solution for q2), restricting Plea1 in working it through personally (possible to infer advantages and disadvantages to doing so)	TRDE TRRR

Post Number Day/time	Post précis	Commentary Mathematics focused	Commentary Actions focused	Code
P9 Friday 11.33pm	<i>Plea1: Thank you. I really understand 2 now. Number 1 is coming to me too ... taking a little more time</i>		Four hours since <i>Plea1</i> started. Assumed to have worked through <i>Help2</i> 's solution to understand the mathematics involved Perseverance – still content to continue working on it	TE LRU LRP
P10 Saturday 2.34pm	<i>Plea1: Thak [sic] you so much. I understand it all now</i>		Completed before the end of the weekend, over a period of 19 hours (including sleeping!) Lets people know that all is well and offers thanks for the help received <i>Plea1</i> perceives that work is understood	TM TE SPP LRU

Table 9.2 Exemplar Thread Two: Posts 1 to 8, Actions focused Commentary and Codes

Post Number Day/time	Post précis	Commentary Actions focused	Code
P1 Thursday 12.03am	Plea2: <i>I just wanted to ask a few questions about the ukmt number theory book. How much prior knowledge does it assume? Are the exercises meant to be challenging? I put particular emphasis on the last question as I find the exercises quite tricky</i>	Posting just past midnight Open to stressing current difficulties	TM LRO
P2 Thurs 8.11am	Deputy Moderator [DM] <i>Well exercises aren't that much fun if they're easy!</i>	Brief reply advising gain pleasure in 'hard' mathematics	TRMA
P3 Thursday 8.22am	HelpA: <i>found the number theory problems to be fairly easy in comparison. ... Just keep practising, and only look at the hints when you're really, really stuck - you'll gain more if you struggle with the question a bit before looking at the hint. They will become easier if you keep hammering away at problems ☺. ... Out of interest, how do you find the inequality problems, and if you have the geometry book, how do you find those problems?</i>	11 minutes after brief reply HelpA – more experienced peer, still at school, offering sincere, genuine advice and encouragement Inviting Plea2 to engage in further the discussion	TB TROD TRMA SPC TROD SPC
P4 Thursday 1.18pm	Plea2: <i>I haven't started the inequalities or geometry yet. I am doing them one by one. I asked ... about the difficulty of the exercises ... spent 30 minutes on one part of the [primes] exercise ... (week2). After ... struggling with the question I looked at the commentary and was extremely put off to know that I had not even been thinking along the lines of the solution ... [Includes scanned image of commentary]. By the way I am in year 10</i>	First part of response immediate to HelpA 's final comment Continues to stress difficulties Giving school year aids helpers to judge level, but also offered here in 'talk'	SPP LRO SPT

Post Number Day/time	Post précis	Commentary Actions focused	Code
P5 Thursday 1.27pm	<i>HelpA: 30 minutes is not long in the grand scale of things. Often you can spend 3-4 hours or more on a difficult problem if you're really getting into it. I know what you mean though about not even thinking along the right lines. Often it's tempting when faced with a solution to think "Wow, I never would've thought of that", but it's best not to think in that way. Instead, make the solution your own! Use the technique in other problems now that you've encountered it ☺. Always look to improve your problem solving 'toolkit' and to add more tools to it. If it's any concellation [sic], I just spent 20 minutes on a question, approaching it in completely the wrong direction, and at the end I arrived back at the initial problem. Annoying, but it happens ☺. I didn't have the required knowledge to solve the problem in fact it turned out. Persistence is key, though once you've bashed away at a problem for a reasonable amount of time, it's not shameful at all to look for hints/solutions ☺ The more problems you have a good go at, the better you will become, I promise! If you keep at it, in six months time I'm sure a lot of problems you struggle with now will be very easy to you.</i>	Just 9 minutes later (during which this long reply has been written) HelpA provides reassurance that Plea2 's state is normal. Offers advice Continues discussion, again reassures Offers more advice and further reassurance	TB SPC TRMA TROD SPC TRMA SPC
P6 Thursday 1.43pm	<i>Plea2: Thanks for the motivation. I was even contemplating giving up working through the books because I thought the exercises were too hard.</i>	Lets HelpA know they have been of great help [continued, authentic encouragement enabled Plea2 to stay on board]	SPP
P7 Thursday 2.33pm	<i>HelpA: You're welcome ☺, never give up!</i> who replies kindly and with a final word of advice!	SPC TRMA
These exchanges have taken place within the space of a morning and early afternoon (a break for lunch?) A little under an hour later HelpB , a second more experienced peer, joins the discussion supporting Plea2 and reiterating some of the HelpA 's advice.			
P8 Thursday 3.21pm	<i>HelpB: 30 minutes definitely isn't a long time when attacking a problem. No doubt your [sic] used to destroying gcse/alevel problems but i actually think it's more fun tackling a longer question. I remember being disheartened when attempting [] question because i couldn't instantly see the answer which is common in Alevel questions, but now i quite like the fact that i have to rack my brains in order to spot the path. It feels more rewarding when you do actually solve it. I've not done either of these books but if they are stretching you that's always a good thing because unfortunately i doubt Alevel will or does. Maybe parts of further Maths possibly</i>	Reiterating HelpA 's advice Reiterating DM 's advice Reassurance as HelpA 's Reiterating DM 's advice Criticism of lack of challenge in school mathematics	TRMA SPC throughout SPO

Table 9.3 Exemplar Thread Two: Posts 9 onwards, Dual Commentaries and Codes

Post Number Day/time	Post précis	Commentary Mathematics focused	Commentary Actions focused	Code
P9 Monday 1.12am	Plea2: <i>In the commentary that I provided in post 4, are you just meant to 'spot' that $2^m - 1$ is of the form $4t + 3$? I ask this because I would never have thought of doing that....</i>	Prove when n is a power of 2, the function $(1/3) \cdot (4^n - 1)$ has a prime factor of the form $4k + 3$ Scanned image sent by Plea1 includes the 'spot'.	Early hours of Christmas Eve. Late in the night posting for 15-16 year old Four days later, Plea2 returns with same problem – indicating inadequacy	TM TA TE LRP LRO
P10 Monday 8.32am	HelpA: <i>Well, it's not *too* hard to spot if you notice that 2^m is always going to be a multiple of 4 for $m > 1$. With more experience a lot of things like that will jump out at you quite quickly ☺</i>	Senses where Plea2 is stuck as clue to $2^m - 1$ is in discussion of 2^m that would lead Plea2 to $4t + 3$ Plea2 does not respond directly to this (thought the 'spot' here is illustrated later by Plea3 in P20 with slight error and corrected in P30).	HelpA [continuity] replies around breakfast time of the same day HelpA suggests what to use to move forward Advice also intended to reassure	TA TRSM TRMA SPC
There were no pleasantries of wishing each other 'Happy Christmas' and no more posts until Plea2 returns on 9 th January when there are five exchanges between Plea2 and DM				
P11 Wed 8pm to P15 Wed 9.02pm	Plea2: <i>seeking justification why $2^m - 1$ is of the form $4t + 3$... has an odd number of primes of the form $4k + 3$ in its prime factorisation. Thanks</i> DM suggests: <i>multiplying two numbers of form $4k + 3$ say $(4k + 3)$ and $(4l + 3)$</i> Plea2 does so, shows working and responds: <i>I think I've got it</i> DM enquires about familiarity with modular arithmetic Plea2 offers thanks.	Multiplication leads to $[36(k+l)+24] + 3$ hence [term] is divisible by 4 and thus of required form Both inputs from DM provide potential for Plea2 to increase their mathematical knowledge	Just over two weeks later and Plea2 is still working on the problem Plea2 is seeking a proof for the justification and in doing so implicitly shows a wish to gain understanding of the fact. 2 minutes between Plea2 asking for and DM giving hint Advice and suggestion of method within DM posts Plea2 includes working implying has seen solution and adds thanks	TE LRP LRO LRC LRU TB TRMA TRSM LRW LRU SPP

Post Number Day/time	Post précis	Commentary Mathematics focused	Commentary Actions focused	Code
P16 Wed 6.02pm to P19 Thurs 4.55pm	Plea2 returns with another statement on a related problem based on $(2^n - 1)$: <i>Can someone show me ... book just gives an example and no justification. I know that it does, but someone show me how to show it generally (like the one above).</i> HelpC states a formula triggers two further messages: DM stressing that is not just for powers of 2 and the formula worth remembering; HelpD suggests generalising $a^n - b^n$	See [Thesis p491] for mathematics involved. None of the suggestions are trivial; all three suggestions shift focus to memory in order to apply a useful fact to achieve a solution	One week later. Wanting to understand why the statement is true, not just accepting of it HelpC veteran poster still at school. Problematic to judge whether necessary for further study and/or balancing instrumental/relational understanding. But all help given has the potential to aid Plea2 's mathematical knowledge	TE LRP LRC LRU TRRR TRMA TRSM TRAM
Plea3 making only their 9 th post joins the thread				
P20 Sunday 4.52pm	Plea3 : <i>I'm in year 9 ☺</i> offers their own solution/method to Plea2 first justification request (P11 above). <i>.... However, after all this, I still don't understand how a number N... [gives own thoughts] ... if I am not mistaken ...or then again I may be wrong. Can you please justify</i>	Solution holds a misconception (or misunderstanding) and a numerical coincidence [see Thesis p491] Problem in understanding Plea3 's mathematical text (even though there is an error) as normal typing cannot produce the precise mathematical notation	Sunday afternoon ☺ – humour but with possible one-upmanship, 'cleverness' as one year younger Plea2 Alternative given but clearly states not understanding, shows own thoughts and wants justification (' <i>Not mistaken</i> ', ' <i>may be wrong</i> ' although open with current thoughts possible suggestion of actually being correct)	TM LRJ SPT SPB TRAM LRW LRO LRU LRC

Post Number Day/time	Post précis	Commentary Mathematics focused	Commentary Actions focused	Code
P21 Sunday 5.24pm to P24 Sunday 8.46pm	<p>HelpD provides example of correct notation DM Gives advice on how to mark-up mathematical text to appear ‘normal’ on the board Also gives very full explanation with numerical example to misunderstanding includes adding and answers own question: <i>why does m have to be more than 1? Does that help? Do post back if you’re still confused</i> Another Veteran Poster points out DM numerical slip: $24=2^3 \times 3$ to which DM replies: <i>Yes, all right, fair enough. Hopefully [Plea3] will understand what I was trying to say despite that. (Curiously, I thought that something was a bit odd when I wrote it, but still didn’t spot it!)</i></p>	<p>Example contains numerical slip/howler /‘silly mistake’ writing $24=2^2 \times 3$</p> <p>[NB This episode is revisited in Chapter Eleven]</p>	<p>Sunday Implicitly signal error DM Gives advice on how mark-up mathematical text to appear ‘normal’ on the board Extending the explanation but making error Invitation to ask for more help</p> <p>A light hearted exchange with an impression of ‘frisson’.</p>	<p>TM TRSE TA</p> <p>TRMA TRRR SPC</p> <p>SPB SPB</p>
Post Number Day/ time	Post précis / commentary			Code
The remaining messages settle down to a discussion started by Plea2 expanding an expression previously suggested by HelpD . Here the exchanges become ‘messy’ and difficult to report here in order. The annotations on the thread [Thesis p491] provide a commentary of the connections, the mathematics involved and a judgement on the quality of help being offered. The main points are listed below with appropriate codes.				
P25 to P32	<p>Seven days after last posting and two days after the DM’s previous post, Plea2 returns and asks: <i>Does $x^{ab}-y^{ab}$ factorise to...</i> [gives suggestion with error of sign in first bracket that they correct], HelpD sends three messages (P26-28), first: <i>on right lines though</i> two minutes apart correcting twice, <i>whoops</i>, a factorisation that is related to Plea2’s attempt. 15 minutes later Plea2 corrects their own post and one minute after Plea3 suggests to HelpD the possible correct factorisation that HelpD had given and in doing so Plea3 corrects own error (made in P20). Another minute later, two minutes since Plea2 last posted, they start: <i>nah... forget that</i> (P32) suggesting second thoughts on what they were about to write down.</p>			<p>LRA LRP TE SPC TRRR LRW SPP SPH</p>

Post Number Day/ time	Post précis / commentary	Code
P33	It is not clear as to which of Plea2/3 or both HelpD is responding to with: <i>nope nothing quite there yet, try doing some simple examples and spot a pattern. May I suggest powers of 3 and higher before trying to generalise</i> . ‘Pattern spotting’ needs to be treated with caution The difference here is that by seeing how the powers in terms appear by trying a few examples, understanding will be built up to write the factorising by detailing the first few terms and then ... to last term or two.	TRRR TRMA
P34 to P37	Plea3 ’s attempt is not correct even though they comment: <i>I have tried this for a few values of a and b and they work fine</i> . HelpA , returning to the thread after one month and some 25 messages later, spots one minor error of sign in first short bracket (Plea3 corrects this) but does not comment on the main second incorrect bracket. HelpA ’s ‘nearly’ offers encouragement. HelpE arrives with own (correct) suggestion mirroring Plea2 ’s (P25) efforts earlier: <i>shouldn’t it be more like this?</i>	LRA LRP TRRR SPC TRDE
P38 & P39	DM suggests a useful technique to simplify notation, which Plea3 uses correctly but fails to correct errors in second bracket.	TRMA LRA
P40	Plea2 returns four days after completing the problem to politely suggest to HelpD that: <i>I believe your correction in post no 82 is partly incorrect</i> and explains why they think this. In the same message Plea2 comments directly to Plea3 : <i>I’m sure your factorisation is wrong. It should be [gives correct form]. I’m with [HelpE] on this one</i> .	TE SPP TRSE SPC
P41	Plea2 then sends another post 8 minutes later (just before midnight) thanking DM : <i>yours was a very helpful hint which made the problem break down much more quickly in this factorisation mess....</i> Factorisation mess seems an appropriate (and humane) description for the intricacies and difficulties of getting both the algebraic manipulation and notation correct.	SPP SPH
P42	Finally two days later Plea3 sends a final message which suggests that they now agree: <i>Thanks I have realised that my equation only works for when a=1, hence my misunderstanding. ☺</i> Interpreting ☺ is problematical: it may for example indicate laughing at own ‘stupidity’ or relief that the problem is now finally sorted. Nonetheless it conveys a ‘happy’ banter type exchange	SPP LRO LRP LRU SPB

9.3 Explanation of Features

This section explains the features accompanied by their respective code index grouped according to the four themes resulting from the interpretative analysis of threads. Two of the four themes concern specific educational aspects related to teaching and learning interactions whose presence is inferred through the interpretation of message content. Although it can, for example, be inferred that something has been learnt or understood, no conclusion can be drawn as to the *degree* to which it has been learnt or understood. Neither can it necessarily be assumed that a teaching strategy adopted was a known pedagogical intention of the helper. For these reasons, these two themes have been labelled as *Features in a Learning Role* and *Features in a Teaching Role*, rather than simply ‘Learning’ and ‘Teaching’. A third theme *Social and Personal* is again the result of interactions but these could be broadly termed non-educational and non-subject specific. The fourth theme, *Temporal*, relates to the medium / web-board structure in which the interactions and the subject study can take place.

9.3.1 Theme One: Features in a Learning Role [Prefix LR]

Table 9.4 [next page] presents the nine features assigned in this theme. The Posting Protocols expect that the person seeking help (the learner) will share their thoughts and include current work on the problem with those offering help. Thus the presence of ‘**openness of current difficulties**’ [LRO] and ‘**showing working**’ [LRW] should permeate throughout any thread. In continuing with a problem as far as seeking help in the first place, some degree of persevering must already be present, but ‘**perseverance**’ [LRP] is further exhibited within the thread by staying in the thread and continuing with the problem until sufficient help had been given. Two features, ‘**seeking re-assurance**’ *that a solution/selected method or presented idea is correct* [LRA], and *seeking whether there is a* ‘**better (alternative) solution**’ *than own obtained* [LRB] are two consequences of knowing that help is at hand. The open access of the board automatically presents opportunities for ‘**joining in to find a solution**’ *to the problem that someone else had initiated* [LRJ]. Participation in AskNRICH provides the opportunities to see others engaging in mathematics. Two features commonplace in such engagement are: *following a hunch or*

‘**intuition**’ for a solution or path taken being correct/wrong [LRI] and the importance of having a rigorous proof, thus *seeking aspects that ‘constitute a proof’* [LRC].

LR – Features in a Learning Role		Explanation and/or Examples from the data
LRA	Seeking assurance whether a solution/chosen method/idea is correct	Posters can have partially-formed or tentative solution <i>‘Does $x^{ab} - y^{ab}$ factorise to:’</i> ExThd2
LRB	Seeking whether there is a better (alternative) solution than own obtained	Similar to but as an alternative to LRA, a poster can wonder whether there other solutions available <i>‘i have got that 3,5,2 works but how do i prove that this is the only answer or find other answers? have i done this correctly? is there a nicer way of solving it?’</i> [also LRA,LRC] CS-P363
LRC	Seeking aspects that constitute a proof	Due to a common usage of AskNRICH as a means to discuss mathematics competitions some posts will be querying aspects of proof, either whether their own attempt is a proof or parts (present or missing) of a written proof <i>‘Can someone show me why $(2^n - 1)$ contains a factor $2^m - 1$ where $n = 2^s * m$ and m is an odd integer. The book just gives an example and no justification. I know that it does, but someone show me how to show it generally’.</i> [also LRU] ExThd2
LRI	Feeling or intuition for solution or path taken being correct/wrong	The web-board provides the opportunity for tentatively expressing thoughts or feelings as to what the solution will be, rather than providing a solution straightaway <i>‘.... Do you think the converse is true?’</i> Response to CS-P125 and 3Thd1
LRJ	Joining in to find a solution to the problem that someone else had initiated	The open nature allows anyone to join the search for a solution, working together <i>‘I am doing the same I still don't understand how ...’</i> ExThd2
LRO	Openness of current difficulties	As with LRW, sharing thoughts is required by the posting protocols <i>‘I know the answers but I can't work out how to get them. Any help is greatly appreciated’.</i> ExThd1
LRP	Perseverance	Continued engagement with a problem <i>‘I'm not sure what happened ... THESE AREN'T THE RIGHT ANSWERS. Thanks in advance’.</i> [also LRO] ExThd1
LRU	Developing signs of (deep/relational) understanding	Evidence within the text that implies a desire to understand or the learner has perceived they understand <i>‘Thank you. I really understand 2 now. Number 1 is coming to me too ... taking a little more time’.</i> [also LRP] ExThd1
LRW	Showing working	As with LRO, including current work is required by the posting protocols <i>‘I got as far as this and it didn't factorise’.</i> ExThd1

Table 9.4 Theme One: Features in a Learning Role

Instances within posts where any of these eight features occur will be explicit within the text. The same cannot be claimed for interpreting content of text to measure the internal process of understanding. Nevertheless, anyone using AskNRICH knowing that no direct

solution will be given must automatically be seeking some degree of understanding to a problem that is more than just being told the answer. Although it might be inferred that the type of understanding sought is beyond a superficial instrumental understanding, it is not always possible to ascertain whether the way that the problem is worked through is more than using a rote technique. Nonetheless, the feature **‘developing signs of (deep/relational) understanding’** [LRU] has been assigned to parts of the text that revealed a desire to understand, or the learner perceived that they now understood – so for example where working presented *showed* understanding, or there were statements such as ‘Got it!’ or along the line of ‘I understand now’.

9.3.2 Theme Two: Features in a Teaching Role [Prefix TR]

This theme was assigned ten features as depicted in Table 9.5 [next page]. Five are used to define different teaching strategies, all of which should have been a result of the Posting Protocol expectation that only hints and explanations should be given that would help the person asking for help to understand. Four of the strategies employed: *a ‘worked solution to a different example’* [TREG]; **‘anticipating difficulties’** [TRAD], *providing ‘specific method’ to adopt* [TRSM] and **‘alternative methods offered’** [TRAM] have names that are self-explanatory. **‘signalling error’** [TRSE] relates to instances where the helper indicates errors either in the working presented or where the learner is showing a misconception. Although the Posting Protocols ask helpers not to provide the solution, there are instances of explicitly providing **‘direct explanation /working through the problem’** [TRDE]. Providing **‘mathematical advice’** [TRMA] relates either to some aspect of a particular mathematical problem or on the process of working mathematically. Both cases provide the opportunity for tools to be added to the mathematician’s toolbox [see Section 11.5.1 Chapter Eleven p19/Thesis p258]. The feature **‘open discussion’** [TROD] refers to general exchanges that remained mathematics focused e.g. discussion on a particular textbook or area preferences. Anyone can offer help, not necessarily correct and/or limited in what it will achieve; and at any time, which might not always be in the most logical sequence: hence the features **‘restricting response’** [TRRR] and **‘overlapping help’** [TROH] respectively. These two latter features are a consequence of the open access, asynchronous nature of AskNRICH, though they are not necessarily disadvantageous as the labels might seem to imply.

TR – Features in a Teaching Role		Explanation and/or Examples from the data
TRAD	Anticipating difficulties in the current problem	A teaching strategy that anticipates a common viewed difficulty [or a known misconception] and seeks to highlight within the help offered <i>'For 2, in a few steps you have divided/multiplied by x, which means that you have to check the case $x=0$ extra'.</i> ExThd1
TRAM	Alternative methods offered	Offering a different way in which the problem could be solved <i>'Another method then the one given above: to find that 2^{m-1} is in the form $4k+3$, just factorise it over four, to give: ...'</i> ExThd2
TRDE	Direct explanation/working through the problem	Although 'against' the posting protocols advice not to give solutions there can be occasions where it might be appropriate to directly work through the solution to the problem or spell out relevant facts <i>'For 2 we consider the following cases: '[and then gives full solution]'</i> ExThd1
TREG	A worked solution to a different example	A teaching strategy that allows a poster to adapt the solution given to a different problem to the one that they are attempting to solve <i>'Basically, find y in terms of x (or vice versa) ... Here's an example... See if you can do it for yours now.'</i> [also TRSM] ExThd1
TRMA	Mathematical Advice	Instances where advice is given either for a particular mathematical problem or a process <i>'I know what you mean though about not even thinking along the right lines. Often it's tempting when faced with a solution to think "Wow, I never would've thought of that", but it's best not to think in that way. Instead...'</i> [also SPC] ExThd2
TROD	Open Discussion	General exchanges that remained mathematics focused <i>'Out of interest, how do you find the inequality problems, and if you have the geometry book, how do you find those problems?'</i> [also SPC] ExThd2
TROH	Overlapping help	Different helpers involved, focusing on different aspects. In ExThd1 , (i) different worked example, (ii) direct explanation and (iii) to highlight original error, suggestion to expand two different expressions
TRRR	Restricting Response	Inherently limited help <i>'For 2 we consider the following cases: ...'</i> [also TRDE] ExThd1
TRSE	Signalling error	Instances where the helper has spotted the error or misconception <i>'you've just made a mistake in expanding the expression...'</i> ExThd1
TRSM	Providing specific method to adopt	Informing the poster of a specific method, even if there are alternatives) to adopt <i>'The method you want to use here is substitution'</i> ExThd1 [although in this case a graphical method would be a viable alternative method to adopt]

Table 9.5 Theme Two: Features in a Teaching Role

9.3.3 Theme Three: Social and Personal [Prefix SP]

Table 9.6 shows the six features allocated to this theme. Two distinguish ‘**banter**’ [SPB], where there is obvious humour but delivered with light-hearted teasing, from ‘**humour**’ [SPH] the genuine neutral witty remark. The Posting Protocols expect respect to be a pervasive feature of AskNRICH. The feature ‘**politeness**’ [SPP] is used where the text explicitly shows what would be considered good manners. However, asking for politeness and respect does not automatically engender a sense of care, thus an additional feature ‘**care for others**’ [SPC] is used to indicate for example kindness to, empathy with, or nurturing of, other AskNRICHers. ‘**non-mathematics talk**’ [SPT] is self explanatory. ‘**opinion**’ [SPO], is reserved for critical comments/judgements, whether about mathematics or not. The latter two features are far more prevalent in the NRICHtalk section on the private part of the web-board. Indeed this was the purpose for which NRICHtalk was set up.

SP – Social and Personal		Explanation and/or Examples from the data
SPB	Banter	Light-hearted teasing In ExThd2 , a numerical slip is pointed out: ‘ $24=2^3 \times 3$ ☺’ receiving the reply: ‘ <i>Yes, all right, fair enough</i> ’
SPC	Care for others	Showing consideration <i>‘See if you can do it for yours now. If you can’t, post your working and we can see where you’ve gone wrong ...’</i> ExThd1
SPH	Humour	Distinguished from SPB as the genuine neutral witty remark <i>‘factorisation mess....’</i> [also SPP] ExThd2
SPO	Opinion	Personal, critical judgments <i>‘but if they are stretching you that’s always a good thing because unfortunately i doubt Alevel will or does’</i> ExThd2
SPP	Politeness	Good manners <i>‘Any help is greatly appreciated. ... Thanks in advance’</i> ExThd1
SPT	Non-mathematics talk	Useful information but not strictly mathematical <i>‘By the way I am in year 10’</i> ExThd2

Table 9.6 Theme Three: Social and Personal

9.3.4 Theme Four: Temporal Aspects [Prefix T]

Four features potentially present in every thread and pervasive throughout AskNRICH were allocated in this theme [see Table 9.7 below]. Although these features are common to CMCs

in general [Henri 1992, Rennie & Mason 2004], nevertheless it remains important to include these when building the characterisation of AskNRICH. An inherent facet of AskNRICH that has a major liberating effect is the removal of time boundaries, captured by two features: ‘**mathematical study present beyond the school day**’ [TM] and ‘**working on a mathematical problem sustained over an extended length of time**’ [TE] where the duration of time spent working on a problem spreads over a longer uninterrupted period. Where the time gap between posts, for example noticeable speed or a long measured reply posted within a short time, is significant to its interpretation, the feature ‘**time between responses**’ [TB] is assigned. The feature ‘**significant influence of asynchronous communication**’ [TA] is used for various specific instances where the medium of AskNRICH as a web-board is interpreted to have an effect on the thread, for example multi-helpers simultaneously posting or any new poster instantly offering help.

T – Temporal		Explanation and/or Examples from the data
TA	Significant influence of asynchronous communication	Incidences of ‘technical’ effects. For example in ExThd1 three helpers have become involved within the first seven messages [also TROH]
TB	Time between responses	When the speed/time between Posts is deemed worthy of note. For example in ExThd1 the detailed worked-through example matching the structure of the original problem arriving within three quarters of an hour of Plea1 requesting help
TE	Working on a mathematical problem sustained over an extended length of time	For example: the four hours that Plea1 in ExThd1 spent on a Friday evening ‘ <i>I really understand 2 now. Number 1 is coming to me too ...</i> ’
TM	Mathematical teaching present beyond the confines of the school day	Posts made to help a learner, posted outside of the normal school day, evidence by posting day/time

Table 9.7 Theme Four: Temporal Aspects

This section has provided an explanation and illustration of the 29 features with the derived code index presented alongside. The next section is a discussion on the general practices revealed by these features and themes.

9.4 Discussion

In order to consider how the findings above contribute to the later overall characterisation of AskNRICH, this section discusses the common practices in terms of teaching and learning interactions under three headings: the medium of AskNRICH in which these interactions

take place; the conversational tone of the interactions (a precursor to an in-depth consideration of conversation-for-education in Chapter Eleven), and Socratic-Style Dialogue and Scaffolding, taking place within the interactions. Although illustrative examples are taken predominately from the two exemplar threads, occasionally additional material from other threads is included.

9.4.1 Medium

Some key features of AskNRICH important to this study are inherently due to the asynchronous, temporal nature of the web-board. Being freed both from the finite time limits of a school lesson and from the confinement of accessing ‘teacher’ help only within school opening hours is crucial to enabling the AskNRICHers to pursue their studies. For example **ExThd1** was started on a Friday evening at a time when the most likely next school contact would be Monday. *Plea1* clearly wanted to solve the problems there and then and has help arriving within the hour. The time interval before help arrives is reliant on a sequence of three events: someone prepared/able to answer has to log on and read the message; the necessary help needs to be compiled, and thirdly, the help post composed and sent. The interval between posts in this thread is short and this is typical of AskNRICH. In this instance the first nine posts span four hours (on a Friday evening no less) and, after the first reply, there is a flurry of posts to-and-fro, for example three helping posts from two contributors (in response to and being responded to by *Plea1*) arrive within a time span of 18 minutes. This is followed by other flurries punctuated by longer periods of quiet (in this case overnight and into the next day) until all is resolved to everyone’s satisfaction.

In AskNRICH postings are made on all days of the week and at all times of the day and night, albeit predominately out of school hours [further evidenced in Chapter Ten]. Indeed, exchanges near the beginning of **ExThd2** are taking place during the Christmas vacation, including Christmas Eve.

The apparent amount of time that the person asking for help is prepared to work on trying to find the solutions can be substantial, as both these thread show. In **ExThd1**, *Plea1* was involved for over four hours on the Friday evening, making a final post at the relatively late hour of 11.33pm. Although it is not possible to know what other things *Plea1* might have

been doing during this time, the number of postings and the work that *Plea1* had needed to do in order to make the next post implies that a substantial proportion was given to working on the problem. Furthermore, *Plea1* has ‘stuck’ at the problem for quite some time. Full resolution on *Plea1*’s part is early Saturday afternoon, well before the next school-day. The working-outside-of-‘normal’-hours and the speed at which help can be offered are natural aids to ‘perseverance’.

In addition to all of the above, the asynchronous nature provides time for reflection, the “slow-down time” [Kyriacou & Issett 2008: 10] or the “Start-Stop-Go” [Tanner & Jones 2000: 29] sequence advocated for classroom practices but, as the authors infer, not always observed. Thus the presence of metacognitive knowledge and skills [discussed further in Section 9.4.3 below] can flourish naturally within the environment of AskNRICH.

As threads can involve a number of individuals deciding to participate, posts can become ‘entangled’ and the sequence of posts appearance might result in a ‘jumbling up’ of help [see **ExThd1-P2, P5, P7, ExThd2-P25 to P42**] as envisaged by Posting Protocol 5 [see Table 8.2 Chapter Eight p6/Thesis p170]. In **ExThd1**, at around 10pm there are three posters involved concurrently, *Plea1*, *Help2*, and the *Moderator*. Posts are coming in quick succession and there is some inevitable asynchronous overlap in the posts. Although the posts appear in a linear time sequence the relevance of message may not necessarily follow this simple timetable [Chapter Eleven addresses this in depth]. In addition anyone can make a post that offers help. In this thread, as evidenced by no comments to the contrary, *Plea1* appears to be unfazed by the number and focus of the helpers and any overlapping of posts. Indeed, when a participant offers help for a question now solved or serendipitously an alternative method, these can be compared against the original for elegance, accessibility etc.. In the case of **ExThd1-P7**, the *Moderator* is taking *Plea1* back to look at the original error some time after strategies have been offered. However, there are no later posts indicating whether *Plea1* did so.

Although the help given can usually be considered of good quality, it is totally reliant on or restricted by the person offering it (i.e. it might not be universally excellent or correct). Obviously the methods proposed for solving the problems also depend on the people posting

and the experiences they have had in solving similar problems in the past. Thus, for example in **ExThd1-P2 Help1** provides an algebraic method that is continued throughout the thread although a graphical method of solution would be possible. In other words the help offered may not be all embracing in terms of methods available. Moreover, although it might be easy to criticise the quality of some of the help posts in **ExThd2** (e.g. **P2,17,27,28,33**), consideration of the quality of help offered across AskNRICH generally demonstrates that this would be unfair. Overall, the whole system is sufficiently robust to overcome any difficulties, any errors will be politely corrected by other posters, or even, in the last resort, by the *Moderator*.

9.4.2 Conversational Tone

The word conversation takes on a specific meaning as further analysis of AskNRICH reveals [see Chapter Eleven], but the tone of the ‘talk’ discussed in this chapter illustrates the varied practices of AskNRICHers, in part revealing their ‘human’ side.

ExThd2 was specifically selected for the quality of advice that one peer gives to another [see for example **P3**] in a situation where the giver can feel an empathy with the receiver, having been in that position only a short while earlier. Although a teacher might offer the same advice, in that instance a distance (power relationship) would inevitably be present and thus the empathy likely to be reduced or lost.

HelpA’s first response [**ExThd2-P3**] directly back to *Plea2*’s question ‘*how much prior knowledge is assumed*’ [**ExThd2-P1**] opens the discussion on the difficulty, or not, of the book. Later in the post *HelpA*’s comment ‘*Out of interest, [other topics]... how do you find those problems?*’ has ‘opened up’ the conversation to include more than the original.

The two examples above have a mathematics focus, the next one carries with it an added critical, personal opinion on the state of school mathematics ‘*if they are stretching you that's always a good thing because unfortunately i doubt Alevel will or does ...*’ [**ExThd2-P7**]. As would be expected the more critical comments are generally made within the private part of

the board [NRICHtalk], but even there they are always delivered with politeness if not without some understandable frustration.

Other talk is ‘looser’, for example: *‘I am doing the same section of the book’* and *‘By the way I am in year 10’* [ExThd2-P4]. These are essentially ‘snippets’ that one might find out about the life of someone within the open access areas of the board, perfectly illustrated by Peter’s explanation [next chapter] as to why he had not replied for a while as *‘the family had been burgled and the computer stolen’* [CS-P126].

The Posting Protocols counsel careful consideration of the use of humour [Table 8.2 Chapter Eight p6/Thesis p170]. Similar care needs to be taken on interpreting whether what appears as a humorous remark (often noticed by the addition of ☺) was intended as such. Given that many of the AskNRICHers ‘compete’ in National Competitions there is always the possibility that some comments are delivered with a natural arrogance. There is, however, no compelling evidence for this in the content of the many thousands of posts read in this study. Moreover, certainly within the more personal private posting part of the board, everyone seems supportive of one another whether the competition scores are high or low. Nevertheless, the features incorporate a distinction between banter, where there is obvious humour but perhaps delivered with some frisson and/or light-hearted teasing, and humour which is reserved for the genuine neutral witty remark. [See also Chapter Eleven].

In giving a clear explanation in ExThd2-P22, **DM** writes (incorrectly) $24=2^2x3$ which by way of banter receives the response $24 = 2^3x3$ ☺. One can imagine people ‘laughing’ at this in a light-hearted way, no-one can seriously believe that **DM** has made a real error but people find fun in pointing out such ‘howlers’. This can be slightly annoying for the person who has made the error and the only course of action is to take it ‘with good grace’: *‘Yes, all right, fair enough. Hopefully [Plea2] will understand what I was trying to say despite that. (Curiously, I thought that something was a bit odd when I wrote it, but still didn't spot it!)’*. **Plea3**’s first post in ExThd2 ending with *‘and I’m in year 9 ☺’* [P20] is a further example of banter, implying (perhaps) being ‘better’ as they are one year younger. These examples are different to genuine (bringing a smile to one’s face) humour. Reading *‘yours was a very helpful hint which made the problem break down much more quickly in this factorisation*

mess' [**Plea2** in **ExThd2-P41**] one can imagine how much of a mess, metaphorically, there had been in working through the problem. Indeed reading the comment for the first time made me laugh-out-loud or as the AskNRICHers write 'lol' [See Chapter Eleven later]. The post: '*Now ive got the first one im motoring through the exercises. who would have thought trigonometry could be this much fun*' used in the opening pages to this thesis is indicative of AskNRICHer humour.

9.4.3 Socratic-Style Dialogue and Scaffolding

The Posting Protocols' entreaty to avoid simply giving a solution encourages AskNRICHers to find teaching strategies other than teacher exposition. Moreover, within AskNRICH, given that anyone can choose to offer help, there is a difference from the classroom situation where essentially just one teacher with their own way of working is available. The classroom teacher may offer different ways of understanding and solving a problem and other classmates may try to help. However, for AskNRICHers, the *only means* of proceeding is being helped by peers, bringing the distinct advantage that multiple helpers may bring multiple strategies and perspectives to understanding and aiding solution. The dominant teaching and learning strategies both invoke the questioning stance of a Socratic dialogue as a means for helpers to scaffold the learner's learning, with the aim that, once the problem has been completed, the learner is in a position, the next time such a problem arises, to undertake the work with less or no help.

Help1's reply [**ExThd1-P2**] in providing a relevant, worked through, related example could be seen as scaffolding **Plea1**. Finding an example was not necessarily trivial, as it required integer solutions i.e. the quadratic equation that will factorise. As previously mentioned [Section 9.4.1], by providing this example, **Help1** has by implication suggested *the* method required, though incidentally in this instance although it is probably the most common it is not the only method that could be used. [In **ExThd2**, **Plea3** offers an alternative method [**P20**] to the one **Plea2** shared (though incomplete)]. Whether it was just fortuitous or not, **Plea1**'s error [**ExThd1-P3**] sets up a cubic equation for which it looks possible that each term can be divided by x and the equation reduced to a quadratic. The correct equation will allow a similar division and **Help2** is anticipating a universal common error (misconception) [Swan 2001] of doing the division in both these circumstances and forgetting that the

equation would also be true if $x=0$. In **ExThd1-P5**, *Help2* a brand new poster, offers help towards the solution by signalling the errors. The Posting Protocols' advice to show working ensures that signalling errors is a common way of offering help, but simply pointing out the error is not the only means. Later in the **ExThd1**, for example, the *Moderator* sets out two expressions to be expanded [**ExThd1-P7**] that, akin to Socratic dialogue, provides *Plea1* with the opportunity to realise their algebraic error.

The use of teaching strategies that offer hints, nudges and advice, that is various degrees of funnelling and focussing [Wood 1994], by necessity help a learner to progress both in the present and providing the means to attempt similar questions, unaided, in the future. That is, the scaffolding help offered enables the learner to move forward within their Zone of Proximal Development [Vygotsky 1978]. The post '*What do you get if you multiply together two numbers of the form $4k+3$ (call them $4k+3$ and $4l+3$)? What form does it take? What if you multiply together four such numbers. Or six?*' [**ExThd2-P12**], scaffolds *Plea2*'s learning. These direct questions offer an idea and suggest a way forward, initially *prompting* questions, specifying/telling (*funnelling*) the format of multiplying the two numbers and other even number of terms, reminiscent of a step-by-step scaffold [Bliss, Askew & Macrae 1996]. However the problem itself requires working with an odd number of terms of the form $4k+3$ (not dealing with 2, 4 or 6 terms) so *Plea2* is required to make their own connection and adapt the number of terms to be multiplied to solve the problem; eventually the *probing* questioning becomes a means of *focussing* on structure. *Plea2*'s next reply [**ExThd2-P13**] indicates that they have been able to complete the problem and '*I think I've got it*' indicates that next time they should be able to do so unaided. Thus the support offered is then *faded* [van de Pol et al. 2010] and the *knowledge transferred* to the learner within a *responsive/contingent, discussion*. Just within this one thread, **ExThd-P22**, **P33** and **P38** contain further examples where the nudge is sufficient for ideas to be taken on board and used in future. Furthermore, anyone just reading some of the posts on AskNRICH (lurking) and doing the mathematics that others are sharing/doing, is presented with a scaffold that can lead to future unaided work [see description of Julia in Section 8.4 Chapter Eight p8/Thesis p171].

Despite the Posting Protocols' stance on not simply providing the answer, such instances do occur. For example, *Help2* [ExThd1-P8] gives the final part of the solution. It is conceivable that, following *Plea1*'s posts, *Help2* made the judgement that *Plea1* is not familiar with taking special care with $x=0$ and this could be understood with the direct explanation of a worked solution. Thus there can be some acceptable reasons for such 'telling'. 'I really understand 2 now' [ExThd1-P9] suggests that *Plea1*'s learning has implicitly been scaffolded by this direct intervention. However, there is simply no way of knowing whether *Plea1* will now be able to do such problems unaided. *Peter*'s direct help with modular arithmetic to R [3Thd2 Chapter Eleven] provides a similar example.

In the Literature Review [Thesis Chapter Seven], there is a discussion on the role of metacognitive knowledge and skills in achieving effective scaffolded learning. Features resulting from coding demonstrates the presence of such knowledge and skills in the AskNRICHers, as evidenced by for example (i) taking responsibility for, and persevering in [LPR], own learning; (ii) the desire to understand [LRU] the mathematics involved (iii) pursuing the notion of proof [LRC], and (iv) discussing the quality of solutions [LRB]. The teaching and learning roles features imply the AskNRICHers' metacognition leads them to be reflective both *in-action* [Schön 1983] and, aided by the asynchronous nature of the web-board [discussed above], at the higher level of *on-action* [Schön 1991].

Overall, the help offered by AskNRICHers to their peers must be considered to be impressive. The Posting Protocols were designed to encourage strategies that evoke Socratic dialogue and scaffolding and the analysis of AskNRICH clearly shows that they are being used and, furthermore, that AskNRICHers' metacognition means that they have both the propensity and the capability of interacting with each other in such a way. The interactions are as pedagogically sound as they are because the AskNRICHers have the ability to reflect on their own learning and, it might be inferred, wish to offer strategies based on how they prefer to learn and perhaps be taught [see Afterword at the end of this chapter].

9.5 Features Summary 2

The Features Catalogue [a concept explained in Section 8.6 Chapter Eight pp16-17/Thesis pp179-180] for this chapter, relating to Teaching Interactions and Learning Interactions, is presented in Figure 9.1.

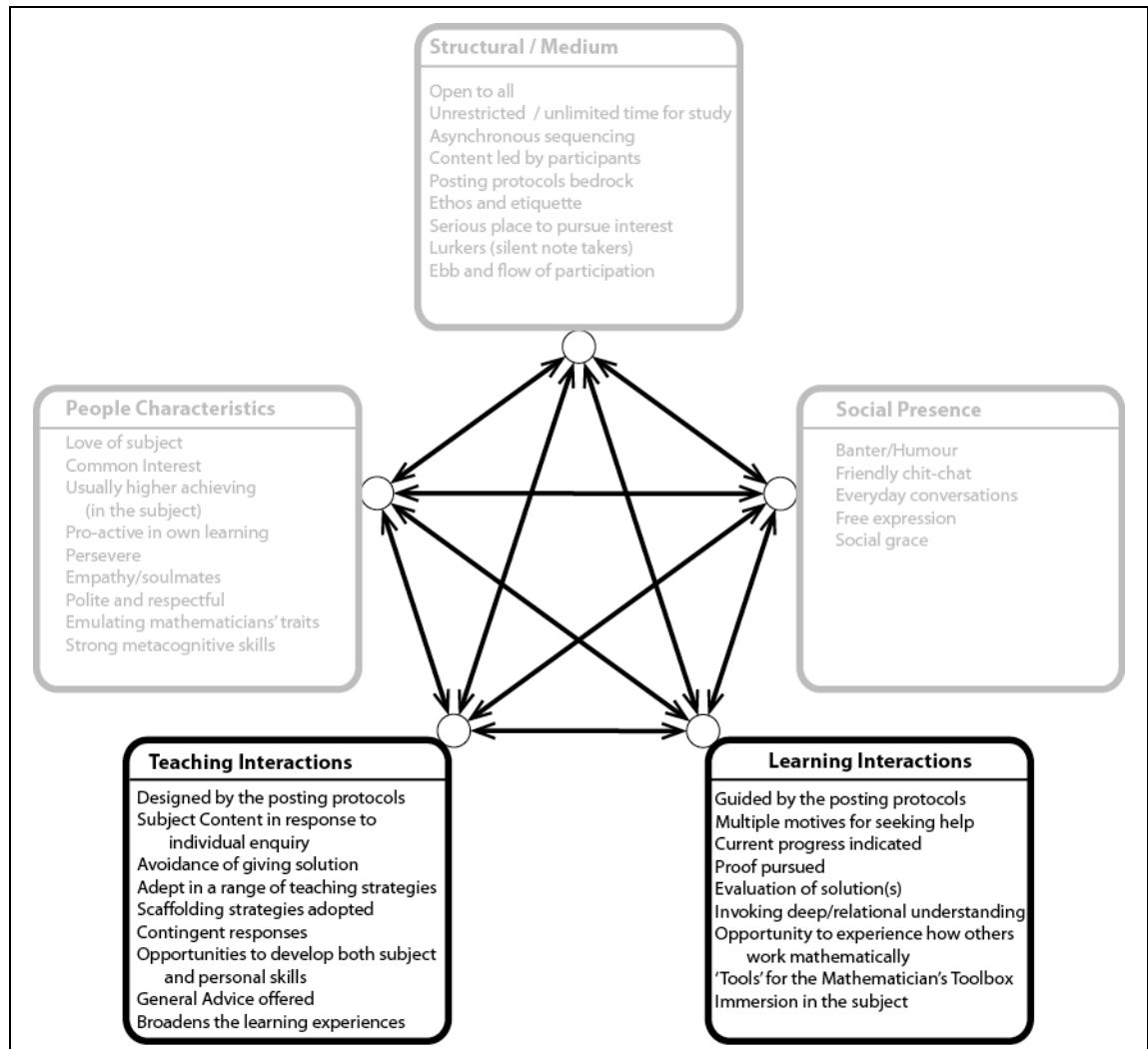


Figure 9.1 Features Catalogue: Teaching Interactions and Learning Interactions

9.6 Conclusions

This chapter has focussed on practices that are 'general' within AskNRICH using the Perspective of Two Exemplar Threads, although as the analysis reveals these practices might well be considered 'remarkable' in the world at large. An overview of the topic and content of the two threads has been presented, accompanied by the outcomes of applying open

coding to the interpretive commentaries constructed from the posts. The features resulting from the coding have been explained in detail under the four themes of: teaching, learning, social and temporal. The general practices exposed by this examination of findings are then discussed, in terms of teaching and learning interactions, in three sections: medium; conversational tone, and Socratic-Style Dialogue and Scaffolding.

The asynchronous nature of the web-board ensures freedom from the constraints of finite lesson time to pursue study outside-of-normal-hours. This enables an individual, within the home environment, both to persevere (and be supported) for an extended period of time and to pursue challenging problems. The nature of a web-board also inherently allows time for reflection at any stage before, during and after interactions. AskNRICH provides opportunities for encountering like-minded peers who, in that moment at least, live and breathe the subject. The consistently high-quality exchanges between equal peers, with evident absence of power relationships, are characterised by a conversational tone of respect and consideration, interspersed with a scattering of witty remarks. The Posting Protocols not only form the foundations for such well-mannered conduct but, in prescribing the way that help should be asked for and provided, foster a Socratic-Style Dialogue. The AskNRICHers', albeit untrained, pedagogical skills are shaped into implementing teaching strategies that scaffold others' proactive, reflective and receptive learning.

This chapter has presented a first Perspective on how young people are using the Internet in their proactive, independent pursuit of mathematical studies beyond the confines of the classroom. The next chapter continues the exploration of AskNRICH by tracking one representative participant as a case study over an eighteen-month intensive period of posting.

Afterword

The AskNRICHers' commitment to the ethos of the Socratic style of interaction and their self-moderation in maintaining the Posting Protocols and seemingly unconscious pedagogy is aptly encapsulated in the following post:

the purpose of posting on this board isn't to give you an opportunity to do whole questions and deny the poster the right to do it themselves. A response like mine, which perhaps nudges the poster into solving the problem, is probably more useful to them than a post telling them exactly how to solve the problem (from which they learn basically nothing). I don't like to nag, but in posting a response like the above you were inviting the (hopefully constructive) criticism

[Posted October 2008, co-incidentally the poster of ExThd2-P17]

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