Chapter Eleven

Playing the Game: Mathematicians' Interactions and Conversations

Perspective Three:

Three Threads on one problem

With many thanks to
NRICH, 'Peter'
and all other AskNRICHers

Doing Mathematics in Different Places: an Exploration of Young People's Activities as they make Independent Use of a Web-Based Discussion Board

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Each chapter has been edited to enable it, as far as is feasible, to 'standalone'.

The chapter numbers and numbering of sub-headings has been left unchanged from the original Thesis.

However, each edited chapter has its own page numbering and any cross-references *within* the chapters and *between* chapters on the NRICH website use these (new) page numbers followed by specifying the page number(s) in the original Thesis chapters.

Where appropriate, references may be given to other chapters (not included on the website) within the full Thesis, either by specifying the Section or providing the Thesis page number(s).

If in a chapter reference is made to any appendices, then the relevant appendix is attached at the end of that chapter.

Each chapter has its own list of references.

[The Thesis title, abstract and acknowledgement pages together with a table of contents for these edited chapters and glossary from the Thesis are also included. The table of contents of the full Thesis appears after Chapter Fifteen].

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Chapter Eleven

Playing the Game: Mathematicians' Interactions and Conversations Perspective Three: Three Threads on one problem

...[using AskRICH] it's great to be able to talk and discuss with other talented mathematicians - an opportunity which I don't really have at school. *[Web-Survey Respondent Male Year 12]*

11.1 Introduction

This chapter is the third of three Perspectives reporting the findings of interpretive analyses of a selection of message threads. The Perspective for this chapter is the analysis of three separate threads [**3Thds**] spread over a eleven-month period, but all discussing the same mathematical question. The first of the two analyses reported in this chapter differs from those reported in the previous two in that it uses a novel, visual technique to *explicitly* represent the network of connections between the thread participants and posts that exists in such a complex melée of interactions.

The purpose of this chapter is to:

i. demonstrate how the process of analysis of threads is facilitated and augmented[1] by the newly designed visual technique, a connection diagram
ii. show, by using both the visual connection diagrams and text-based interpretative commentaries on the threads, how the AskNRICHers' exchanges can be considered as conversation-for-education
iii. present a review of literature on what it is to work as a mathematician, however a mathematician might be defined [originally Thesis Section 2.8 pp53-56]
iv. present the results of further analysis of the threads that demonstrates that the AskNRICHers display traits that can be considered to be attributable to professional mathematicians' ways of working

The work reported in this chapter continues to address questions presented in previous chapters whilst specifically addressing two further questions: *What types of interactions are*

[1] Or even '(e)nriched'!

shown between the participants as they engage with mathematics? In what ways does the behaviour of AskNRICH participants emulate the working practices of professional mathematicians?

The theoretical underpinning of the reporting in this chapter of the AskNRICHers' interactions and conversations is based on van Lier's [1996: 167] work on contingent conversation-for-education. The literature on Mathematicians [presented in Chapter Two of the Thesis, but reproduced within this chapter as an additional section, Section 11.x] underpins the relation of traits occurring within AskNRICHers' exchanges to those of professional mathematicians in a social setting.

The next section of this chapter introduces the Three Threads. The remaining three sections follow the division used above in setting out the purpose of this chapter in discussing: (i) diagrammatic representations of interactions, (ii) conversation-for-education and (iii) & (iv) combined, mathematicians: people-who-do-mathematics.

11.2 The Three Threads

This section starts with a short introduction to the ideas that initiated the work and the rationale for using the Three Threads. It continues by presenting the mathematical problem that was the subject of the threads, the participants involved and an account of how the threads were analysed.

11.2.1 Background and Rationale

AskNRICH's full title, the **Ask-a-Mathematician** service, reflects the intention that there would be 'mathematicians' available to provide help. The web-survey response quoted at the opening of this chapter, together with my increasing awareness of the web-board environment and the mathematical culture in which I am situated, triggered the idea of considering the ways that AskNRICHers' behaviours emulate that of professional mathematicians.

When I came across the three threads, I was reminded of my own experience of talking with other colleagues around the table during ‘coffee times’ discussing a current mathematical problem of interest. The inference made here is that my colleagues and I, in talking about and doing mathematics in this way, consider ourselves mathematicians. Clearly, as the chapter’s opening quotation shows, in the same way (some) AskNRICHers, talk and discuss mathematics and also identify themselves as mathematicians. The contributions made in the three threads conjured up the vision of a set of people periodically dropping in and out during ‘coffee breaks’ to *talk about* and *do mathematics*, just as in the physical world people may gather round a table to do the same thing.

The three threads, posted under different titles though actually all on the same problem, involved twelve AskNRICHers, contributing 38 posts. The number of posts and the number of participants were of a manageable size providing a bounded situation for analysis. The presence of the two participants across all three threads suggested a further potential dimension of analysing the interactions, both within a single thread and across threads. Thus the AskNRICHers’ discussions have some inherent similarities to the intermittent coffee-table chats.

11.2.2 The Mathematical Problem

The mathematical problem [see Figure 11.1] comes from the 2005 British Mathematical Olympiad [BMO] paper and is in two parts: firstly proving a statement and secondly seeing if the statement would be true if considered ‘the other way round’.

> Let n be an integer greater than 6.
> Prove that if n - 1 and n + 1 are both prime,
> then $n^2(n^2 + 16)$ is divisible by 720.
> Is the converse true?

Figure 11.1 The 3Thds Question

The problem concerns a traditional piece of mathematics, but the work required is at a level of difficulty that it is most likely to be beyond that found in the classroom. That is, it is neither readily available nor common within the school environment although the AskNRICHers starting each thread are of school age as indeed are some of the helpers.

11.2.3 The Participants

The problem appeared in *Onwards and Upwards*, the appropriate section for the level of difficulty, each time initiated by younger[2] secondary aged pupils who were using the problem in preparation, and as practice for, a forthcoming BMO paper. School national examinations would not have questions that reach this level. Some respondents offering help had either used the problem as practice the previous year or indeed had taken the BMO paper in which it had first appeared. The first and third threads were started by participants aged 14 to 15; the second thread was started by someone a year younger. Twelve different AskNRICHers contributed, two across all three, all the others to only one of the threads.

11.2.4 Starting to Analyse the Threads

To start the analysis, a full prose narrative account was made of the three threads and two accompanying interpretative commentaries were made for each of the 38 posts, one on the mathematics and one on actions. For all three threads both the two interpretative commentaries and the prose account were used to derive the response types for each post or part of post. The post, poster, response type and a synopsis and/or comment on the interaction were then tabulated. This table was then augmented with the information on which participants could be linked to each interaction. The full table that provided the information from which connection diagrams of all three threads could be constructed appears in the appendix at the end of this chapter [Appendix 11.4].

11.3 Diagrammatic Representations of Interactions

This section explains the role of the diagrams and how they complement the text-based techniques to provide a clearer, more comprehensive and rich result of the analysis. It then presents in diagrammatic form, followed by a discussion, the interactions and response types of both the first thread and ***John***'s contributions to all three threads.

[2] Younger in the sense that the people who initiated the thread were in school years where the mathematics curriculum being studied would be at the level for asking questions in the (lower) *Please Explain* Section.

11.3.1 Purpose of the Connection Diagram

Text-based descriptions do not explicitly model the complex network of connections formed by posts that do not necessarily follow the same simple sequence as the chronological order of appearance on the web-board. It is, for example, quite common to see a new reaction resulting from a post further back in the sequence. Similarly, the types of interactions embodied in posts remain implicit within the text. [Thesis Chapter Six Section 6.3.4.5 pp129-131 describes the connection diagrams and typology of response devised in order to explicitly portray the jumbled, interwoven network of posts. An extract of Section 6.3.4.5 is presented as an appendix to this chapter [Appendix 11.x]]. However, the response type and connection information in the diagrams cannot convey the quality of the message text and thus the diagrams are *complementary* to the textual analysis. Nevertheless, this pictorial representation greatly facilitates and enhances analysis that demonstrates the presence of conversation-for-education [van Lier 1996: 167] by being able to explicitly see and thus gain a better grasp of the network of interactions.

Moreover, the connection diagrams aid consideration of AskNRICHers' mathematical traits. They enhance the comparison of traits with those of professional mathematicians by providing a more vivid image of the AskNRICHers as 'people with personalities' and hence traits, even though the evidence for the traits actually comes from the textual analysis.

However, it is important to recognise that the design of the connection diagrams is a simple offering for the three threads, adequate for the present purpose, to improve the depth and quality of analysis leading to the characterisation of AskNRICH. The finalised diagrams were not intended for any other purpose such as, for example, finding patterns in threads. They are a representation of my interpretation, devised after careful deliberation and closure, and naturally will always be open to scrutiny and re-interpretation by others.

11.3.1.1 Connection Diagram for Thread One

Table 11.1 below lists the posts, posters, message texts, response types and synopsis for **3Thd1** alongside the connection diagram. The response type column lists each response by its type and the posters involved (their names are abbreviated by omitting "Help").

Appendices 11.5 and 11.6, included at the end of this chapter, contain a fuller version of the thread with the addition of information on the interval between posts and the connection diagrams for each of the **3Thds** separately and for all **3Thds** combined, respectively.

The connection diagram in Table 11.1 shows 22 entwined responses, including examples of all five types, within a thread that involved only five AskNRICHers and just ten posts. Thus the complex structure of the connection network representing a complex melée of interaction is immediately clear.

The interactions whose response type is easiest to categorise are those that are a **Direct Response** [**DR**] as they always connect just two participants. Although, as signalled above, the diagram does not convey content quality, it is important to be mindful that many of these responses will contain the pedagogical exchanges that scaffold the learning [see Chapters Nine and Ten].

The nature of the web-board dictates that the first, trigger post, is open, sent out into the ether. However, the participants' common interest also engenders other posts addressing everyone in general. Both kinds of post are designated as **Open Response** [**OR**].

The other three response types are ways of recording instances of following/picking up on other posts. One of these three, **Picked Up Response** [**PUR**] records participants who pick up **OR**s, either from the trigger or other post(s). **My Response** [**MR**] has been used to record instances where a participant provides their own solution, to an idea (usually) presented to the person who has asked for help; for example in **3Thd1-P3** it is suggested to ***Peter*** that he may like to find a counterexample, which results in two other participants joining in, **3Thd1-P6&7**, with their own counterexamples. A **Follow On Response** [**FR**] records a participant who makes a direct reference to a post, having picked up on a suggestion from a different poster. For example, in **3Thd1-P9** ***HelpD*** joins in to correct ***HelpA***'s counterexample, **3Thd1-P6**, but this could only be done if ***HelpD*** was aware of the possibility of looking for a counterexample suggested by ***HelpB*** in **3Thd1-P4**.

Post	Poster	Message text	Response Type	Synopsis of interaction / comments
P1	***Peter***	*Let n be an integer greater than 6. Prove that if n - 1 and n + 1 are both prime, then $n^2(n^2+16)$ is divisible by 720. Is the converse true? i have managed to prove the first part of the question using the fact that all primes are of the form 6n-1 and 6n+1. when i tried to prove the converse i cant do it. i know that 2 and 3 divide n and n is of the form 2mod5 3mod5 or mod5. from here where do i go? thanks*	OR PU by A & B	Completed first part of question but cannot do second part in finding if converse is true
P2	***HelpA***	*Do you think the converse is true?*	DR to Peter	Suggests starting with an intuitive approach – 'feeling' whether it is true or not true
P3	***Peter***	*i presume that it isn't but im not very sure*	DR to A	Responds by saying that he assumes that it not true, but is not sure
P4	***HelpB***	*If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample.*	DR to Peter	Connects ***Peter***'s solution from the first part of the problem and suggests looking for a counterexample
P5	***Peter***	*thanks i ve got it now,* *for anyone who's interested one counter example is 48.*	DR to B OR PU by A & C	Has found, and shares, 48 as a counterexample
P6	***HelpA***	*or 24 ☺*	DR to Peter MR fr B4 OR PU by D	'Smugly' (via emoticon) suggests 24 would also do (in fact it does not)
P7	***HelpC***	*Or if you really want to do no work whatsoever when it comes to multiplication just use 720*	DR to Peter MR fr B4 OR	Gives the 'blindingly-obvious-once-someone-has-pointed-it-out' solution of 720
P8	***Peter***	*lol i totally missed that*	DR to C	Amused (lol - laughs out loud) at missing the obvious
P9	***HelpD***	*Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺*	FR to B4 DR to A	Politely suggests that 24 'does not quite cut' it as a counterexample
P10	***HelpA***	*Sorry haha, I was thinking that all numbers 0 (mod 6) worked. Good job i didn't make that mistake when I took the paper last year!*	DR to D OR	Laughs at own error and shares mistaken thoughts

Connection Diagram

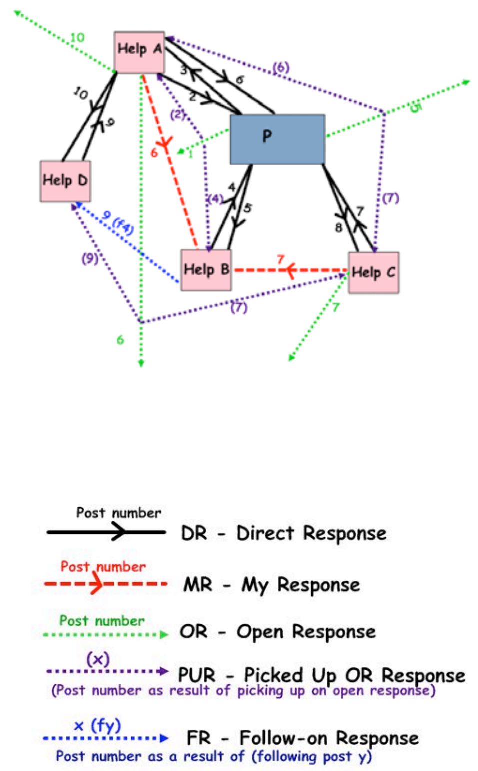

Figure 11.2 Connection Diagram for 3Thd1

Table 11.1 3Thd1: Posts with Response Type and Thread Connection Diagram

The following sub-section illustrates a different use of a connection diagram in considering the participation of one poster, ***John***, in multiple threads.

11.3.1.2 Connection Diagram of One Participant's Interactions Across Threads

Figure 11.3 [next page] illustrates the responses and interactions determined by the analysis of ***John***'s three contributions [**3Thds-P7**, **P21** and **P37** respectively]. ***John***'s three posts (in italicised text) are given in Table 11.2, together with a context and explanation of the mathematics or interaction.

Thread & Post Number & Date and Post Text		Comment
3Thd1-P7 Jan 2007	In reply to ***Peter***'s counterexample of 48 (strategy suggested by ***HelpB*** **3Thd1-P6**) and ***HelpA***'s (which turns out to be incorrect) 24 ☺ *Or if you really want to do no work whatsoever when it comes to multiplication just use 720*	When n=720, n-1=719 is prime but n+1=721 is not prime n^2 must automatically be divisible by 720 and thus the requirement that: $n^2(n^2 +16)$ is divisible by 720 is met 'without doing any work' This is a 'blindingly-obvious-once-someone-has-pointed-it-out' solution of 720 .
[3 minutes later ***Peter*** replied: *lol i totally missed that*: indicating his amusement at missing the obvious]		
3Thd2-P11 Mar 2007	Responding to request from ***R*** (the poster who has asked for help): I don't really understand this PETER. Sorry. Does anyone know a different way to give a hint/explain P's hint? (20 minutes later) *If n-1 and n+1 are prime, what are the possible remainders when you divide n by 5?*	[***R*** has returned the next evening to continue to try and solve the problem] This post offers a hint which connects with ***HelpE***'s [**3Thd2-P3**] questions from the previous evening to ***R***'s own reply [**3Thd2-P4**] to ***HelpE*** and ***Peter***'s [**3Thd2-P9**] afterthought of mod5
3Thd3-P11 Nov 2007	Responding to ***S*** (the poster who has asked for help) I've just realised that my counter example is exceedingly wrong as while 720 is divisible by 720, $720^2(720^2+16)$ isn't. (10 minutes later) quotes the above and replies: *Yes it is!* $720^2(720^2+16)/720 = 720(720^2+16)$	***HelpC*** should know that 720 is correct as it was they who suggest that it as a counterexample in thread one (which so amused ***Peter***)

Table 11.2 John's (HelpC's) Involvement across all Three Threads

John's interventions are seemingly considered contributions responding to others' posts and are potentially far reaching in their effect on others' learning. In **3Thd1**, ***John*** picks up on a suggestion offered by ***HelpB*** of looking for a counterexample. Following ***Peter***'s offer of 48 as one possible counterexample and ***HelpA***'s seemingly rather smug, though actually incorrect, response of 24 being even better as it is smaller, ***John*** comes in with his own solution of 720 and '*no work*' comment. This is delivered with a certain degree of humour and receives an equally humorous as well as admiring response from ***Peter***. Interestingly ***John*** did not correct ***HelpA***'s error of 24, even though the two posts were made five minutes apart. This is perhaps a fine illustration that not all can be ascertained from latent content!

In **3Thd2**, ***John***'s contribution, building on two earlier posts, attempts to move ***R*** on in their solution. Although ***John***'s post is in response to ***R*** asking if anyone else can help as they are not understanding ***Peter***, the remaining exchanges on the thread are all between ***R*** and ***Peter***. In **3Thd3**, ***John*** has taken no part until ***S*** suddenly returns to announce that their own solution of 720, which is of course precisely the same counterexample as ***John*** had so 'cleverly' made in the first thread, '*is exceedingly wrong*'. Again ***John***'s intervention is delivered with humour, reminiscent of a pantomime exchange. One can almost hear the laughter in ***S***'s reply, though of course this can only be a conjecture!

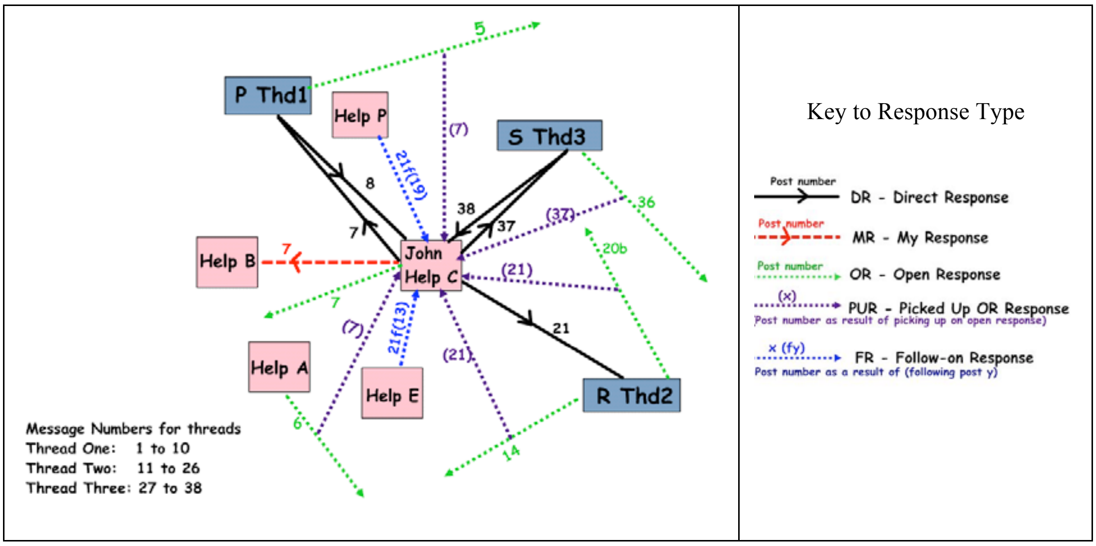

Figure 11.3 Responses and Interactions resulting from John's (HelpC's) contributions to the 3Thds

These excerpts involving ***John*** not only highlight the return in **3Thd3** of the 'clever' counterexample he offered in the first, but also convey the cyclical nature of the conversation that to me was 'typical' of how mathematicians might speak to each other 'round' a coffee-table. Co-incidentally and completely unrelatedly, the coffee-table is the same scenario that van Lier [1996: 168] uses in his discussions on conversation types, stating that what others might perceive as idle chat needs to be defended for the advantages it brings. Indeed the exchanges within the three threads, whilst not idle, do indeed demonstrate the usefulness of seemingly light-hearted chat.

The next section now uses the connection diagrams for all **3Thds** to analyse and relate the AskNRICHers' exchanges to Conversation-for-Education.

11.4 Conversation-for-Education

The 'lines' going in all directions, succinctly captured in the connection diagrams, shows visually the chaotic, in the sense of jumbled and interwoven, nature of the exchanges. Moreover, the need for five different response types and the requirement for a many-to-many, or multi-mapping between post and response type, is a further sign of complexity. In **3Thd1**, for example, seven out of the ten posts have been allocated to more than one response type [see Table 11.1 earlier]. There are several possible multi-mapping scenarios: different response types may be allocated as a result of the entire content of the post or even just one fragment of it, or several different (discrete) fragments of the post each results in the allocation of one or more response types. Table 11.3 illustrates four allocations given to ***HelpA***'s brief post [**3Thd1-P6**]: **MR** as it was made because of ***HelpB***'s suggestion of looking for a counterexample; **DR** responding directly to ***Peter***'s solution, **OR** left for others to read and indeed **PUR** as it is later picked up by ***HelpD*** (to tell ***HelpA*** that the example was incorrect).

Thread & Post Number and Post Text		Comment
Thd1 P6	***HelpA***: or 24 ☺. Including ☺ gives the inference that 24 is a better answer than ***Peter***'s 48 as 24 is smaller solution	My Response from P4, ***HelpB***'s suggestion that ***Peter*** looks for a counterexample Direct Response to ***Peter*** (who had given 48 as a counterexample) Open Response Picked Up later by ***HelpD*** who has spotted that 24 as a counterexample is incorrect as it does not satisfy divisibility by 720

Table 11.3 Example with One Post Assigned More Than One Response Type

In Table 11.4 ***Peter***'s response [**3Thd1-P5**], has two fragments: he thanks ***HelpB*** [**DR**], and then shares his solution which in this case in turn results in having different types allocated [**OR**] that is also picked up by [**PUR**] ***HelpA*** & ***HelpC***.

Thread & Post Number and Post Text		Comment
Thd1 P5	***Peter**: thanks i ve got it now* *for anyone who's interested one counter example is 48*	Direct Response to ***HelpB*** Open Response Picked Up by ***HelpA*** & ***HelpC***

Table 11.4 Example with Two-Part Post Assigned Multiple Response Types

Thus so far the connection diagrams provide evidence visually for the flexibility, unpredictability and symmetry of the exchanges that van Lier [1996: 175] states to be properties that define a conversation-for-education. The information necessary to construct the connection diagrams is extracted from the interpretive commentaries. This section continues to discuss conversation-for-education through a further iteration of analysing the AskNRICHers' exchanges using the connection diagrams and the interpretative commentaries together.

The conversations of two AskNRICHers (***Peter*** and ***John***) who contributed across all **3Thds** vary in format in the same way that 'ordinary' conversations may. ***John***'s three short responses are all in a helping role. In **3Thd1** ***Peter*** is in the role of learner, in **3Thds2&3** he is a helper. **3Thd2** ends up by ***Peter*** striking up an intense one-to-one conversation with ***R*** as ***Peter*** persists, even though ***R*** has asked generally if someone else can help as he apologetically explains that he is not understanding ***Peter***'s explanations. In addition to acknowledging ***R***'s predicament: *'since i don think that you understand modular arithmetic (dont worry about this) i shall write …'* [**3Thd2-P12**], ***Peter***'s subsequent teaching strategies [see Chapters Nine and Ten], respond to ***R***'s ideas and work, until ***R*** had '*Got it!'* [**3Thd2-P15**].

In **3Thd1-P5**, although ***Peter*** now has a solution to the problem, which could have ended the thread at **3Thd1-P4**, he openly shares it, thus continuing the conversation. Later, ***John***'s clever solution of 720 [**3Thd1-P7**] is instantly recognised in ***Peter***'s response: *'lol i totally missed that'* [**3Thd1-P8**]. This statement, although short, conveys laughter, humour, and admiration, qualities of a congenial atmosphere in which to engage with others.

Although ***John***'s responses might superficially appear minimal, suggesting 720 in **3Thd1-P7** and returning with it in **3Thd3-P11** provides insight for everyone as well as specifically contributing to ***Peter***'s and ***S***'s engagement with the problem. ***John***'s apparently sudden intervention at a crucial moment, picking up on the ideas and help of others, that he too could have contributed, conveys the sense of someone capable of giving cohesion to the conversations, further contributing to an amiable environment.

van Lier [1996] chose the word contingency to illustrate that, in free conversation, what is said is dependent upon what [others] have to say and what they chose to say is unpredictable; thus potentially there is an equal distribution of rights and duties. For the AskNRICHers, the content of the exchanges (the talk) is entirely in the hands of those AskNRICHers who choose to contribute *at that moment in time*. Thus the AskNRICHers initiate and manage their own conversations; each chooses what they wish to write, although they should do so within the bounds of the Posting Protocols. Even if the protocols remove complete freedom, they nevertheless facilitate conversation-for-education and promote pedagogical interactions of the type that are positioned *towards* the outer reaches of the contingency line on van Lier's diagram [1996: 179].

The analysis of the **3Thds** shows that the AskNRICHers' conversations are positioned towards the outer ends of the other six radii of van Lier's diagram: (1) If not all talk is truly conversational, it is at least dialogic as exemplified by ***Peter***'s and ***R***'s exchanges in **3Thd2**, and certainly not monologic. (2) There is always the potential for the talk to be substantially symmetrical rather than asymmetrical; in all three threads the person initially asking for help interacts as an equal with those offering the help. (3) Although the problems posted on AskNRICH, as in the case of the three threads, are often based on mathematics examinations or tests, the underlying ethos of being involved with the problems is always one of process-orientation with the emphasis on engagement and academic and personal growth. (4) The openness with which an AskNRICHer can show any difficulties and a helper checking whether an explanation is or is not being understood, ensure that the exchanges thrive on a proleptic understanding gap and (5) an exploratory teacher role. Again ***Peter***'s and ***R***'s exchanges in **3Thd2** provide an example of this. (6) Finally, by initiating and continuing to pursue posts to their own satisfaction, AskNRICHers determine their own actions. Thus, taken together with the examination of contingency in the previous paragraph it can be seen that the pedagogical interactions in the **3Thds** tend towards the least restricted type of van Lier's [1996: 194] four pedagogical interactions, designated as transformational.

Furthermore, similar arguments may be applied to the **ExThds** and many of the **CSThds** and thus much of the activity in AskNRICH provides / promotes opportunities to engage in transformational pedagogy.

From Figure 11.3 [p9 above] it can be seen that ***John***'s three contributions, all picking up on open responses, link with five different AskNRICHers. ***John***'s first response [**3Thd1-P7**] is unlikely to have been made if other AskNRICHers had not already been talking about counterexamples. The most serendipitous response of all was ***John*** in **3Thd3-P11** assuring ***S*** that they really were not 'exceedingly' wrong. It could not be predicted whether ***John*** would see ***S***'s remark or not, but that he did should have helped ***S***'s confidence. When ***S*** had realised their error, hopefully they did not mind others knowing that they had made a 'silly' mistake. They were after all in good company with ***HelpA*** of **3Thd1** who had originally thought that 24 was a counterexample. Such to and fro, 'free-fall', conversations constantly suggesting, correcting and debating are part of being a mathematician, the focus of the next two sections of this chapter.

The first of these two sections [Section 11.x] provides some useful context using Section 2.8 [Thesis Chapter 2 Literature Review I] of the thesis. The purpose of this review is to highlight, from a variety of literature resources, a number of traits associated with a person who may be regarded as a mathematician. The review deliberately avoids the difficult issue of attempting to define a mathematician, merely considering traits associated with a person who may be regarded as a mathematician.

11.x Mathematicians

> Of course, by mathematicians, we mean more than just the members of AMS [American Mathematical Society]; we mean the ***people who do mathematics***. Some mathematicians are children; some would never call themselves mathematicians. [Cuoco, Goldenberg & Mark 1996: 384 my emphasis]

A Google search for the exact phrase 'definition of a mathematician' yielded only 31 results and no serious, widely cited definition. It produced such 'gems' as: "[the person] you don't want to meet at a party" [Barrow 1992: 5] and "a blind man in a dark room looking for a black cat which isn't there" attributed to Charles Darwin [various sources including Nahin

2006: 352] and thus no helpful results to reproduce here. Dictionary definitions such as, for example, "an expert in or student of mathematics" [OED online nd] are similarly unhelpful.

Phillips found that asking both children and adults to describe a mathematician elicited a myriad of simple descriptions [Upitis, Phillips & Higginson 1997]. The list of responses (my ordering) that were routinely given are as follows: "someone who is good at maths; not a person who just does computations; someone who likes to solve problems; a person who studies patterns of numbers; a person who see maths in everything; a person who forms hypotheses and works on answers using numbers; a very logical, ordered person; a person who uses the creative strategies to solve the logic problems; a person who forms hypotheses and works on answers using numbers; a very logical, ordered person and a person who uses the creative strategies to solve the logic problems" [p139] as well as: "someone who works with numbers; a genius or a brain; like an accountant or a banker and a person who teaches maths at university" [ibid]. I have re-ordered the list so that the first eight are descriptors that can also directly apply to school pupils[3].

Although these descriptors are not overly informative, some of these simple descriptions, such as "a person who studies patterns" recur in other literature examined later. However, Phillips, who refers to herself as a primary school teacher, researcher, and mathematician, goes on to develop her own definition:

> My belief is that a mathematician is someone with a special way of viewing his or her world … has the ability to stop and enjoy mathematical beauty … can generalise from specifics and who can generate specifics from generalisations … can create a multitude of strategies to solve a problem … Hence, professional life is an open-ended adventure for a mathematician. [p140]

In addition to substantiating the views expressed in the literature reviewed in previous sections of Thesis Chapter Two, Phillips' definition concurs with Burton's work [1999] in which the world of professional (paid to undertake research) mathematicians is described as: "a world[4] of uncertainties and explorations and the feelings of excitement, frustration and satisfaction associated with these journeys, but above all a world of connections, relationships and linkages" [p138]. In other work Burton [1995, 2004] identified the

[3] The remaining four may well be present amongst the AskNRICHers either now or in the future!

[4] Peter's opening quotation in Chapter Ten illustrates just such a world.

aesthetic feeling gained from doing mathematics and the nurturing of intuition and insight as important parts of the process of coming to know mathematics.

Whereas Burton's focus is on Professional Mathematicians, the ultimate goal of Cuoco et al. [1996], a group of academic mathematics educators, was developing a school mathematics curriculum. The quotation that appears at the start of this section is a footnote in their paper discussing the "habits of mind" of people who create mathematics. The authors highlight the gap between a subject that is studied in school called mathematics, and the way that mathematics is created and applied outside of the school environment [ibid: 375]. School mathematics is disjointed in that it tends to consist of a set of discrete skills studied independently, as illustrated by the original analogy of the apocryphal footballer who learns to dribble but never plays the game, attributed to Wiggins [cited in Greeno, Collins & Resnick 1996] or the woodworker who practices the joints but never makes a [coffee] table [Schoenfeld 1989].

Cuoco et al.'s [1996] proposal intended to help pupils to learn and adopt some of the ways that "mathematicians *think* about problems" [p376] … "mental habits that allow students to develop a repertoire of general heuristics and approaches that can be applied in many different situations" [p378]. Habits of mind that students should acquire are those of: *Pattern Sniffers, Experimenters, Describers, Tinkerers, Inventors, Visualisers, Conjecturers and Guessers* [pp378-384]. Although each habit is illustrated by a mathematical example, the authors claim that these habits are generic. From this they progress to listing habits that are not so common outside of mathematics, i.e. ways that mathematicians approach things: *Talk Big & Think Smal*l (i.e. consider special cases), *Talk Small & Think Big* (simple problems can give rise to deep and/or general theories), *Use Functions* (studying the mechanism of change), *Use Multiple Points of View, Mix Deduction and Experiment, Push the Language, Use Intellectual Chants* (meditating or reflecting on ruminations that occur when working away at a problem)[5]. There are some striking similarities between this list and Phillips' prose account of what she feels is being a mathematician, for example Phillips' 'generalise / specifics …' [Upitis et al. 1997: 140] parallels 'Talk / Think Big / Small …' here.

[5] Not content to stop here, Cuoco et al. assemble further lists for a Geometers' special stash of tricks of the trade [p389] and a special collection of habits of mind for algebraists - people in 'algebra mode' [p393].

In the preface of *'How to Think Like a Mathematician'* Houston [2009], a university lecturer in mathematics[6], presents some friendly, practical advice [px] aimed at undergraduates on 'doing' and 'working' on mathematics[7]: "It's up to you; Be active; Think for yourself; Question everything; Observe; Prepare to be wrong; Develop your intuition; Collaborate and Reflect".

The motivation for publication Houston gives is similar to that of Cuoco et al. in wishing students to experience exciting mathematics: "I aim to make [students] free to explore, give them the tools to climb the mountains and give them their own compasses so that they can explore other mathematical lands" [ibid: xi]. Houston too recognises the limitations of school mathematics in not promoting such habits of mind. Although expressed in a different way to Cuoco et al.'s listing of habits of mind, Houston's friendly advice is suggestive of how to acquire such habits. Houston's inclusion of "intuition" makes an immediate connection to Burton's work above. Equally the advantageous reasons that he gives for "prepare to be wrong" reflect Burton's difficult but rewarding journey.

Indeed, the situations and people perspectives in /from which the authors present their arguments are all different: Phillips (general public and teacher); Cuoco et al. (curriculum development); Burton (professional research mathematicians); and Houston (teaching undergraduates mathematics). Nevertheless, they all portray traits which form overlapping and consistent pieces of an indistinct, incomplete picture[8] of that imprecisely defined creature: a mathematician.

11.5 AskNRICHers Emulating Professional Mathematicians?

> When discussing the process of mathematical discovery, mathematicians now openly acknowledge making illogical leaps in arguments, wandering down blind alleys or around in circles and formulating guesses based on analogy or on examples that are hidden in the later, formalized exposition of their work. [Epp 1994: 257]

[6] This makes Houston a mathematician according to one criteria on Phillip's list and a candidate for Burton's study.
[7] Each piece of advice is accompanied with reason and explanation – including "But don't trust [Intuition] completely".
[8] one might say Turneresque.

This section focuses on the second of the two questions presented in this chapter's introduction: *examining ways in which the behaviour of the AskNRICHers emulates the working practices of professional mathematicians*. This is addressed by a revisiting and further analysis of the threads to draw out instances that demonstrate behaviours and activities, hereafter combined and labelled traits, attributable to the way that mathematicians work [presented in the previous section] as exemplified in the quotation from Epp above.

Although AskNRICHers post from, and in, an isolated location they are also working together within a virtual environment. This 'working together' led to the vision described at the beginning of this chapter of people metaphorically sitting round[9] a coffee-table talking about and/or doing mathematics. The contention is that this is little different from the way that professional mathematicians work round a real table during coffee breaks, a well-recognised tradition as illustrated in Bollobás' [2006] book: *'The Art of Mathematics: Coffee Time in Memphis'*.

The coffee-table scenario was examined using the content of the three threads.
Section 11.2.4 has already reported on how the initial intensive analysis of the three threads, to draw connection diagrams and to establish the presence of conversation-for-education [van Lier 1996: 167] was carried out. When undertaking the further analysis reported below, a reflective standpoint [Brown 2001] was adopted for revisiting the interpretations made of the posts, further informed both by the literature presented in the previous section and my "tacit knowledge … knowing-in-action" [Schön 1983: 49] of what it means (to me) to be a mathematician.

In order to report the findings I compiled a list of traits that, as Cuoco et al. [1996] stated about their collection of habits, is neither exhaustive (mathematicians may well do more than this) nor exclusive (other people may well do some or all of these). Nonetheless all the traits have been present during my own 'doing' and 'talking about' mathematics with colleagues. The traits presented below are partitioned into three sections to reflect those that can be considered as: (a) techniques for the mathematician's toolbox, (b) customary occurrences when engaged in mathematics and (c) social graces embedded within the

[9] 'Round' seemed appropriate as John's suggestion of 720 in **3Thd1** came 'full circle' in **3Thd3**.

exchanges. The last of these adds more detail to the similar features first discussed in Chapter Nine under the Social and Personal theme [Section 9.3.3 Chapter Nine p17/Thesis p200]. Some of the traits in the list and others may have already been observed in the work reported in the previous two chapters, but here the traits reported spring only from the content of the **3Thds**' 38 posts now analysed for evidence of the coffee-table analogy.

For simplicity of presentation, the findings appear *solely* in a tabular form with the discussion element of this part of the chapter integrated into the table. Thus it is necessary for the reader to follow the content of the tables. In the first two tables each trait has an entry that consists of:

- illustrative example(s) extracted from the posts
- example(s) in the literature reporting the same trait
- discussion of the trait and/or the part it plays in emulating mathematicians ways of working

The final table builds on the work of Chapter Nine and thus contains only illustrative example(s) and discussion.

The metaphor of a *toolbox* containing mathematical tools, also sometimes labelled an arsenal of techniques [Houston 2009], is in widespread use [Black Douglas nd, Wolf 1998] and commonly understood. The AskNRICHers use this metaphor, for example [**ExThd2-P5** Chapter Nine]: *'always look to improve your problem solving 'toolkit' and to add more tools to it'*. Schoenfeld [2006: 500] draws a similar analogy between his passion for cooking, necessitating the gathering together of a range of implements, with his other passion of solving mathematical problems. Table 11.5 describes three strategy items that can be placed in the toolbox.

11.5.1 Techniques for the Mathematician's Toolbox

Findings: Techniques for the Mathematician's Toolbox
Trait 1: Follow Hunches/Intuition
Example The first response to ***Peter***'s request for help: *'Do you think the converse is true?'* [**3Thd1-P2**] is inviting ***Peter*** to follow a hunch/intuition. **Literature** *Develop your intuition – But don't trust it completely* [Houston 2009: x]. Intuition is one of five categories in the epistemological model for coming to knowing mathematics [Burton 1999, 2004]. 83% of Burton's study group supported the notion of intuition or insight. One of five common characteristics of Mathematical Creativity [Sriraman 2008: 1]. **Discussion** Intuition is advantageous when the hunch is correct, less so when not. Nevertheless, there is often an inexplicable and unquantifiable feeling that one is on the right track whilst working on a mathematical problem. Even if it is difficult to quantify intuition the feeling for something to work mathematically comes from making an educated guess (with the presumption that it may be wrong). However ***Peter***'s reply *'i presume that it isn't but im not very sure'* [**3Thd1-P3**] indicates: (i) a still tentative feeling and not yet prepared to follow one's hunch and (ii) a possible interpretation by ***Peter*** that the wording of the problem suggests the converse not being true. If the converse were true it would be natural as a question setter perhaps to simply ask for a proof, or in fact most likely to set the whole question up as an 'if and only if'.
Trait 2: First search for Counterexamples
Example ***HelpB***: '*... you can construct a counterexample'* [**3Thd1-P4**] coincides well with a hunch that the converse is not true. **Literature** *...where the true mathematician has a chance to shine. ... given any statement try to find a counterexample* [Houston 2009: 93]. Promotion of using counterexamples in proof strategies [Stylianides nd; Stylianides & Stylianides 2009; Zazkis & Chernoff 2008]. Students' views on rigour of counterexamples [Simon & Blume 1996]. **Discussion** Looking for a counterexample is a common starting point in proving something is not true. If a counterexample can be found it is a quick and succinct way of completing a proof. If a counterexample cannot be quickly found, relatively speaking, generally there is a move to alternative, more complex methods but this decision is not taken lightly.
Trait 3: Always consider special cases (especially involving zero)
Example ***HelpG****: 'One small thing you've missed - n can be 0 mod 5, but that gives you* n^2 *is 0 mod 5 so you're still fine'* [**3Thd2-P6**]. **Literature** The translation to special cases is almost automatic (Talk Big & Think Small) [Cuoco et al. 1996: 384]. **Discussion** Part of a mathematician's toolbox is to ensure that any special case is carefully considered. The number zero can be problematic, for example division by zero is undefined. It either disappears for example when 'or mod5' should strictly be '0mod5' [**3Thd1-P1**] though admittedly no-one picked up on this, or is omitted, not especially considered in a proof [**3Thd3-P5**] which was picked up in ***HelpG***'s response above.

Table 11.5 Examples from the 3Thds of Strategies for the Mathematician's Toolbox

11.5.2 Customary Occurrences when Engaged in Mathematics

When mathematicians are working on problems, either individually or as a group, there are instances that can commonly occur attributable to either people and/or the situation. Opportunities to experience such traits arise through being active, observing and collaborating as advised by Houston [2009], taking part in the mathematical journey [Burton 2004]. Table 11.6 details seven such traits that mathematicians have or things that they do that can be seen in the three threads. Having these traits is not exclusive to mathematicians, as already stated, but mathematicians do have these traits and the AskNRICHers are working on mathematical problems.

Table 11.6 Examples from the 3Thds of Customary Occurrences when 'Doing Mathematics'

Findings: Customary Occurrences when Engaged in Mathematics
Trait 4: Thinking Out Loud
Examples ***R***: *'Is there a way to narrow it down further? Or can m have multiple values?* [**3Thd2-P13**] is followed nine minutes later by: *'Assuming m can have multiple values … Yes, so n can be expressed as…'* [**3Thd2-P14**]. **Literature** The habit of noticing something and wondering why [Cuoco et al. 1996: 387]. *Talking is a good way of getting things done* [Burton 2004: 130]. **Discussion** ***R*** raises question(s) and before any reply arrived proceeded to answer them in the style of a rhetorical question. Just asking a question (saying it out loud) can provide the way forward, especially if it is likely that someone can answer it for you if necessary.
Trait 5: Amusing Howlers – Glaring Errors
Examples ***HelpA***: or 24 ☺ [**3Thd1-P6**]. ***S***: *'I've just realised that my counterexample is exceedingly wrong'* [**3Thd3-P10**]. **Literature** *Don't worry about being wrong* [Houston 2009: x]. **Discussion** On the contrary 720 is **not** exceedingly wrong [**3Thd3-P10**], neither is 24 a counterexample [**3Thd1-P6**], huge but simple mistakes to make (howlers). We all make them, and we neither mind making them nor mind seeing others do something silly. Maybe mathematicians have the confidence not to mind but life would be duller if we did not have howlers sometimes.

Findings: Customary Occurrences when Engaged in Mathematics

Trait 6: I wish I had thought of that!

Examples

HelpC: *'Or if you really want to do no work whatsoever when it comes to multiplication just use 720'* [**3Thd1-P7**].

Literature

Collaboration – work with others if you can [Houston 2009: x], *benefit from experience of others* [Burton 2004: 130].

Discussion

The obvious only becomes obvious when it has been made so by someone else or a subsequent thought. 720 is itself a comparatively high number but given that statement is already divisible by 720 then divisibility is assured. (n+1 = 721 is divisible by 3 so not prime). My reaction was similar to ***Peter***'s *'lol i totally missed that'* [**3Thd1-P8**] when one realises the 'of course' nature of the value given. There is always admiration and respect when a colleague comes up with the obvious when nobody else has considered it yet.

Simplicity is the ultimate sophistication [Leonardo da Vinci].

Trait 7: Scribbled working

Examples

See ***S***: *'This is what I've done …'* [**3Thd3-P4**].

Literature

… develop the habit of writing down thoughts … [Cuoco 1996: 379].

useful to formulate written and oral descriptions [Cuoco et al. 1996: 379].

See also quotation by Epp [1994] cited at the beginning of this section.

Discussion

To communicate in writing via the web-board, the level of detail in any explanation or working out will be relatively high when compared to how people might physically sit close together and scribble on the back of an envelope. Nevertheless informal working out definitely making sense to the writer will probably make sense to another mathematician. This is very different from the expectation that many teachers have in a classroom, often a consequence of the emphasis on method marks in an examination. ***S***'s explanation is succinct and clear, even if some working is lost between the lines e.g. ***S*** fails to include the step that as n^2 is divisible by 3^2 then $16n^2$ is divisible by 3^2 which is (strictly) needed to state therefore and indeed some parts omitted (the case of zero in Table 11.5).

Trait 8: Having Afterthoughts

Examples

Peter: *'sorry ive realised you can write this as mod5'* [**3Thd2-P9**] one minute after posting an explanation using multiples of 6 and mod 30 (probably connected/considered by 5x6=30).

S: *'I've just realised that my counterexample is exceedingly wrong'* [**3Thd3-P10**].

Literature

Any definition of a mathematician should probably include the attribute (or defect) of not being able to leave well alone [Brakes 1995: 388].

See account of working on a handshake problem [Rowland 2003].

Discussion

Problems can 'nag' away in one's mind even after proposing a solution, often subconsciously one is looking for a 'better' solution, whatever better might mean in this context. ***S***'s afterthought indicates that having one is not necessarily fortuitous (as 720 is actually correct)!

Findings: Customary Occurrences when Engaged in Mathematics
Trait 9: A Moveable End Point to any Finished Solution
Examples ***Peter***: *'thanks i ve got it now, for anyone who's interested one counter example is 48'* [**3Thd1-P5**] is not the end of the thread with other offers arriving. In **3Thd3** two participants (***ANP1*** & ***ANP2***) who add points beyond the immediate problem (that has been solved). This opens up conversation [see LRIII p147] and has the potential to add further knowledge: *'If the converse were true, then it would be a really, really fast way to find big prime numbers!' 'Not to mention being a proof of the twin prime conjecture!'*[**3Thd3-P8&9**]. **Literature** This is a variation on 'not leaving alone' [Brakes 1995] above, but the distinction between this and 'afterthoughts' is that here other people are the ones who are returning to the problem. This is similar to some of the advantages of collaborating given by Burton's [2004] mathematicians: e.g. increase in quality and quantity of ideas, get into areas that one may not have thought of. **Discussion** ***Peter***'s post [**3Thd1-P5**] suggests the problem is concluded but others remain fixed on the problem finding their own examples. (Even though ***Peter***'s post above indicates that he is satisfied he still looks back at the thread and comments back to ***HelpC***). Fortuitously in minimising incorrect solutions remaining as errors, ***HelpD***'s posts some six hours later pointing out that 24 is not a counterexample and again ***HelpA*** responds. In **3Thd3** ***S***, having found the required solution with satisfaction, like ***Peter*** returns an hour later with the comment: *'By the way, for it to be a proof of the twin prime conjecture…'* [**3Thd3-P10**] although whether this had had any contribution to the howler mentioned above remains open to question. When a solution is found to a problem it is not necessarily the end point. Someone might at any time (immediately or much later) return to it and start another conversation. This might be even more prevalent within a virtual environment as new posts are flagged as such, and thus potential intrigue can draw the correspondents (and lurkers) back. Using the two examples that illustrate the on-going nature of a solved problem, the three participants (***ANP1***, ***ANP2*** & ***HelpD***) who extend the conversation, though regular posters, had not contributed to the thread before the solution had been shared. In this sense they 'lurked' within these threads.
Trait10: Tutoring on Unfamiliar Territory
Examples ***Peter***: *'since i don think that you understand modular arithmetic (dont worry about this) i shall write in a basic algerbraic form'* [**3Thd2-P12**]. **Discussion** There can be situations where the mathematics needed to solve a problem is new or unfamiliar to a participant in the discussion when someone in the group may resort to a detailed explanation or even some direct teaching. ***Peter***, after trying with some minimal hints, decided to introduce ***R*** to modulo arithmetic using a fairly didactic manner. The explanation given eventually became sufficient for ***R*** to complete the solution successfully. Some 'coffee-table' discussions between professional mathematicians would include explanations, presented to colleagues/peers in a similar way as ***Peter*** did here, seeking to explain in a way that ***R*** would find understanding, rather than just telling.

11.5.3 Social Graces Embedded within Exchanges

A collegiate group like that proposed of mathematics 'people' sitting round a table during a coffee break is likely to include some social and personal exchanges that contribute to a friendly atmosphere. Similar exchanges can be seen within the **3Thds**, see Table 11.7 below: a sense of camaraderie appears, evident through a combination of banter, humour, admiration, praise, politeness, success accompanied by exhilaration and personal asides. The headings in Table 11.7 evolved from the intensive analysis described earlier in this chapter and provides further findings that relate to the social graces of the delivery of the responses.

Findings: Social Graces Embedded within Exchanges
Trait 11: Banter/Humour
Examples ***HelpA***: or 24 ☺ (seemingly smug?). ***HelpC***: Or if you really want to do no work whatsoever when it comes to multiplication just use 720 ***Peter***: lol i totally missed that [**3Thd1-P5-7**]. 'Pantomime' like response by ***HelpC***: *'Yes it is!* to ***S***'s *'while 720 is divisible by 720,* $720^2(720^2+16)$ *isn't'* [**3Thd3-P10&11**]. **Discussion** See Section 9.3.3 p17/ Thesis p200 for distinction made between banter, light-hearted teasing as in ***HelpA***'s comment and humour, the genuine neutral witty remark, interpreted as present in the other examples above.
Trait 12: Politeness
Example **R**: *'I don't really understand this PETER. Sorry. Does anyone know a different way to give a hint/explain Peter's hint?'* [**3Thd2-P10**]. **Discussion** Although the posting protocols ask for politeness, the comment above is indicative of the sensitivity that people show to each other when realising that someone is genuinely trying to help but is not succeeding.
Trait 13: Admiration
Example [See 'blindingly-obvious' remark in Table 11.1 above]. ***Peter***: *'lol i totally missed that'* [**3Thd1-P7**] in response to the simplicity of choosing 720. **Discussion** Suggestion of admiration made more explicit by using the shorthand text for laughing-out-loud
Trait 14: Pleasure at Success
Example ***R***: *'Got it!'* [**3Thd2-P15**]. **Discussion** Image of ***R*** jumping off their chair, punching the air and so pleased that at last and after a struggle the problem was solved.
Trait 15: Praise
Example ***Peter***: *'yes well done this completes the proof. ... yes anyway. well done'* [**3Thd2-P16**]. **Discussion** A justly apt congratulatory post, implicitly recognising the work that ***R*** has put in. (It is always good to have one's endeavours praised and the additional recognition of some hard work finely accomplished).
Trait 16: Personal Comments
Examples ***Peter***: *'i remember fondly this question. this was my first bmo question i completed. arrr memories ...'* [**3Thd2-P16**]. ***Peter*** (at 7.38pm) *'I think that there is a nicer way but this is still nice and simple and im tired at the moment'* [**3Thd2-P8**]. **Discussion** The first is a personal reminiscence of fond memories that offers a sociable 'joining-the-club' feeling. In keeping with many of the other threads analysed, there is a personal comment at the end of each of these three threads, though in the case of the first and the last it is in response to having made a 'howler'.

Table 11.7 Examples from the 3Thds of Social Graces Embedded within Exchanges

Although Houston [2009: x] makes it clear in his list of advice that there is no competition in collaborating, Burton's [2004] research suggests that professional mathematicians can still experience competition even within a collaboration and/or co-operation situation [pp131-134]. Any competitive element or one-up-man-ship in the posts has, after some consideration, not been included as a feature since it is to some extent problematical to determine where it appears. Having read a very large number of posts, including the web-board's private area, I have concluded that the care that participants show for each other is much more explicit than any attempt to be competitive. Moreover, there appears a respect for those who are the most talented (or rather, score highly in any competitions) and a great deal of compassion for the many who actually score few marks[10].

This section has examined ways in which the AskNRICHers working together could be considered to emulate the working practices of professional mathematicians in a social setting. The findings have supported the argument, alluded to in Section 10.3.2.2 [Chapter Ten p11/Thesis p223], that because the AskNRICHers are immersed in a rich mathematical environment, engaging with others who are enthusiastic about the subject and experiencing their mathematics, they will themselves engage in activities in ways similar to professionals in the field.

11.6 Features Summary 4

The Features Catalogue [a concept explained in Section 8.6 Chapter Eight pp16-17/Thesis pp179-180] for this chapter, relating to Social Presence, is presented in Figure 11.4 below.

The term 'Social Presence' was used as the identifier for this Catalogue as the Features listed in Figure 11.4 are strongly similar to some that Garrison and Anderson [2003: 51] classified[11] as social presence. The term Social Grace was adopted to portray the camaraderie in a social setting conveyed in the proposal of mathematicians sitting round a coffee-table. The banter, humour, friendly 'chit-chat', often accompanied with a pe ering of emoticons and texting abbreviations are liberally sprinkled throughout the threads is part and parcel of everyday conversations taking place within an environment that allows free expression. These qualities are pervasive throughout AskNRICH and provide a major part of the cohesion that binds the AskNRICHers together.

[10] In the 1989 Putnam competition in the USA the median score was 0 out of a possible 120 which was not unprecedented [Larson 1994: 33].

[11] A classification that used the indicators of expression of emotions, use of humour and self-disclosure for the Affective category and vocatives, inclusive pronouns and phatics and salutations for the Cohesive category. The collaborative model underpinning of the Open Communication category is incompatible for AskNRICH.

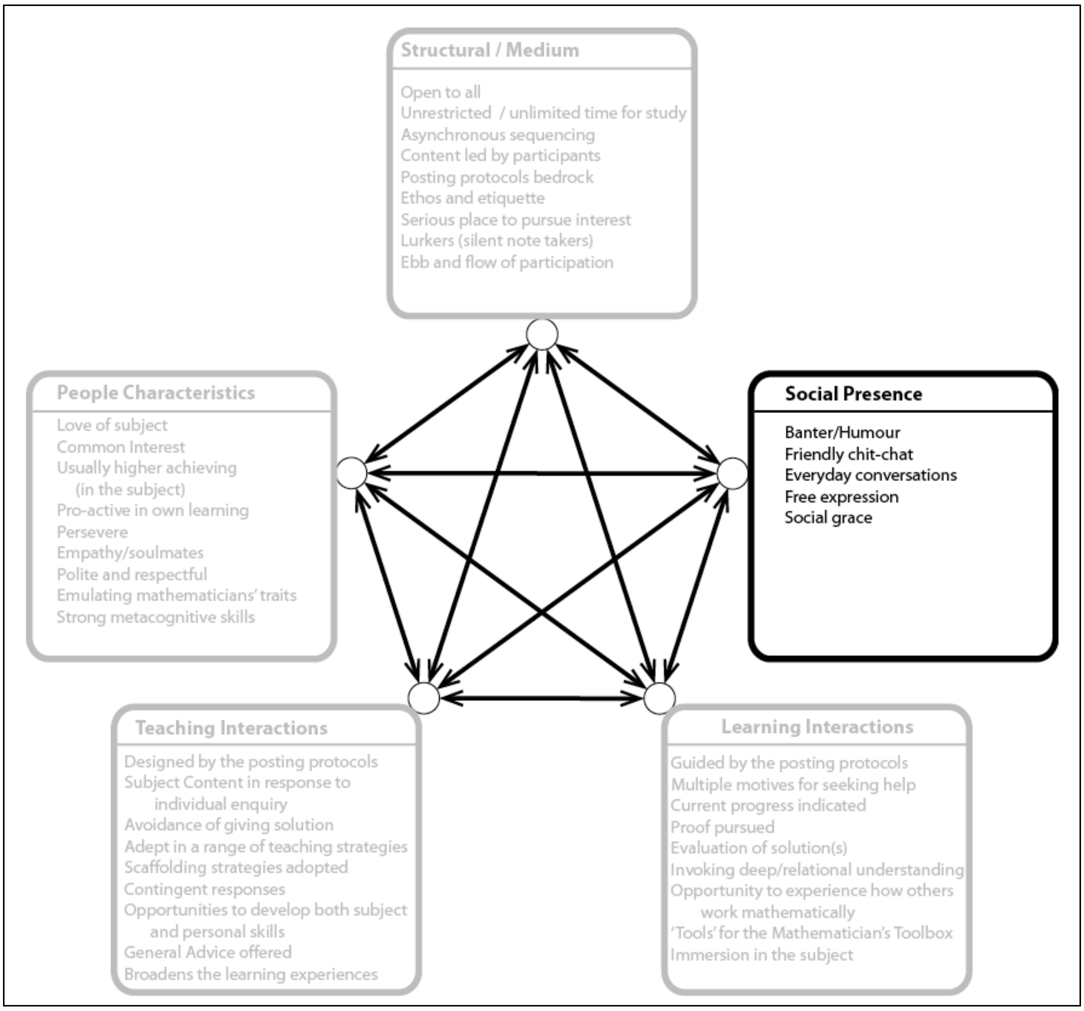

Figure 11.4 Features Catalogue: Social Presence

11.7 Conclusions

This focus of this chapter has been on the mathematical activities, interactions and exchanges within three specific threads all on the same problem that had appeared over a eleven-month period on AskNRICH.

The visual mapping of the Three Threads confirmed that the complexity of the network of interactions within the AskNRICHers' conversations was well beyond that of simple turn-taking. The jumbled and interwoven nature of the exchanges made clear by the visual mapping is used to argue that these are contingent conversations which thus met the criteria to be considered 'conversation-for-education' [van Lier 1996: 175]. Further consideration of the threads in relation to individual components of contingency leads to the claim that the type of exchanges also tend towards the most free of pedagogical interaction that is called transformational [van Lier 1996: 180].

Further analysis of the Three Threads made against perceived ideas of what it might mean to be a mathematician (prefaced by a short literature review outlining a range of possibilities) resulted in establishing a number of 'Traits', a collective label for behaviours and activities. These traits each belonged to one of three groupings. One group brings together the techniques that an individual can add to their own (mathematician's) tool-box, comprising of a set of traits made possible where there is a mixture of the experienced and less experienced, expert and (relative) novice. The other two groups emerged as more focused on the person: customary occurrences when engaged in mathematics and social graces embedded within exchanges that help to maintain the desire to meet. The findings show that the AskNRICHers' work, activities, interactions and exchanges emulate those of professional mathematicians in the coffee-table analogy. Moreover, the camaraderie revealed by the investigation of that analogy combined with the to-and-fro, free-fall conversations highlights the Social Presence amongst the AskNRICHers.

This chapter has reported the third and final Perspective used to explore AskNRICH. Its findings are combined with those from the other two Perspectives in a wholistic view presented in the next 'Interlude' chapter.

Postscript

> I became a mathematician by falling in love with mathematics
> [Papert 2006: 581]

And as another very famous mathematician, Erdös, was apparently fond of saying:

> A mathematician is a machine for turning coffee into theorems
> [Hoffmann 1998: 7]

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van Lier L. (1996). Interaction in the Language Curriculum: awareness, Autonomy & Authenticity. Harlow: Pearson Education.

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6.3.4.5 Forming a Diagrammatic Representation of Interactions and Connections

Threads capture every word that has, with deliberation, been written down and thus represent the totality of the AskNRICHers' talk and conversations. The web-board's linear sequencing of posts, determined solely by the time of the posting, hides the complex nature of the network of interactions. The interpretive commentary accompanying each post recognises the intricacies of AskNRICHers engaging with each other by considering a post in relation to the whole thread. Nevertheless, the analysis of the Three Threads Perspective highlighted the potential benefits of a complementary investigation using a simple visualisation of the interactions and connections between all participants and posts, both within and across the threads.

Although visual mappings are not a new concept, the purpose, rationale and implementation of the diagrammatic representation needed for this study are different to those of previously reported studies [Thesis Chapter Five Section 5.5 p98]. The unique nature of AskNRICH fosters forms of free exchanges whose subtleties and complexities, not seen in other studies, need to be accommodated and clearly, explicitly, visible.

A type of visual mapping, a **connection diagram,** and an associated **typology of responses** were devised for this study. These diagrams portray interactions between participants in terms of the five response types explained and illustrated in Table 6.7 [next page]. The response typology was derived, initially using the three threads, through considering: the chronological order of the individual posts; the immediate interaction that resulted from a post, and the overall actions evident within the thread. As an example, the connection diagram portraying the first of the three threads is shown in Figure 6.3 below. The labelled rectangular boxes indicate the individual participants, the lines represent responses and are colour and line-style coded to show their type. Alongside each line is a number indicating the post that formed the response [Section 11.4 p10/Thesis p253 explains in detail the multi-mapping between post and response type].

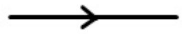
Type of Response	Description	Example of Response [Taken from **3Thd1** used in Chapter Eleven]	Line Representation
[DR] Direct	the reply to a statement/question from one participant to another posting protocols determine that direct responses are formative in nature	***HelpB***: *If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample*	
[FR] Follow-on	from a participant who makes a direct reference to a post, having picked up on a suggestion from a different poster	***HelpD***: *Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺*	
[MR] My (Mine)	from a participant, other than the originator asking for help, offering their own solution having picked up on another's hint	***Help A***: *or 24 ☺*	
[OR] Open	a statement/question offered to anyone (who may or may not respond) 'out in the virtual world' including the first post of any thread	***Peter***: *for anyone who's interested one counter example is 48*	
[PUR] Picked Up (an) Open Response	from a participant who has picked up on an OR (Open Response), either from the trigger or other post(s)	***HelpC***: *Or if you really want to do no work whatsoever when it comes to multiplication just use 720*	

Table 6.7 The Five Response Types with Examples and Line Representations

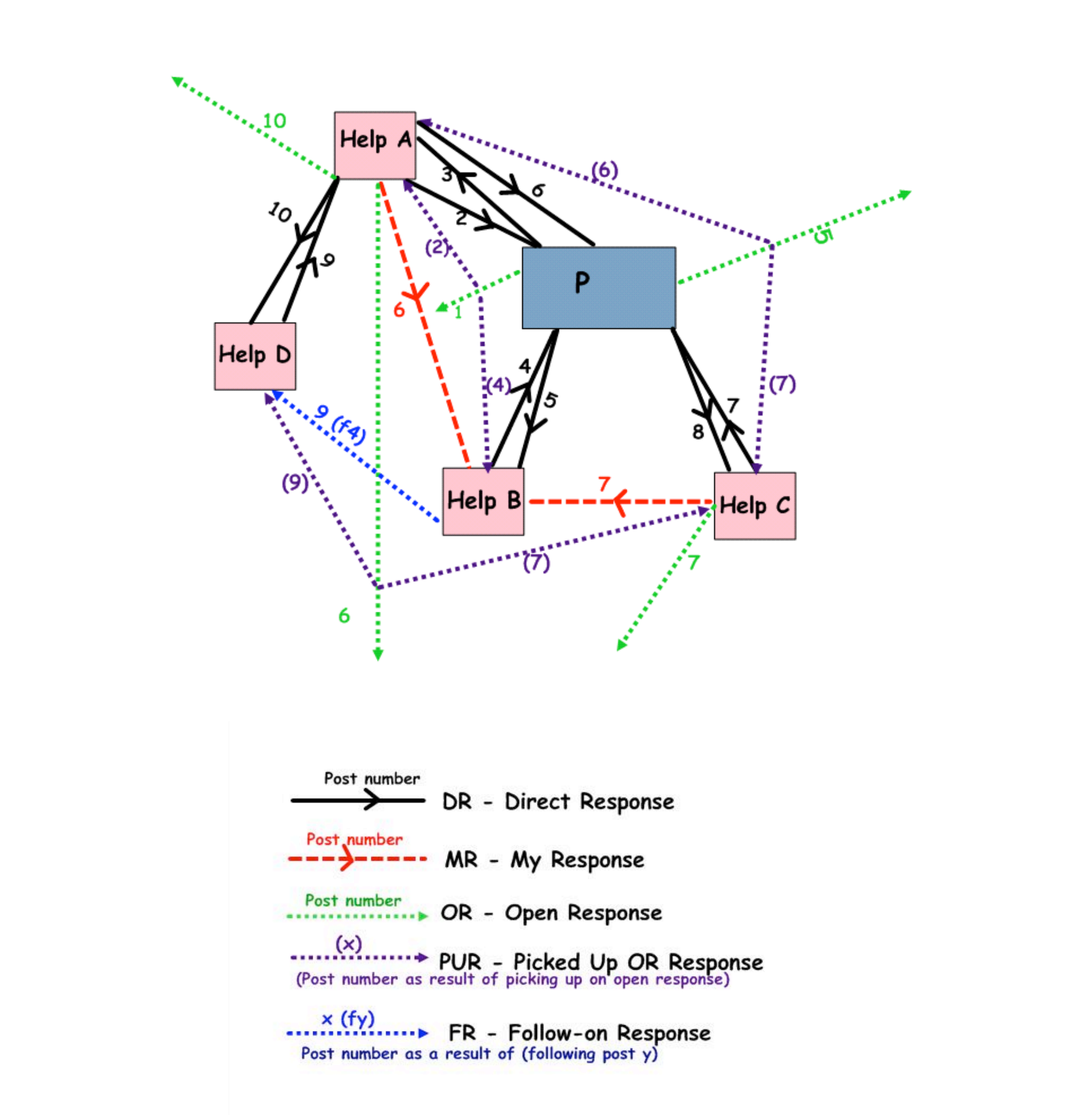


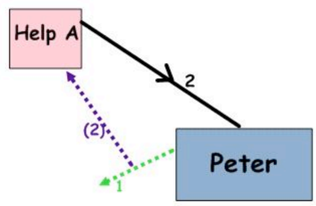
Figure 6.3 Connection Diagram for 3ThD1

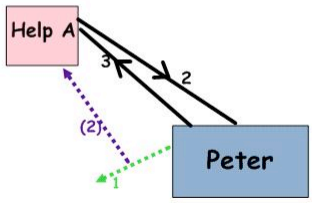
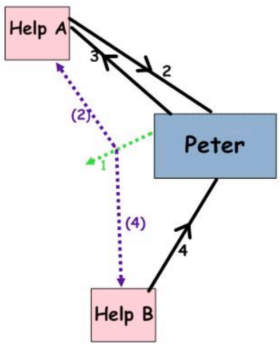
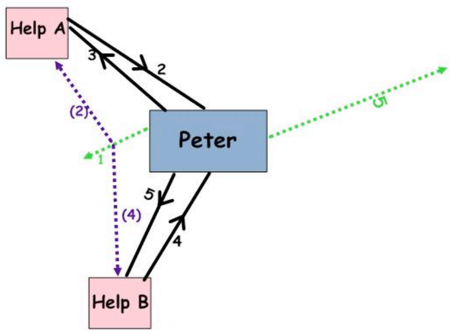
The following two Appendices [Thesis Appendices 6.5 and 6.6] present a series of connection diagrams tracking the progress of a thread, from two different viewpoints, as each post arrives.

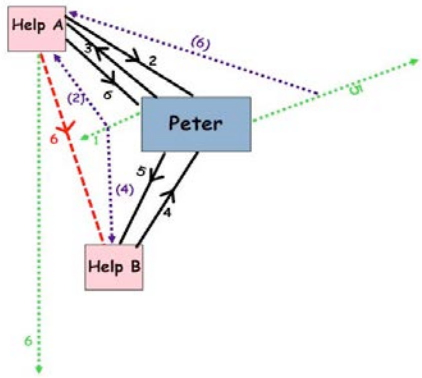
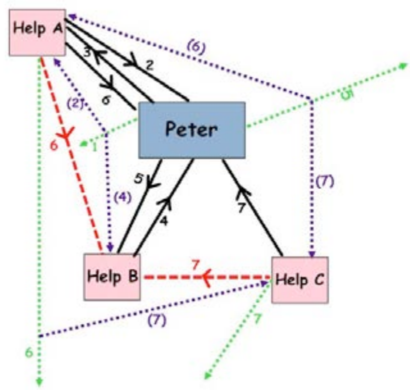
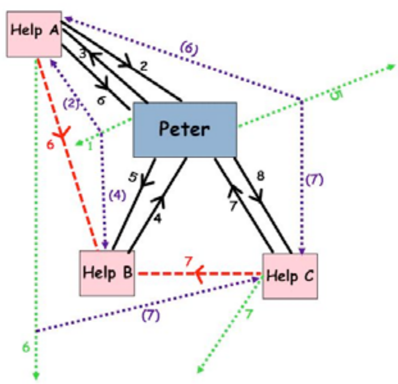
In this Appendix a sequence of Connection Diagrams, together with the associated post text, synopsis and response types, as each post of Thread One of the Three Threads arrives on the web-board.

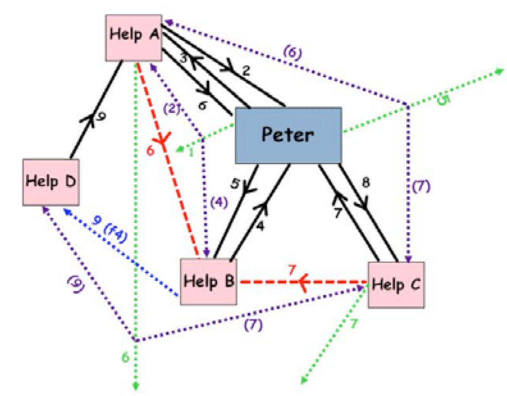
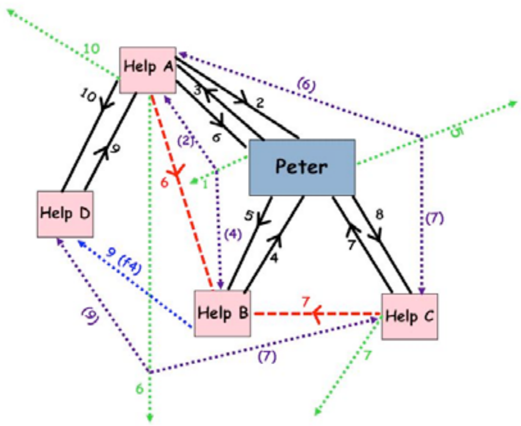
The entry for each post in this table follows the format below. The labelling of response type and the graphical conventions and symbols used in connection diagrams have been given above.

Post No.	Participant and Post text		
	Response type	Connection Diagram	Synopsis of interaction / comments

1	Peter: Let n be an integer greater than 6. Prove that if n - 1 and n + 1 are both prime, then $n^2(n^2 +16)$ is divisible by 720. Is the converse true? i have managed to prove the first part of the question using the fact that all primes are of the form 6n-1 and 6n+1. when i tried to prove the converse i cant do it. i know that 2 and 3 divide n and n is of the form 2mod5 3mod5 or mod5. from here where do i go? thanks		
	OR [PUR by A & B]		Peter has completed the first part of question but cannot do second part in finding if converse is true
2	Help A: Do you think the converse is true?		
	PUR from OR Peter's Post 1 DR to Peter		Suggests starting with an intuitive approach – 'feeling' whether it is true or not true

3	Peter: i presume that it isn't but im not very sure		
	DR to A		Responds by saying that he assumes that it not true, but is not sure
4	Help B: If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample.		
	PUR from OR Peter's Post 1 DR to Peter [MR from A & C] [FR from D]		Connects Peter's solution from the first part of the problem and suggests looking for a counterexample
5	Peter: thanks i ve got it now, for anyone who's interested one counter example is 48.		
	DR to B OR [PUR by A & C]		Has found, and shares, 48 as a counterexample

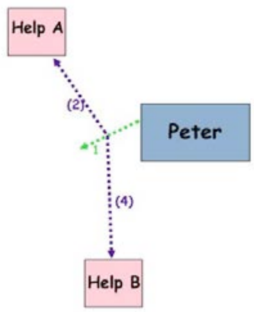
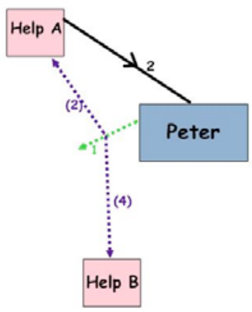
6	Help A: or 24 ☺		
	DR to Peter MR from B’s Post 4 OR [PUR by C & D]		‘Smugly’ (via emoticon) suggests 24 would also do (in fact it does not)
7	Help C: Or if you really want to do no work whatsoever when it comes to multiplication just use 720		
	PUR from Peter’s Post 5 DR to Peter MR from B’s Post 4 PUR from OR A’s Post 6 OR		Gives the ‘blindingly-obvious-once-someone-has-pointed-it-out’ solution of 720
8	Peter: lol i totally missed that		
	DR to C		Amused (lol - laughs out loud) at missing the obvious

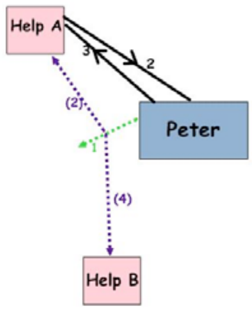
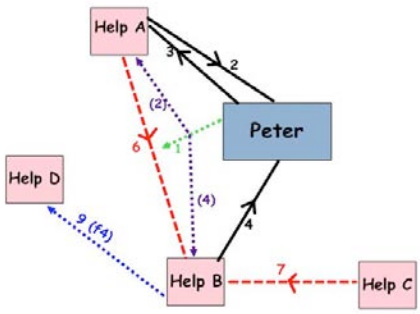
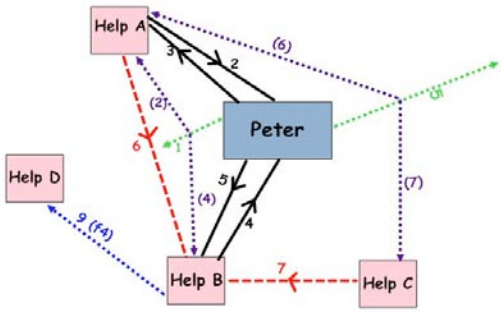
9	Help D: Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺		
	FR from B's Post 4 PUR from A's Post 6 DR to A		Politely suggests that 24 'does not quite cut' it as a counterexample
10	Help A: Sorry haha, I was thinking that all numbers 0 (mod 6) worked. Good job i didn't make that mistake when I took the paper last year!		
	DR to D OR		Laughs at own error and shares mistaken thoughts

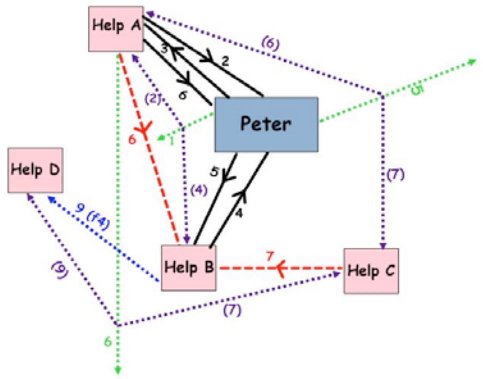
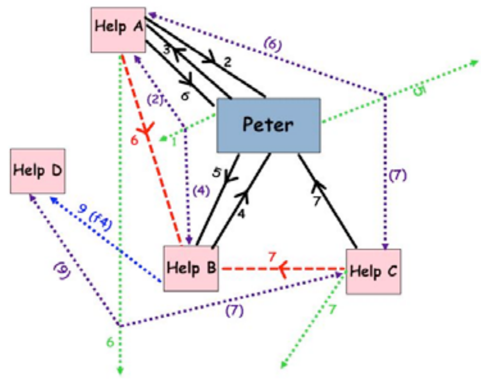
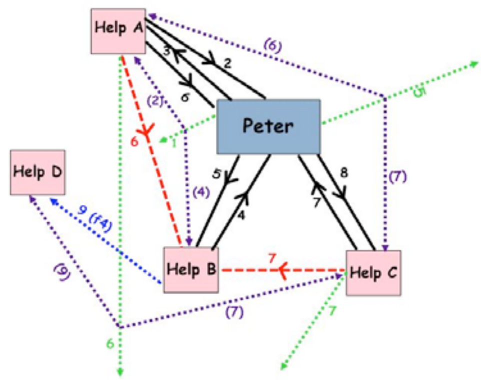
In this Appendix a sequence of Connection Diagrams, together with the associated post text, synopsis and response types, as each post of Thread One of the Three Threads arrives on the web-board. This alternative view differs from that of the previous appendix [Appendix 6.5] in that here it shows the situation when the post has arrived, including its later consequences within the thread.

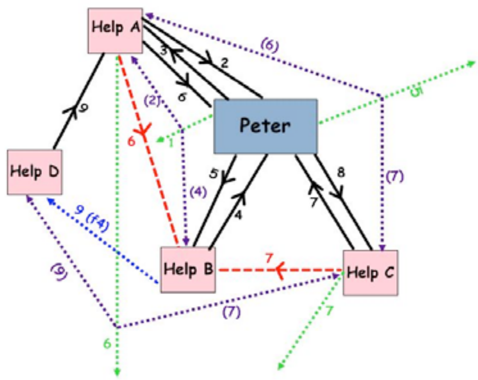
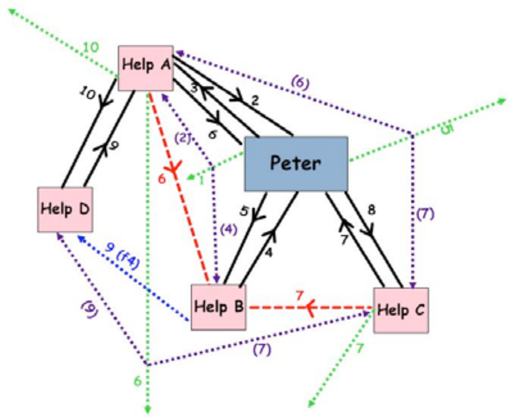
The entry for each post in this table follows the format below. The labelling of response type and the graphical conventions and symbols used in connection diagrams have been given above.

Post No.	Participant and Post text		
	Response type	Connection Diagram	Synopsis of interaction / comments

1	Peter: Let n be an integer greater than 6. Prove that if n - 1 and n + 1 are both prime, then $n^2(n^2 +16)$ is divisible by 720. Is the converse true? i have managed to prove the first part of the question using the fact that all primes are of the form 6n-1 and 6n+1. when i tried to prove the converse i cant do it. i know that 2 and 3 divide n and n is of the form 2mod5 3mod5 or mod5. from here where do i go? thanks		
	OR [PUR by A for Post 2 & by B for Post 4]		Peter has completed the first part of question but cannot do second part in finding if converse is true
2	Help A: Do you think the converse is true?		
	PUR from OR Peter's Post 1 DR to Peter		Suggests starting with an intuitive approach – 'feeling' whether it is true or not true

3	Peter: i presume that it isn't but im not very sure		
	DR to A		Responds by saying that he assumes that it not true, but is not sure
4	Help B: If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample.		
	PUR from OR Peter's Post 1 DR to Peter [A for Post 6 & C for post 7 provide MR] [D for Post 9 provides FR]		Connects Peter's solution from the first part of the problem and suggests looking for a counterexample
5	Peter: thanks i ve got it now, for anyone who's interested one counter example is 48.		
	DR to B OR [PUR by A for Post 6 & C for Post 7]		Has found, and shares, 48 as a counterexample

6	Help A: or 24 ☺		
	DR to Peter B's Post 4 allowed MR OR [PUR by C Post 7 & by D Post 9]		'Smugly' (via emoticon) suggests 24 would also do (in fact it does not)
7	Help C: Or if you really want to do no work whatsoever when it comes to multiplication just use 720		
	PUR from Peter's Post 5 DR to Peter B's Post 4 allowed MR PUR from OR A's Post 6 OR		Gives the 'blindingly-obvious-once-someone-has-pointed-it-out' solution of 720
8	Peter: lol i totally missed that		
	DR to C		Amused (lol - laughs out loud) at missing the obvious

9	Help D: Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺		
	B's Post 4 allowed FR PUR from OR A's Post 6 DR to A		Politely suggests that 24 'does not quite cut' it as a counterexample
10	Help A: Sorry haha, I was thinking that all numbers 0 (mod 6) worked. Good job i didn't make that mistake when I took the paper last year!		
	DR to D OR		Laughs at own error and shares mistaken thoughts

Key
- DR: Direct Response to poster
- FR: Follow Up response to earlier message
- MR: My/Mine Response/Solution from (as a result of) earlier message/posting
- OR: Open Response/Message (to all AskNRICHers)
- PUR an open response that is Picked Up by specific poster(s)

Table A Thread One

Post	Day/time	Time Gap	Poster	Message	Response Type & Persons Involved	Synopsis of interaction / comments
Thd1 P1	Saturday 11.13am	-	Peter	Peter: Let n be an integer greater than 6. Prove that if n - 1 and n + 1 are both prime, then $n^2(n^2 +16)$ is divisible by 720. Is the converse true? i have managed to prove the first part of the question using the fact that all primes are of the form 6n-1 and 6n+1. when i tried to prove the converse i cant do it. i know that 2 and 3 divide n and n is of the form 2mod5 3mod5 or mod5. from here where do i go? thanks	OR PUR by A & B	Completed first part of question but cannot do second part in finding if converse is true
Thd1 P2	Saturday 11.14am	1min	Help A	Help A: Do you think the converse is true?	DR to Peter	Suggests starting with an intuitive approach – ‘feeling’ whether it is true or not true
Thd1 P3	Saturday 11.37am	23 mins	Peter	Peter: i presume that it isn't but im not very sure	DR to A	Responds by saying that he assumes that it not true, but is not sure
Thd1 P4	Saturday 11.39am	2 mins	Help B	Help B: If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample.	DR to Peter	Connects Peter’s solution from the first part of the problem and suggests looking for a counterexample
Thd1 P5	Saturday 11.58am	19 mins	Peter	Peter: thanks i ve got it now, for anyone who's interested one counter example is 48.	DR to B OR PUR by A & C	Has found, and shares, 48 as a counterexample

Post	Day/time	Time Gap	Poster	Message	Response Type & Persons Involved	Synopsis of interaction / comments
Thd1 P6	Saturday 12.02pm	4 mins	Help A	Help A: or 24 ☺	DR to Peter MR fr B4 OR PUR by C & D	'Smugly' (via emoticon) suggests 24 would also do (in fact it does not)
Thd1 P7	Saturday 12.07pm	5 mins	Help C	Help C: Or if you really want to do no work whatsoever when it comes to multiplication just use 720	DR to Peter MR fr B4 OR	Gives the 'blindingly-obvious-once-someone-has-pointed-it-out' solution of 720
Thd1 P8	Saturday 12.10pm	3 mins	Peter	Peter: lol i totally missed that	DR to C	Amused (lol - laughs out loud) at missing the obvious
Thd1 P9	Saturday 6.59pm	6 hrs 49	Help D	Help D: Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺	FR to B4 DR to A	Politely suggests that 24 'does not quite cut' it as a counterexample
Thd1 P10	Saturday 7.11pm	12 mins	Help A	Help A: Sorry haha, I was thinking that all numbers 0 (mod 6) worked. Good job i didn't make that mistake when I took the paper last year!	DR to D OR	Laughs at own error and shares mistaken thoughts

Table B Thread Two (One and a half months later)

Post	Day/time	Time Gap	Poster	Message	Response Type	Synopsis of interaction / comments
Thd2 P1	Monday 7.05 pm	-	R	Let n be an integer greater than 6. Prove that if n-1 and n+1 are both prime, then $n^2(n^2+16)$ is divisible by 720. How would I start this question? I thought of trying to split 720 up into its prime factors, 2,2,2,2,3,3,5. However, this didn't get me anywhere. Can anybody give me any hints?	OR PUR by P & E	Asks for help on the first part of the problem
Thd2 P2	Monday 7.07 pm	2 mins	Help Peter	what form do primes greater than 6 take. as soon as you see what to do this is very simple so i shall leave the hint at that	DR to R	Responds with key question (based on what he had used to answer the question mentioned in Thd 1 P1)
Thd2 P3	Monday 7.07 pm		Help E	If n-1 and n+1 are prime, what can you say about n (nothing too deep - just divisibility sorts of things)?	DR to R	Provides different, (probably) more manageable key question
Thd2 P4	Monday 7.14pm	7 minutes	R	R: My thoughts: • n must be even, because primes above 6 are all odd, so (n-1) must be odd, so n is even. • n must also be divisible by 3, because (n+1) and (n-1) are both **not** divisible by 3. • I think that there must be some way to work out the last digit of n by knowing that (n-1) and (n+1) are not divisible by 5, but either n^2 or n^2+16 must be divisible by 5 (so the product is divisible by 720). I can't work out how though.	DR to P & E OR PUR by C & F	Responds with his thoughts to Help E's questions. Feels that he needs to show that one of two specific terms must be divisible by 5
Thd2 P5	Monday 7.31pm	17 mins	Help F	Help F: The first two points suffice. n is even, and n is divisible by 3, can you make a slightly stronger statement about n and use this as the base of your argument?	FR to E3 DR to R	Responds to R thoughts using Help E's key questions
Thd2 P6	Monday 7.32pm	1 mins	R	I think n must end in 2 or 8. This is because n^2+16 must divide by 10, so n^2 must end in 4, so n must end in 2 or 8	DR to E	Although this message follows Help F, the time gap suggests that for P6, R may have continued to work on his thoughts (see P4).
Thd2 P7	Monday 7.34pm	2 mins		N must divide by 6, and end in 2 or 8, so n can be expressed as (some multiple of 30)+(12 or 18). I'm not sure how to write this algebraically though	DR to F	For P7 it is more likely that R could be responding to Help F's comment and suggestion

Post	Day/time	Time Gap	Poster	Message	Response Type	Synopsis of interaction / comments
Thd2 P8	Monday 7.38pm	4 mins	Help Peter	if you know modular arithmetic try putting multiples of 6 where 6kmod30+1 and 6kmod30-1 are both possibly prime as n. i think that there is a nicer way but this is still nice and simple and im tired at the moment. hope this helps. if you dont know modular arithmetic mod 30 is just a multiple of 30 and 6kmod30 is 6k + mod30	DR to R	Returns to revive own method of solution hinted at in P2 above. Mentions modular arithmetic for the first time. Personal feeling that there are other methods that could be employed but feels own choice is simple. Implies feeling tired is currently effecting own ideas
Thd2 P9	Monday 7.39pm	1 mins		sorry ive realised you can write this as mod5	DR to R	Afterthought – mulling over problem after posting
Thd2 P10	Tuesday 5.07pm	Next evening	R	(P10a) I don't really understand this Peter. Sorry. (P10b) Does anyone know a different way to give a hint/explain Peter's hint?	DR to P OR PUR by C	Politely responds to Peter to say that they have not understood his help and asks if anyone else can either explain Peter's hint or suggest an alternative way of obtaining the solution
Thd2 P11	Tuesday 5.27pm	20 mins	Help C	If n-1 and n+1 are prime, what are the possible remainders when you divide n by 5?	FR to E3 FR to P9 DR to R	(Same person as in Thd 1). Offers a hint which connects with Help E's questions, R's own message 4 and Peter's afterthought of mod 5
Thd2 P12	Tuesday 5.38pm	11mins	Help Peter	since i don think that you understand modular arithmetic (dont worry about this) i shall write in a basic algerbraic form 6k - cannot be prime because 6 divides it 6k+1 - maybe prime 6k+2 - cannot be prime because 2 divides it 6k+3 - cannot be prime because 3 divides it 6k+4 - cannot be prime because 2 divides it 6k+5 - maybe prime where k > 0 so for x to be prime x is in the form 6k-1 or 6k-1. so if n-1 and n+1 are both prime n = 6k so then this proves your statement earlier that 6 divides n. so now you must show that n^2 or n^2+16 is divisible by 5 now n is in the form 5k, 5k+1, 5k+2, 5k+3, 5k+4 for which of these is it true that n-1 and n+1 is prime? now square n [contd.] and sub in n^2 = 5k + m where you know m and then prove what that n^2+16 or n^2 are divisible by 5	DR to R	Decided that R is unfamiliar with modular arithmetic (though should not worry about this) and provides a detailed algebraic solution that will take R part way there – thus leaving R to complete the final part.

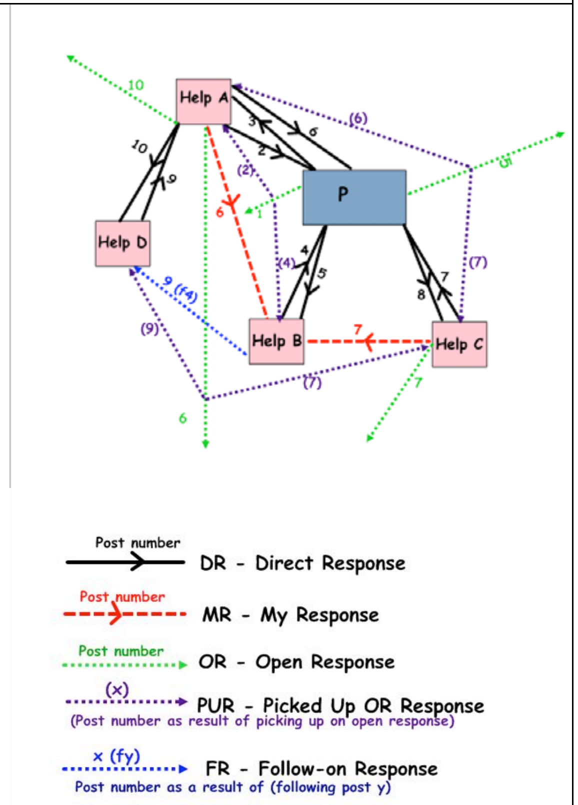
Post	Day/time	Time Gap	Poster	Message	Response Type	Synopsis of interaction / comments
				i hope this makes sense.		
Thd2 P13	Tuesday 5.48pm	10 mins	R	OK. 5k+1, because n-1 would divide by 5 and 5k+4, because n+1 would divide by 5. Is there a way to narrow it down further? Or can m have multiple values.	DR to P	Responds to Peter's work by continuing to move some way towards the final solution, but not quite completed yet.
Thd2 P14	Tuesday 5.57pm	9 mins		Assuming m can have multiple values: For m=0, n^2 will equal $25k^2$, so it will be divisible by 5. For m=2, n^2 will equal $25k^2+20k+4$. Therefore, $n^2+16=25k^2+20k+20$, so it will be divisible by 5. For m=3, n^2 will equal $25k^2+30k+9$. Therefore, $n^2+16=25k^2+30k+25$, so it will be divisible by 5. Yes, so n can be expressed as 5k+m, where k is a positive non-zero integer, and m equals 0, 2, or 3	DR to P	Peter provides further detailed help which allows R to reply …
Thd2 P15	Tuesday 6.02pm	5 mins		Got it! n is divisible by 6, so n^2 is divisible by *36*. n^2 is divisible by 4, so n^2+16 **must** also be divisible by *4*. Either n^2 or n^2+16 must be divisible by *5*, from the last post. *36*5*4*=**720**	DR to P OR	**'Got it!'** Elation/Relief in succeeding. R completes the solution having shown that one of the two specific terms must be divisible by 5 (as R had predicted in P9 above)
Thd2 P16	Tuesday 6.05pm	3 min	Help Peter	yes well done this completes the proof. i remember fondly this question. this was my first bmo question i completed. arrr memories ... yes anyway. well done	DR to R	Congratulates R and adds a personal memory

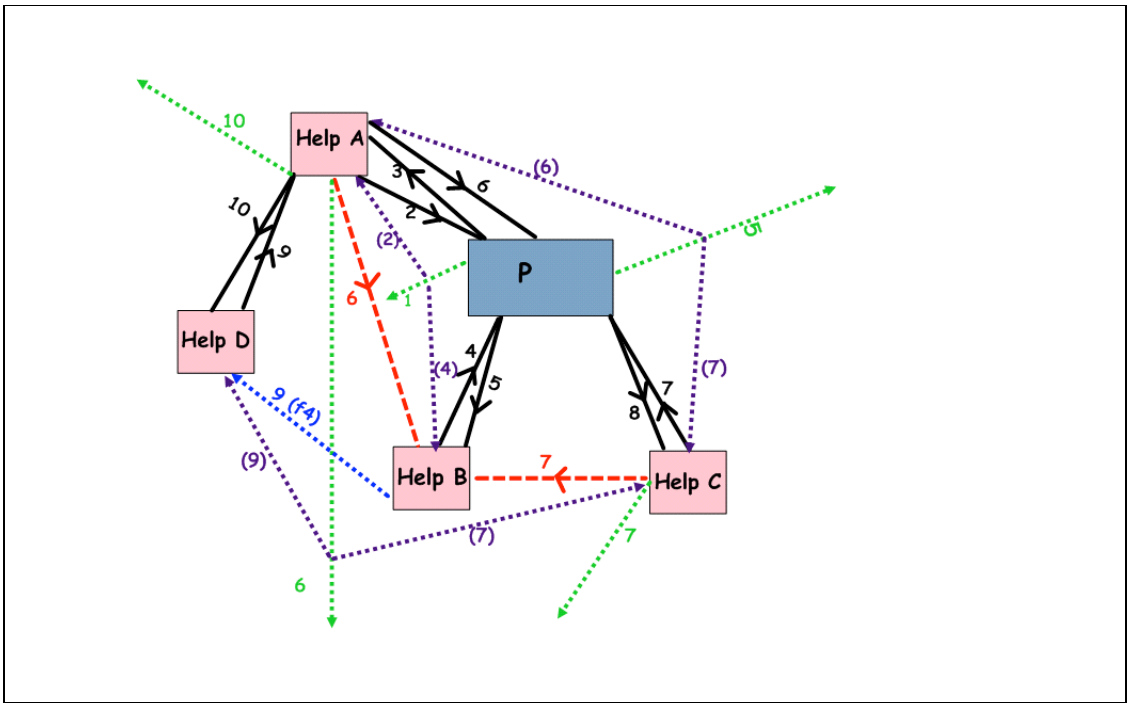
Table C Thread Three (Eight months later – the next academic year)

Post	Day/time	Time Gap	Poster	Message	Response Type	Synopsis of interaction / comments
Thd3 P1	Thursday 4.33pm	-	S	n be an integer greater than 6 where n - 1 and n + 1 are both prime. Prove that $n^2(n^2 +16)$ is divisible by 720. I've written 720 as prime factors, and I've proved that the expression is divisible by all of the prime factors except 5. Can anyone give me a hint?	OR PUR by G & P	Like R above is stuck on the first part of the question though has already worked to the stage where there is a need to show divisibility by five
Thd3 P2	Thursday 4.43pm	10 mins	Help P	Peter: consider n mod5, what can it/cant it be?	DR to S	First to respond (as with R) with a repeat of the mod 5 hint.
Thd3 P3	Thursday 4.45pm	2 mins	Help G	Write n as 5k+a and consider which numbers a can be given n-1 and n+1 are both prime (so not divisible by 5) and substitute these possible expressions for n into $n^2(n^2+16)$. (Alternatively/equivalently if you know about modular arithmetic consider possible values of n mod 5) You said you showed it is divisible by the other prime factors of 720 - did you make sure you showed that it was divisible by them to the right power (ie that 3^2 and 2^4)?	DR to S	Provides a more detailed response and help which includes within other ideas the same hint as Peter's
Thd3 P4	Thursday 5.03pm	18 mins	S	P4a: sorry about the formatting, I am putting the backslashes in, but they don't seem to be working P4b: This is what I've done - we know that n must be divisible by 2 and 3 for the numbers on either side of it to be prime. $n^2(n^2+16) = n^4+ 16n^2$. n^4 is divisible by 2^4 and 3^4 (so is also divisible by 3^2). $16n^2$ is divisible by 2^4x2^2 (so is also divisible by 2^4). Therefore, $n^2(n^2+16)$ must be divisible by 3^2 and 2^4 n-1 and n+1 can't be 0 mod 5, therefore n can't be 1 or 4 mod 5. This means it must be 2 or 3 mod five, so it's square must be 4 mod5. Therefore n^2 + 16 is 4+1 mod 5 = 0 mod 5. P4c: Thanks everyone. I can't believe I didn't see that...	OR DR to G OR OR	Apologises for not knowing how to format mathematical text to appear properly on the board. Shares working to show that problem has been solved (actually, not quite completely solved)
Thd3 P5	Thursday 5.08pm	5 mins		Sorry again, I've just realized that you need to put + in …	OR	Apologies again for lack of formatting but explains what they had been doing wrong

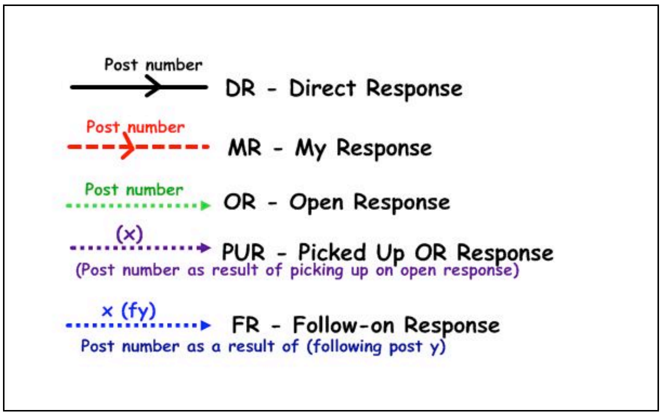
Post	Day/time	Time Gap	Poster	Message	Response Type	Synopsis of interaction / comments
Thd3 P6	Thursday 5.08pm	5 mins	Help G	One small thing you've missed - n can be 0 mod 5, but that gives you n^2 is 0 mod 5 so you're still fine. Have you got the second bit about the converse as well?	DR to S	Responds to highlight the missing part that will complete S's solution and enquiries of S whether they have solved the second part of the question
Thd3 P7	Thursday 6.58pm	1 hr 50 mins	S	The converse is false because 720 is divisible by 720, but 720+1=721 has a factor of 7	DR to G	Responds with the same 'blindingly-obvious' counterexample of 720 that Help C had given in Thd 1 and to which Peter had 'lol'
At this stage the problem has been fully resolved (or so it would appear) Two posters (ANP1 & 2) new to the thread join in the discussion						
Thd3 P8	Thursday 7.41pm	43 mins	ANP (1)	If the converse were true, then it would be a really, really fast way to find big prime numbers!	MR fr G6 DR to S OR PUR by ANP (2)	Adds an extra 'conversational' comment about why it is 'useful' that the converse is not true
Thd3 P9	Thursday 7.44pm	2 mins	ANP (2)	Not to mention being a proof of the twin prime conjecture!	DR to S & ANP (1)	Responds to comment above by mentioning twin prime conjecture
Thd3 P10	Thursday 8.46pm	1 hr 2 mins	S	P10a: I've just realised that my counter example is exceedingly wrong as while 720 is divisible by 720, $720^2(720^2+16)$ isn't. So is seems that converse is false, but finding a counter example might be hard. Maybe by trying to solve $n^4 + 16n^2 - 720n=0$, but I feel there should be something like a proof by contradiction. P10b: By the way, for it to be a proof of the twin prime conjecture you'd also have to prove that there are infinitely many multiples of 720 that can by written as the product of a square number and that number +16.)	OR PUR by C DR to ANP (1) & ANP (2)	Returns to announce that the 720 previously mentioned was 'exceedingly wrong' and does not work as a counterexample [though from Thread One it obviously does work]. S shows new thoughts in trying to find the solution and has complicated the problem by obtaining a quartic equation and considering that perhaps proof by contradiction is now required. The message concludes with a response about the twin prime conjecture.

Post	Day/time	Time Gap	Poster	Message	Response Type	Synopsis of interaction / comments
Thd3 P11	Thursday 8.56pm	10 minutes	Help C	**quote:** I've just realised that my counter example is exceedingly wrong as while 720 is divisible by 720, $720^2(720^2+16)$ isn't. Yes it is! $720^2(720^2+16)/720 = 720(720^2+16)$	DR to S	Picks up S's exact words '… (720) isn't' and replies 'Yes it is!' Help C should know as the first person to suggest that it as a counterexample in thread one (which so amused Peter).
Thd3 P12	Friday 5.05pm	Next day	S	For some reason I was thinking of $720^2 + (720^2+16)$. Lets just hope I didn't make a mistake like that on the BMO today.	DR to C OR	Responds to explain how they had come to make the error (inextricably interpreting incorrect operation sign and concludes that they hope they did not do anything as silly as that in the test taken earlier in the day.

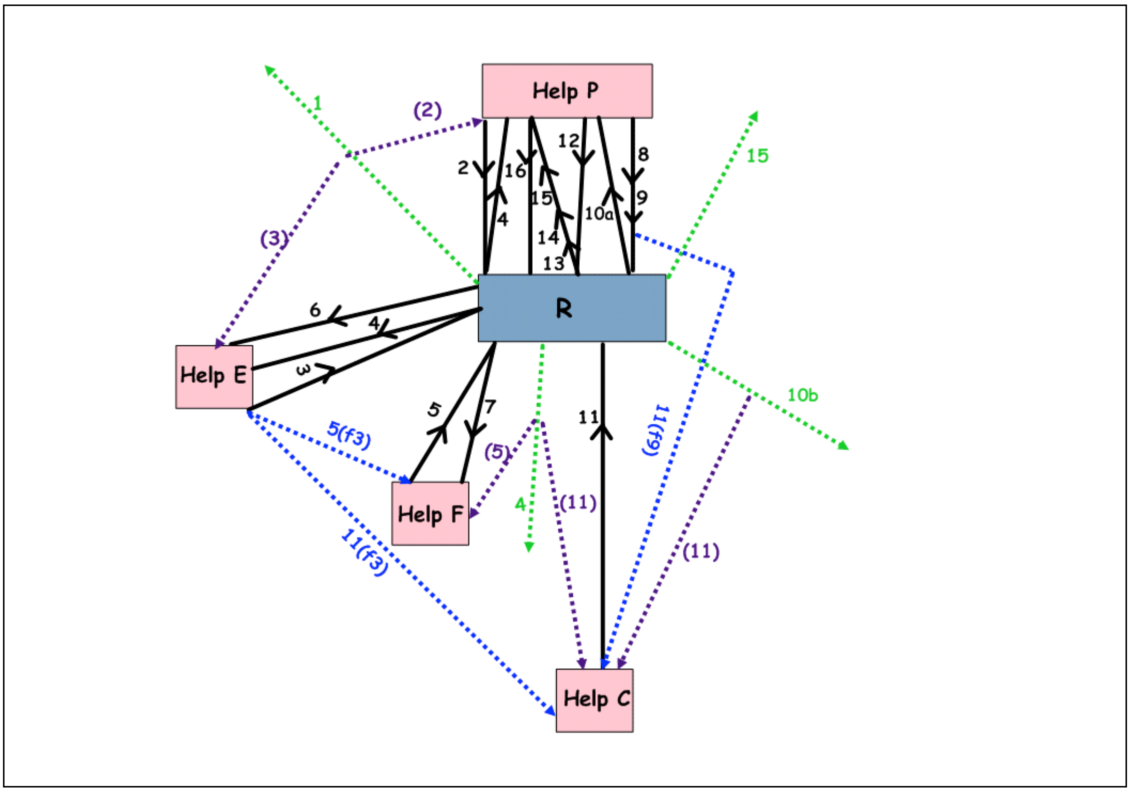
Post	Gap	Poster	Message text	Response Type	Synopsis of interaction / comments	Connection Diagram
P1		Peter	Let n be an integer greater than 6. Prove that if n - 1 and n + 1 are both prime, then $n^2(n^2+16)$ is divisible by 720. Is the converse true? i have managed to prove the first part of the question using the fact that all primes are of the form 6n-1 and 6n+1. when i tried to prove the converse i cant do it. i know that 2 and 3 divide n and n is of the form 2mod5 3mod5 or mod5. from here where do i go? thanks	OR PU by A & B	Completed first part of question but cannot do second part in finding if converse is true	
P2	1min	Help A	Do you think the converse is true?	DR to Peter	Suggests starting with an intuitive approach – 'feeling' whether it is true or not true	
P3	23 mins	Peter	i presume that it isn't but im not very sure	DR to A	Responds by saying that he assumes that it not true, but is not sure	
P4	2 mins	HelpB	If you look back over your proof, you used the fact that ALL primes are 6n-1 and 6n+1. However, is the converse of *this* true? Are all 6n-1 and 6n+1 prime? Using this, you can construct a counterexample.	DR to Peter	Connects Peter's solution from the first part of the problem and suggests looking for a counterexample	
P5	19 mins	Peter	thanks i ve got it now, for anyone who's interested one counter example is 48.	DR to B OR PU by A & C	Has found, and shares, 48 as a counterexample	
P6	4 mins	HelpA	or 24 ☺	DR to Peter MR fr B4 OR PU by D	'Smugly' (via emoticon) suggests 24 would also do (in fact it does not)	
P7	5 mins	HelpC	Or if you really want to do no work whatsoever when it comes to multiplication just use 720	DR to Peter MR fr B4 OR	Gives the 'blindingly-obvious-once-someone-has-pointed-it-out' solution of 720	
P8	3 mins	Peter	lol i totally missed that	DR to C	Amused (lol - laughs out loud) at missing the obvious	
P9	6 hrs 49 min	HelpD	Not to be a spoil sport, but I don't think 24 quite cuts it as a counterexample ☺	FR to B4 DR to A	Politely suggests that 24 'does not quite cut' it as a counterexample	
P10	12 mins	HelpA	Sorry haha, I was thinking that all numbers 0 (mod 6) worked. Good job i didn't make that mistake when I took the paper last year!	DR to D OR	Laughs at own error and shares mistaken thoughts	



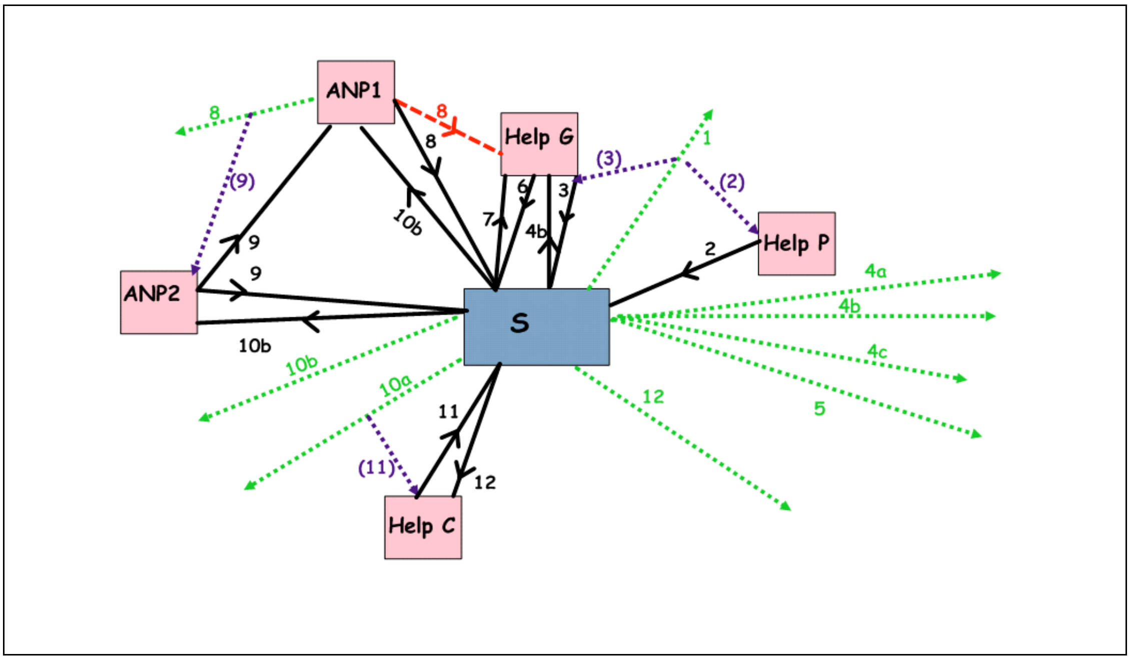
Connection Diagram for 3Thd: Thread1



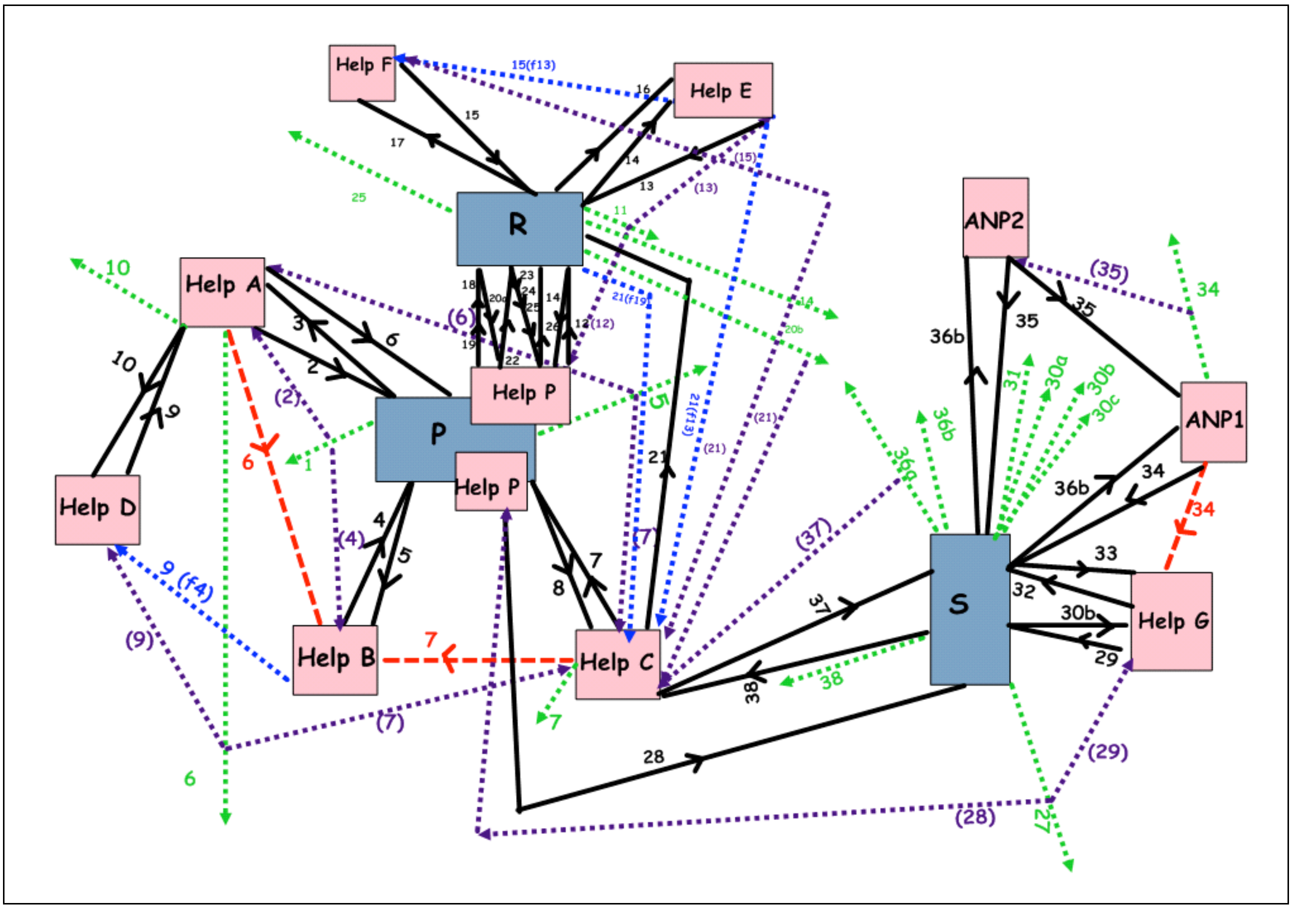
Key for Connection Diagrams



Connection Diagram for 3Thd: Thread2



Connection Diagram for 3Thd: Thread3



Combined Connection Diagram for all 3Thds