# Chapter Ten

## Doing Mathematics in Local Isolation

## Perspective Two:

## Case Study of an AskNRICHer

With many thanks to
NRICH, 'Peter'
and all other AskNRICHers

Doing Mathematics in Different Places: an Exploration of Young People's Activities as they make Independent Use of a Web-Based Discussion Board

Libby Jared

PhD Thesis

King's College London

Supervisor Dr Mary Webb

Each chapter has been edited to enable it, as far as is feasible, to 'standalone'.

The chapter numbers and numbering of sub-headings has been left unchanged from the original Thesis.

However, each edited chapter has its own page numbering and any cross-references *within* the chapters and *between* chapters on the NRICH website use these (new) page numbers followed by specifying the page number(s) in the original Thesis chapters.

Where appropriate, references may be given to other chapters (not included on the website) within the full Thesis, either by specifying the Section or providing the Thesis page number(s).

If in a chapter reference is made to any appendices, then the relevant appendix is attached at the end of that chapter.

Each chapter has its own list of references.

[The Thesis title, abstract and acknowledgement pages together with a table of contents for these edited chapters and glossary from the Thesis are also included. The table of contents of the full Thesis appears after Chapter Fifteen].

Dr Libby Jared

October 2014

Chapter Ten

# Doing Mathematics in Local Isolation
# Perspective Two: Case Study of an AskNRICHer

*I love maths that makes me think and being able to go into a world inside of my mind and then the feeling of satisfaction when I have solved a problem I have been trying for ages.*
*[Peter aged 16, email communication]*

## 10.1 Introduction

In the previous chapter the common practices and use made of the web-board by the AskNRICHers were determined using the Perspective of analysing two exemplar threads. This chapter is the second of three Perspectives reporting the findings of interpretative analyses of a selection of threads. This Perspective is a single case study of one representative user, Peter, through the 151 retrievable threads in which he participated over an eighteen-month period. The analyses of Peter's interactions are based on an in-depth examination of the 1875 posts in those threads, including the 484 that he made. AskNRICHers like Peter work in isolation, confined to a local environment and thus generally unable to physically meet with peers with similar ability and enthusiasm for the subject. As the concluding remarks of Chapter Eight explain [pp17-18/Thesis pp181-182], the AskNRICHers find, remotely, like-minded others that they can engage with and enjoy rich mathematical experiences, no longer alone.

The purpose of this chapter is to:

i. present a statistical analysis of Peter's posts that reveals his pattern of use of AskNRICH
ii. use two threads to examine Peter's engagement and interactions when in a learning role
iii. reflect on eleven threads on Mathematical Induction [MI] to follow Peter's progress in mastering the topic, from starting out in a learning role and moving onto a teaching role
iv. use a variety of selected threads to scrutinise a number of ways that Peter interacts in a teaching role

Thus the findings of this case study contribute to further addressing the questions: *Who are the participants? Why do they participate? How is AskNRICH typically used? What characteristics do participants of AskNRICH exhibit as they pursue their interest in mathematics? What mathematics teaching and mathematics learning roles are manifested within AskNRICH? What types of interactions are shown between the participants as they engage with mathematics?*.

The case study focuses on the participation of one AskNRICHer who at times is seeking help whilst at other times is offering help. Analysis of threads involves both the case study subject and all the other AskNRICHers participating in the thread. Within AskNRICH, help may be offered by more experienced, equal, or by less experienced peers. van Lier's [1996: 194] conceptualisation of an individual's four-part ZPD was selected as the theoretical underpinning in the reporting for this chapter. The four parts of the individual's ZPD ("assistance from more capable peers or adults"; "interaction with equal peers"; "interaction with less capable peers" and "inner resources: knowledge, experience, memory, strength") may be exemplified in the individual's multiple activities.

The remaining part of this chapter is in four sections that mirror the division used above in setting out its purpose. Thus the section that follows introduces Peter, including some background generated by an email interview with him, and then examines the patterns of his participation in AskNRICH.

## 10.2 Background Information for the Case Study

Although school pupils can come to AskNRICH, post a query but hardly stay, there is a core of prolific and veteran status posters who do participate over a period of time – in some instances for years [see Section 8.4 Chapter Eight pp7-12/Thesis pp171-176]. Although there are other participants who have contributed a greater number of posts, Peter's posts were a manageable number to study in depth.

In this chapter Peter's involvement with AskNRICH over an eighteen-month period, is analysed. During this time, November 2006 to May 2008, Peter made 501 posts across some

150 different threads. Towards the latter part of 2007, email communication with Peter (preceded by telephone contact with his parents to gain informed consent) provided additional background material to this case study. Peter was contacted by email in November 2007 and agreed to answer some questions.

This section continues by using Peter's responses to the email correspondence just mentioned to describe his approach to, and motivation for studying mathematics. This is followed by numerical data on Peter's postings to indicate the volume and type of posting (asking for or giving help) and the days and time of day the postings were made.

### 10.2.1 Introducing Peter in his Own Words

Peter[1] reported that he started to use the NRICH site as he wanted more out of his mathematics studies than he was then able to have at school. Moreover, as his comment below implies, he was experiencing a degree of frustration but had the motivation to be proactive in searching for an alternative resource that would suit his needs. His comment below additionally suggests that when he lost the website's address he had determination, in attempting to relocate it. Peter was thus interested in the subject and wanted to do more:

> *... about a year and a half ago when I first became interested in maths. I was bored and wanted to further the level of maths I did. I then temporarily stopped using the site and forgot the name and then spent a couple of weeks typing enrich[2] into Google and searching all of the pages. At the end of the summer a year ago I then finally found the site again and began to use the site regularly. I completed a load of problems of the site.* *[email communication]*

From the comment below, made during his first term of his final year of compulsory school, it can be deduced that Peter sat his GCSE mathematics examination a year early (then aged 15) and gained an A* grade. Peter described his school mathematics experience as one where, for him, the pace was too slow and the work unchallenging. His own learning was taking him far beyond the school syllabus and he was now involved in self-teaching. Such

[1] In November 2007, Peter was aged 16.

[2] The addition of the 'e' caused the difficulty – though in 2011 a google search on enrich would have nrich as the first hit.

experiences have much in common with other participants, evidenced by similar remarks appearing in various postings.

> *My maths lessons were last year really boring for me as I found them too simple and moving at too slow a rate. At the end of year 10, I sat my maths GCSE and got an A*. I also took core 1 maths and achieved 95%. I taught myself all of the core modules in my spare time and am now working on mechanics. In lessons at school I am teaching myself the rest of the modules for maths A-level. I shall also teach myself further maths after this.*
>
> *My maths teacher is very helpful and helps me with things I am stuck on whenever he can. The math department as a whole is willing to be very flexible to allow me to further my mathematics education. For example, I was allowed to drop ICT and instead sit in the maths department teaching myself maths from a text book.* *[email communication]*

Although Peter's special needs do not appear to be directly addressed by his school, Peter is sympathetic towards the efforts of his teacher and has gone to some lengths to negotiate his own timetable.

Peter had been asked to describe how he learnt his mathematics, and, if he was teaching himself new material, how he did this. Three interesting points stand out in his response below. First is his wish to let things settle, a key strategy frequently adopted by mathematicians encountering either challenging problems or new work. Secondly, Peter shows maturity in metacognitive self-reflection in realising what helps him study. Finally, Peter's revelation that the need to let things settle is generally only necessary with harder material beyond A-level, he finds the latter causes few problems in his overall understanding:

> *I do teach myself all the new material that I learn. I do this through reading through text-books, leaving it to settle for a week or so then doing as many questions I can. I don't always leave it to settle but find it most helpful when learning harder material, for example. When learning A-level this is not necessary since I can normally understand it pretty much straight away.* *[email communication]*

Indeed in another part of the same email Peter remarked:

*most A-level work is just solving the same problem with different figures.*

*[email communication]*

For Peter, studying work beyond A-level appears to be a natural state. In the email below he reveals that ‘Analysis’ and ‘Number Theory’ were amongst the topics he was most interested in, which are usually first met at undergraduate level. ‘Olympiad maths’[3], as its names implies, is formed of the most challenging problems, far beyond A-level standard, many of which are on number theory and geometry. Peter, like many of the AskNRICHers, rising to the Olympiad’s challenge, uses the web-board to discuss past problems[4]. Peter is not alone in his ‘passionate’ hatred of geometry an emotion that probably arises for the period when formal Euclidean geometry became substantially unfamiliar territory in UK schools [Jones 2002].

*When I’m not doing A-level school maths I teach myself Olympiad maths except for geometry which I hate with a passion. I also look at some analysis though not much since I have not completed enough maths in other areas to get to the really interesting stuff. My favourite type of maths is number theory closely followed by algebra.*

*[email communication]*

The comment above also demonstrates Peter’s awareness of his own limitations. Although obviously a high-attainer in the subject, Peter can nevertheless still recognise and accept that he has yet to experience the pre-requisite mathematical topics to be able to succeed at the level of Analysis he aspires to. Here again there is further evidence of strong metacognitive skills providing the basis for ‘deep learning’ to flourish.

Having shown Peter’s avid interest in pursuing his mathematical studies this chapter now turns to a quantitative account of his postings to AskNRICH.

### 10.2.2 Numerical Data on Peter’s Postings

Given an individual’s posting name, the AskNRICH search facility returns all their posts that are still-retrievable, grouped by web-board section and then thread title, but listed in an

---

[3] National school-age mathematics competitions culminate in the most successful scorers being invited to be part of the British Mathematics Olympiad [BMO] Team.

[4] Three Threads in next chapter provides an example of AskNRICHers doing this.

apparently random order. Clicking on a specific post will display the entire thread, at which point it is possible to extract further information on the post including: post number, date, day and time. Only at this stage can the posts be sorted into chronological order and the only means of doing this is manually. In this case study the earliest retrievable post was Peter's ninth on 25th November 2006, the last was number 501 on 1st May 2008, the intervening period spanning the final two years of Peter's compulsory schooling. These posts appeared in 151 different threads. For ethical reasons only postings on the open-access mathematics sections are included in the reporting of this case study. Table 10.1 provides statistical data on threads involving Peter.

| | | |
|---|---|---|
| November 25th 2006 | Earliest retrievable message (9) | Year 10 (14 to 15 years old) |
| May 1st 2008 | Last message (501) | Year 11 (15 to 16 years old) |
| | | |
| Total number of threads retrieved (involving Peter) | | 151 (including 16 'private' threads) |
| Total number of threads retrieved in mathematics sections | | 135 (89.4% of total threads) |
| Total number of Peter's posts retrieved | | 484 (96.6% of total posts) |
| Total number of Peter's posts in one single private thread | | 104 (20.1% of all retrieved posts) |
| Total number of Peter's posts retrieved in mathematics sections | | 345 (90.1% if thread above disregarded; 71.3% of all retrieved posts;) |

Table 10.1 Duration and Counts of Peter's Participation in Threads

Of the 151 threads, above, 135 (89.4%) appeared on the open-access sections. 484 of Peter's 501 posts were retrievable of which 345 (71.3%) were in the open mathematics sections. This percentage figure is misleading as Peter spent 104 posts participating in the Private Section of the board, in a fun, not mathematics related, word-association game. If this thread is disregarded then the study is working with 90.1% of all Peter's retrievable posts within the mathematical sections spread across 135 different threads. These 135 threads contained a total of 1875 posts by all participants, including Peter, though similar to above, ignoring one outlier thread, this becomes 1474 posts, all of which were considered when undertaking the analysis that formed this case study. Table 10.2 enumerates the number of threads and posts, according to Peter's role, assigned one of four categories, determined by examining the post and thread. Crucially, although the number of *threads* (85) in the category where Peter only offers help is nearly twice that of the other three categories added together, examining the number of *posts* involved shows a much more even division. Peter's involvement in threads where he is in a learning role involve more posts per thread; the number of Peter's posts in the threads where he is asking for help (176 at least), just exceed the number of posts where he is offering it (169 at most).

| Type of thread | Number of Threads | Number of Posts | Percentage of Posts |
|---|---|---|---|
| Thread started by Peter requesting help | 37 | 159 | 46.1% |
| Thread within which Peter asks a subsidiary question | 9 | 17 | 4.9% |
| Thread where Peter offers help | 85 | 156 | 45.2% |
| Thread where Peter offers help and asks a subsidiary question | 4 | 13 | 3.8% |
| Total | 135 | 345 | |

Table 10.2 Breakdown of Peter's Participation in Mathematical Threads

Table 10.3 below summarises Peter's posting pattern in terms of day and time of the first appearance of a post by Peter within the thread. The results demonstrate that Peter's use of AskNRICH was essentially an out-of-school activity. There was greater activity per day at weekends accounting for just over half of all his postings. The majority (85%) of weekday postings were made early morning, late afternoon or evening i.e. outside of normal school hours. Where there is activity during the day, this was mostly on dates that were likely to be during school holidays. Table 10.3 also shows only three first posts made after 10pm, indeed an examination of Peter's subsequent posts in threads reveals that Peter tended not to post much after 10.30pm. However this should not be taken as implying that no work is undertaken later than this; for example, ***Peter-Post318*** mentions being up until 1am the previous evening, trying to solve a problem started at 10pm.

| Day of posting (all 484 posts) | Sunday (113 posts) 23.3% | Saturday (98) 20.2% | Weekdays (273) 56.5% |
|---|---|---|---|
| | | | |
| Time of first post in the 62 mathematics weekend threads:<br>Earliest 8.37am Latest 11.12pm<br>54 threads were between 10am-10pm: 23 between 10am-2pm; 31 2pm-10pm.<br>6 threads were earlier than 10am & 2 later than 10pm | | | |
| | | | |
| Time of first post in the 73 mathematics weekdays threads<br>Earliest 7.16am Latest 10.14pm<br>65 threads were between 10am-10pm: 11 between 10am-4pm; 23 4pm-6pm; 31 6pm-10pm<br>7 threads were earlier than 10am & 1 later than 10pm | | | |

Table 10.3 Peter's Posting Patterns (Day and Time)

This section has introduced Peter through his own words and by quantitative analysis of his participation using a specially created catalogue of Peter's posts. The remaining sections of the chapter portray Peter's mathematical experiences as an AskNRICHer by presenting the findings of the analysis of all retrievable threads that contain his posts.

The following section analyses Peter's experience when engaged in a learning role and begins with reporting the findings from a detailed analysis of two sample threads where Peter is seeking help. Although within AskNRICH anyone can join in to offer help, the threads used to portray Peter in a learning role involve helpers who are more experienced, either peers or older people. In this respect then Peter's position is in the zone of van Lier's [1996: 194] four-part ZPD labelled '*assistance with more capable peers or adults*'. Equally, Peter's declared 'self-studying' as reported above, intrinsically also places him in the zone labelled '*inner resources*'.

## 10. 3 Peter Engaged in a Learning Role: Viewed through Two Sample Threads

All 46 threads where Peter is asking for help were read and any two could potentially have been selected as samples. In the end, the first selected was Peter's first retrievable thread (November 2006), a deliberate choice as it was the first. The thread was based on an Olympiad question, different to his school studies. The choice for the second sample was based on it being around the time when Peter had become established at asking for help, and in contrast to the Olympiad questions, based on A-Level mathematics, 'normal' school mathematics, even if Peter was studying this earlier than usual. In the event it was about the twentieth that Peter had initiated asking for help (May 2007). The two threads are précised below in Tables 10.4 and 10.5 respectively.

### 10.3.1 The Sample Threads

Peter was around 14-years-old at the time he initiated the first thread [**T1**] and at the start of a period of sustained posting. Peter's opening message, on a Saturday, had him explaining that he is reading a Number Theory book and, using the definition given for a prime number, he had come up with an idea that he knew could not be true but could not see why; a situation that might be considered as a self-aware misconception or self-induced cognitive conflict [Swan 2001]. During the exchanges, Peter receives a comprehensive explanation of the rigorous definition with links made back to other less rigorous definitions used in school.

| Post Number | Précis of message thread<br>[Comments in square brackets is additional commentary relevant to analysis] |
|---|---|
| P1 | ***Peter-Post9*** **Saturday 1.39pm**<br>*hi ive*[5] *just began to read an introduction to number theory and inequalities and i seem to have fallen at the first hurdle.* [Book defines a prime number]<br>*later* [the author] *proves that this also means that p only divides by ±1 and ±p but i am still confused. surely this means* [using the book's definition] *that 6 is a prime as 6\|42 and 6\|6 but not 7. clearly i have over looked something. please explain. thank you.* |
| P2 | ***HelpA*** **1.43 pm [4 minutes later]**<br>*Yes, but if we choose m and n to be 14 and 3, 6 goes into neither 14 nor 3, but goes into 14*3 = 42.* |
| P3 | ***Peter-Post10*** **1.46pm [4 minutes later]**<br>*Thanks. i new that it would be something basic like that i had overlooked. i thought it meant for any choice of multiples in which case all numbers would be prime. i get it now. cheers.* |
| P4 | ***Deputy Moderator [DM]*** **1.47pm [1 minute later]**<br>[Writes a very full explanation, only eight minutes after Peter's first post, as to the reason why the definition does not imply that 6 is a prime number. Extends the examples to illustrate the mathematical sense of 'or' which can imply 'both'].<br>*Does that help? Do post back if not!*<br>*By the way, it's integer, not interger. I don't want to be picky, but it's sometimes useful to know the correct spelling as it makes looking things up easier!*<br>[Continues to discuss the formal, rigorous definition in the book in relation to school definitions of prime numbers]. |
| P5 | ***DM*** **1.52pm**<br>*Oops, it took me so long to write that that* ***HelpA*** *got there first!* |
| P6 | ***HelpA*** **1.52pm**<br>*Better to get a full explanation after 10 minutes than a one-line sentence after 5!* |
| P7 | ***Peter-Post11*** **3.14pm**<br>*thanks to both of you. i get it now. just out of interest, others who have read the book how did you find it. ive never looked at number theory before but im finding it a bit harder to understand than other maths i have looked at. thanks to all* |

Table 10.4 Synopsis of First Sample Thread [T1]

The second thread [**T2**] results from Peter attempting to teach himself A-Level Mathematics and meeting trigonometrical equations for the first time. The help provided is 'light touch' in response to Peter asking only for a 'few subtle hints' [**T2-P1**]. During the exchange, Peter is given additional explicit technical help over how to post mathematical expressions so that they appear as 'normal' text [see **T2-P7** below].

[5] Apart from 'interesting' spellings, in common with some others used to a modern-day texting life, Peter does not use capital letters. Peter's spelling and punctuation will be normally left as it appeared in the posts.

| Post Number | Précis of Message Thread<br>[Comments in square brackets is additional commentary relevant to analysis] |
|---|---|
| P1 | ***Peter-Post385*** **Saturday 4.25pm**<br>*can some one help me with this problem. this is the first trigonometrical equation i have done so please take it slowly and drop me a few subtle hints. prove that: tan(45'+A/2)=(1+sinA)/cosA = cosA/(1-sinA) where 45' means 45 degrees sorry for the lack of formatting but i tried to put it in latex and it didn't work. thanks for any help.*<br>[Here formulae text written using only standard keyboard that can be open to confusion. The AskNRICH board has instructions on how to use a mathematical text (LaTeX)]. |
| P2 | ***Help1*** **[Team Member] 4.49pm**<br>*Hi Peter: For the first equality, do you know the formula for tan (x +y)? Do you need help with the second equality?*<br>*To write in LaTeX, start your line with \[, end with \], and write maths in the middle! (There's a slightly more comprehensive guide here.)* |
| P3 | ***Peter-Post386*** **5.12pm**<br>*if i sort the first equality then ill give the second a go.*<br>*Yes I do know the formula to expand tan(X+Y)*<br>*I have tried doing this and meant to post my workings here but forgot.* ☺<br>[Provides workings – all correct].<br>*from here i tried a variety of things but each one has failed, quite possibly because of a lack of competence on my part. can you nudge me from here please.* |
| P4 | ***Help2*** **5.20pm**<br>*Might help if you write sinA and cosA in terms of tan(A/2).* [A succinct but key hint]. |
| P5 | ***Peter-Post387*** **5.50pm**<br>*as i guessed i failed because of a lack of competence on my part when trying the correct option. i did this before but I think I must have gone wrong short of the mark. ill put it down to experience. if anyone is interested i did the following:*<br>[Shares solution although there is a small error writing 1-t not t-1in the final line].<br>*thanks* ***Help2*** *and* ***Help1***. [Misspells latter's name].<br>*ill post back if i can't get the second one* |
| P6<br><br><br>P7 | ***Help1*** **6.10pm**<br>*Almost - have another look at your very last line.*<br>*Great stuff otherwise!*<br>***Help1*** **6.13pm**<br>[Additional technical advice on even better use in marking-up mathematical text distinguishing between ordinary text and italicised script for variables]. |
| P8 | ***Peter-Post388*** **6.37pm**<br>*i put t-1 not 1-t like it should be ... and ... spelt your name wrong.*<br>*Now ive got the first one im motoring through the exercises. who would have thought trigonometry could be this much fun. thanks again* |
| P9 | ***Help1*** **6.56pm**<br>*Lol, I was referring to the 1-t, but that too!*<br>*Good luck with the rest of the problems* |

Table 10.5 Synopsis of Second Sample Thread [T2]

### 10.3.2 Observations on Learning Opportunities

The discussion below of the analyses of the two sample threads focuses on drawing out Peter's learning opportunities as an AskNRICHer, presented in four sub-sections.

#### 10.3.2.1 Mathematics Challenge

In both threads, all the work being undertaken is far in advance of the syllabus/curriculum intended for school pupils of Peter's age. For example, as already stated, **T1** requires a more rigorous definition of a prime number than is usually found in school. Furthermore, the underlying principles contained within the definition are also beyond school study. The further contributions [**T1-P4**] by **DM** provide connections with known school definition of prime numbers thus extending general mathematical knowledge. Thus in relation to van Lier's [1996: 179] types of Pedagogical Interactions this fits on the cusp of the 'freer' transaction/transformation. The topic of **T2** is most likely to be met during A-level studies, two years later than Peter's school year. Here Peter is trying to learn how to manipulate trigonometrical identities/equations. He is gaining mathematical knowledge through knowing formulae [**T2-P2**] and the hint to rewrite in terms of half angles [**T2-P4**], a common technique that facilitates algebraic manipulation across a range of similar problems. Therefore, in this instance the thread fits a less contingent, more restricted, type of pedagogical interaction somewhere between (good) IRF Questioning and Transaction.

#### 10.3.2.2 Experiencing Other People's Mathematics

The emphasis in this sub-section is on the opportunities for Peter to be immersed in an wholistic mathematical experience through the interactions with others who participate in offering help. Such 'one-step removed' experiences are a variant of Sawyer's [2006: 4] contention on enhanced learning opportunities through engaging in activities similar to professionals within the field. This is a theme that is returned to in the next chapter.

In the first thread, Peter immediately gains a mathematical experience through ***Help1***'s comment providing an example that counters Peter's idea and demonstrates the definition [**T1-P2**]. Just four minutes after ***Help1*** has replied, Peter is introduced (by ***DM***) to the need for more rigorous mathematics [**T1-P4**]. The ensuing exchange, a contender for a contingent conversation [van Lier 1996], a focus of Chapter Eleven, between these two helpers [**T1-P4-6**] about speed of reply versus depth of definition, provides Peter with an unplanned learning opportunity to consider relative merits of ways of 'doing mathematics'. The discussion on the merits of both the 'quick-fix' response and a more measured relational deliberation, connects the common non-rigorous definition with the mathematically rigorous. There is, however, no evidence to indicate whether or not Peter has noted this. Nevertheless, the ideas conveyed in this exchange would have a place in a mathematician's toolbox [see Section 11.5.1 Chapter Eleven p19/Thesis p258]. In the second thread, the advice [**T2-P7**] about italic and non-italic font being intrinsic to assuming variables and ordinary text respectively highlights, at least to mathematicians, an important difference. In this instance, the advice is explicit and thus it can inferred that Peter should have noticed it and potentially have a new tool. ***DM***'s message [**T1-P4**] asking for the word integer to be spelt correctly is not strictly experiencing mathematics and could be judged as a reprimand, though it is gently accomplished and accompanied with a firm, precise, (mathematician's) reason as to why the correct spelling would be useful!

#### 10.3.2.3 Exploiting Thinking and Understanding

This sub-section highlights instances where Peter's current thinking and understanding can be exploited by others, that in turn, provide him with the opportunity to develop his thinking and understanding further. At the start of **T1** there is clear evidence in the way the message had been phrased that there has been careful thought prior to posting. Having met in a book a new and rigorous definition of a prime number, Peter had realised [**T1-P1**] that the interpretation he is making could not be correct. Hence Peter was thinking and understanding that he had a misconception that led to a contradiction [see earlier reference to a self-inflicted cognitive conflict]. Even when the misunderstanding had disappeared, Peter continued to think about the principles involved by acknowledging that his (initial and incorrect) idea would mean every number being a prime [**T1-P3**], rather than quickly moving on with an unquestioning acceptance. This is an example that can be categorised as conceptual (deep) rather than surface thinking. The detailed definition [**T1-P4**] has provided the opportunity for relational understanding [Skemp 1987]. In the second thread there is some evidence that Peter is determined to understand, in the 'work-things-out-for-himself' sense, as he asks only for a hint as he encounters a new topic [**T2-P1**]. By experiencing/doing similar questions there is provision to make gains in understanding, though with the evidence available, the understanding gained can only be claimed to be at least instrumental [Skemp 1987]. Nonetheless, Peter's actions in each of these threads map neatly onto the elements of 'Learning Knowledge Deeply' listed by Sawyer [2006: 4].

#### 10.3.2.4 Reaching out to other AskNRICHers: following Ethos and Etiquette

This sub-section focuses on instances within Peter's posts conducive to his and others' learning, rather than just his own, and relates to the ethos and etiquette of the web-board. Although Peter's direct interactions in the two threads considered above was with more capable peers and adults, given the open-access to the web-board, his interactions could be considered additionally related intrinsically to two other zones of van Lier's [1996: 194] multiple ZPD: "interaction with equal peers" and "interaction with less capable peers". The relation to these two zones is explored fully later in Sections 10.4 and 10.5 where Peter takes on a teaching role, but some parts of posts resulting from Peter's adherence to the Posting Protocols [set out in Appendix 8.1 Chapter Eight pp20-21/Thesis pp467-468] provide some initial indirect examples.

So for example:

- giving a clear exposition of the problem and asking for an explanation [**T1-P1**]
- showing what he is able to do by being open in sharing his current confusions [**T1-P1**] and limitations [**T1-P7 T2-P3,5&7**]

ensures that Peter articulates his current state, both to himself and to others that will come to help or 'lurk'.

The following three examples illustrate adherence to the protocols creating a pleasant, sharing atmosphere within AskNRICH:

- apologising for forgetting to share his work in the first message [**T2-P3**]
- always being polite throughout, with a constant stream of '*please*' and '*thank you*', [**T1-P1&7**, **T2-P1&8**] and a more contemporary expression of gratitude of '*cheers*' [**T1-P3**]
- sharing his solution with others who might be looking at the exchange [**T2-P5**]

There are further noteworthy personal touches, falling outside the protocols, which result in AskNRICH being a 'happy place' in which to learn:

- suggesting that it his own lack of competence that is causing the problem [**T2-P3**]

- attempting to draw other people in by asking if anyone else is reading the book [**T1-P7**]
- a further relaxation with 'friends' with the use of the ☺ emoticon [**T2-P3**] and humour – '*motoring through*' and '*who would have thought that trigonometry could be this much fun*' [**T1-P7**][6]
- a light hearted (lol) exchange with ***Help1*** [**T2-P9**] whose intention had been to focus Peter back to 'the last line' of the mathematics, not the mis-spelling of ***Help1***'s name

The observations made here and in the preceding three sub-sections all add to and further exemplify the features and discussions presented in the previous chapter.

So far, Peter's learning role has been considered through two sample threads. This chapter continues using a series of threads on Mathematical Induction [MI] to investigate Peter's transition from learning the topic to taking on a teaching role, helping others who are subsequently encountering it.

## 10.4 From Learning Role to Teaching Role: Experiences of using Mathematical Induction

During examination of all threads involving Peter, those involving MI stood out because of both the number of threads and the quality of the learning and teaching evident in the posts, especially for someone of Peter's age. This bounded set of threads provided the opportunity to track Peter's mathematical progress in learning the topic and follow Peter's transition from a learning role to a teaching role. These threads, which again also typify AskNRICHers engaging in contingent conversations [van Lier 1996], can be related to all four parts of van Lier's multiple ZPD [ibid: 194] through Peter's interactions with more, equal and less capable peers and Peter's observable inner resources.

My own teaching experience leads me to consider that voluntarily, pursuing rigorous MI proofs beyond the standard series proofs is not the norm for most school students. As will

[6] the quotation selected as an introduction to the entire thesis.

become apparent from the threads, it could be said that Peter [aged only 14-15 years] appeared in the nicest sense of the word, *obsessed*, with this topic. Additionally, Peter recalled, unprompted, his MI experiences in his later email interview:

> *Without the Internet I would have struggled to learn new maths as I wouldn't have been able to find the most interesting areas of maths to buy books on and study further. For example, I taught myself a lot of number theory from the Internet before realising that I was very interested in it. I also use a lot of Internet articles and e-books to learn new maths, for example I learnt [mathematical] induction of the Internet (using Vicky Neale's[7] article on NRICH).* *[email communication]*

Eleven separate threads were used for the analysis. Table 10.6 below provides a précis of these threads based on an interpretation of the texts. The second column of the table indicates Peter's progression that can be related to van Lier's [1996: 194] four-part ZPD: starting with the self-study of the subject that brought him to learning from more able peers, to working with equal peers and on through to teaching less experienced peers, gaining increased inner resources in the process.

### 10.4.1 Threads Involving Mathematical Induction

As mentioned above, Peter is considerably younger than the normal age for meeting Mathematical Induction – some five years before it is expected to be part of a repertoire of proof strategies. However, as soon as the term 'mathematical induction' is mentioned Peter is proactive in finding out more.

[7] ***DM*** who also made the final exchanges in the first of the Mathematical Induction threads.

Table 10.6 Peter's Progression through Eleven Threads on Mathematical Induction

| Thread | Progression | Interpretive summary of events evident within the thread |
|---|---|---|
| One<br><br>November | The term Mathematical Induction is introduced | The first time the term is mentioned to Peter is in response to a thread started by his 29th post where a helper asks the question: *Do you know Induction? I'll start you off ….* |
| Two<br><br><br>November | Peter's first attempt at using MI<br><br>Posts 47-65 excluding 49 | Four days later Peter begins a new thread [see Appendix 10.7 for full text] calling it mathematical induction. Has been shown a proof (which he gives) but at a particular stage stops understanding it. Help quickly came, enabling the comment: *I'll sleep tonight now*. The next day asks if anyone can recommend a site he could visit and whether others have found it difficult when it comes to constructing one's own proof (rather than reading someone else's). The latter receives some nine different people helping. Two people set questions to practice whilst another offers the three steps always required in the formal proof. The ***Deputy Moderator*** [***DM***] points Peter to an article on the NRICH site [see email communication above]. |
| Three<br><br><br>January | Peter's second attempt at MI, eight weeks later<br><br>Posts 160-164 | Peter wonders whether he has covered the relevant material to be able prove an inequality using the technique. Receives an algebraic hint and a reminder that just needs induction arguments. Peter sends a solution wondering whether it can constitute a proof and a new helper replies *not quite* and again lays out the three steps. 15 minutes later Peter returns having done it though the working out is spasmodic with some gaps. The thread concludes with Peter recommending a web address that he had found useful *'especially if you have taught yourself'* |
| Four<br><br>February | A non-standard use of MI and debate between two other AskNRICHers on visual explanation versus MI<br>Posts 169,171&172 | A week later Peter starts the thread asking for a hint on a chess-board problem. After someone suggests simply looking at a chessboard, the answer is obvious. Two undergraduates discuss proving the problem using mathematical induction, with Peter 'lurking' – evidenced by a final comment. |
| Five<br><br>February | Approving someone's solution<br>Post 173 | Again this shows Peter 'lurking' as he offers congratulations to someone else who is in the early stages of trying out doing the proof: *yes that's correct, well done.* |
| Six<br><br>February | MI is not strictly needed<br><br>Posts 183,184&187 | Having started a thread on a four-part sequence question, one helper suggests that one way of solving it might be to use Induction. Peter admits that he is a bit confused on using it if not in the usual format, though has remembered the three necessary steps. |
| Seven<br><br><br><br>February | Peter offering help (for the first time) to a newcomer wishing to know about MI<br><br>Posts 255,257,259&260 | Three months later Peter succinctly gives the three steps to a new poster. He also provides an example of the proof concluding with an explanation behind the principle of MI. ***DM*** offers same article link as to Peter earlier. The new poster remains unsure so Peter reiterates the reasoning behind the three steps and promises: *to try and find a good exercise that [he] had used when he was leaning about the topic.* Four minutes later he posts the web reference, prefaced with the words: *here we go*. |
| Eight<br><br><br>March | Investigating an alternative proof which uses MI<br><br>Posts 314&315 | Another month later Peter starts a thread stating he has solved a problem using modular arithmetic but wanted to try it with mathematical induction. He shares his incomplete proof, using the three steps. Asking for a *'gentle push'* two people offer a little help and Peter realises his proof. |

| Thread | Progression | Interpretive summary of events evident within the thread |
|---|---|---|
| Nine<br><br>April | Peter voluntarily using MI<br><br>Posts 346-348 | A regular poster poses a problem and though help has been given after a week the problem is unresolved. But the words mathematical induction have been introduced seemingly from nowhere. In cross-posting Peter offers to try a proof by induction at the same time as a team member suggests: *staying away from induction for now (for several reasons)* [**AskNRICHer-Post421**]. However Peter decides to try it out and having sought confirmation in recognising an error, eventually succeeds, gaining praise from the team member: *Peter, Your proof by induction is great – well done. I don't usually like counting rectangles, but you have done it in a neat way* [**AskNRICHer-Post439**]. |
| Ten<br><br>May | Using MI on de Moivre's Theorem<br><br>Posts 399-405 | Peter posts a query[8] connected to using the binomial expression within a trigonometrical formulae proof and has done the base case (step one of the three required) but is unable to move forward. The first person offering help mentions using de Moivre's theorem. Peter admits that the book actually had given two hints – not only induction but also de Moivre's theorem but as he had never heard of the theorem wants to stick with using induction. Help returns with two further hints and stating de Moivre's theorem. 15 minutes later Peter begins his next post: *proof of de Moivre's Theorem: (a lot easier than I thought it would be ☺)*. At the start of the next thread on a different topic, Peter indicates that he has solved the original problem. |
| Eleven<br><br>May | Using MI to prove a pattern spotted<br><br>Posts 410&411 | The thread has been started by someone asking about how to find the formula for the sum of n rows of Pascal's Triangle. Peter's response begins with a comment that he has spotted a pattern: *summing the first few rows i noticed that the sum is* $2^{n+1}$*-1. now we want to prove this formula for all n. using induction there is a simple proof and i havent attempted any other method.* |

### 10.4.2 Analysis of Peter's Progress in Studying Mathematical Induction

The following analysis has been made based on a consideration of episodes evident within the sequence of eleven threads showing Peter progress as he engages with a new topic. Comparisons with what might happen within a classroom setting when learning any new topic is made where appropriate.

In **MI-T2**, Peter realises (through thinking and practice) that he lacks a full understanding of the proof. When he asks for help, he receives help from no less than nine people, all more experienced AskNRICHers, on what to do, is given further problems to try and is steered into completing the three step formal proof. These same processes would occur when the subject was taught in the classroom, but Peter has a fuller, more enhanced experience garnered from help from many teachers, not just one. Peter's posts at the end of the thread suggest that he is successful in solving the problem, from which it might be inferred that the

[8] During this thread Peter achieves veteran status [see Chapter Eight p9/Thesis p172].

topic has been learnt and understood. However, as **MI-T3** shows, after an interval of some eight weeks, the three steps have to be given again. It would be reasonable to assume therefore that the topic has neither been learnt nor fully understood and to some extent forgotten. Peter's comment at the end of the thread supports this inference:

> *because I have only done a few and it's been quite a while since I last did [one].* ***[Peter-Post164]***

In the mathematics classroom this would be addressed or pre-empted by the teacher referencing previous work. However by the end of the thread [**MI-T3**] the posts show progress in both learning and understanding the topic, although even as late as **MI-T8**, when he provides an incomplete proof, Peter has not yet fully grasped it. Consolidation and practice, a strategy recommended by the Cockcroft Enquiry [DES 1982 paragraph 243], comes, for example, in **MI-T4**, **MI-T6** and **MI-T8**, when re-imagined as exercise questions.

The discussion within **MI-T4** is of particular interest in two ways. Firstly, it provides further strong evidence of experiencing other people's mathematics as in Section 10.3.2.2 above. As the thread develops, two team members choose to employ MI to solve the problem. Secondly, it provides an authentic but unusual situation, rather than the normal set of routine number based exercises from a textbook, in which mathematical induction can be used to solve the problem. Thus the posts initiated within the thread provide Peter with an alternative and additional viewpoint of how MI can be implemented in problem-solving; a further strategy to place in the 'tool-box'. Later **MI-T6** and **MI-T9** show MI being used by Peter in different contexts and although in **MI-T8** an alternative proof has already been found, Peter is seeking a MI solution.

**MI-T5** marks a temporary departure for Peter from only asking for help as he offers congratulations on another AskNRICHer's successful solution. However, it is **MI-T7** that clearly sees Peter taking on the teaching role and offering resources that he had previously found helpful. It might therefore be inferred that the topic has now been learnt and understood, but given Peter's later requests in **MI-T8** and **MI-T9** for help, further learning on his part has still to take place. Nonetheless, the final thread in the sequence **MI-T11** again has Peter totally in a teaching role. Peter provides a pattern spotting formula and assures the person asking for help, a less experienced AskNRICHer, that it can be proved using MI (as he has done it!).

The posts within **MI-T10** suggest that Peter is moving towards 'mastering' the topic. At first glance Peter appears not to have mastered the topic since he asked for help, unable to move beyond the first step using the base case. However when a helper suggests using de Moivre's theorem, which Peter has not heard of, as an alternative method, Peter first proves the theorem using MI rather than applying it to the problem. He then completes the original problem using MI:

> *btw incase anyone is bothered I solved the question i posed earlier.*
> *Thankyou very much to anyone who helps, your all great resources ☺*
>
> ***[Peter-Post406]***

It was this thread that led to my earlier portrayal of Peter's interest as 'obsession' with the topic.

This section has used a bounded set of threads that involved a sequence of MI related problems in which Peter's increasing inner resources of knowledge, experience and memory enabled him to make the transition from asking for help to offering it. He has involved newcomers and offered 'old hands' an additional insight into the topic [**M1-T9**]. Thus all four parts of van Lier's [1996: 194] multiple ZPD have at some point been evoked within these threads. Furthermore, however, the threads illustrate that Peter's pursuit of understanding and a quest to understand underlying principles connects with 'Learning Knowledge Deeply' [Sawyer 2006: 4], 'Making Connections' [Ofsted 2008, Upitis et al. 1997], and the portrayal of 'Adam' in Anthony [1996]. By way of further example, in **MI-T10** having proved de Moivre's theorem using MI, Peter then asks how de Moivre's theorem is applicable to the original problem. Moreover the linear progression of understanding through the sequence of threads emulates that postulated by Byers and Herscovics [1977: 26] in their four-part model of understanding: informal knowledge, initial conceptualisation, gaining precision and finally formalisation.

The next section continues to follow Peter focusing on his participation when the primary purpose of posting is to offer help.

## 10.5 In the Role of Helper

Peter offered help almost from the start, with his seventh retrievable post [***Peter-Post15***], five days after re-establishing contact with AskNRICH. Peter made contributions to all three mathematics sections, offering varying help to AskNRICHers with less, the same and more experience. This section presents findings resulting from studying and analysing all 89 threads that included Peter in a helping role. The findings are reported under three main sections: Teaching Strategies; Helping but Learning, and paralleling Section 10.3.2.4 on the learning role, Reaching out to other AskNRICHers but this time *perpetuating* ethos and etiquette.

### 10.5.1 Teaching Strategies

Analysis of Peter's helping posts showed that, when he had expert knowledge that he could pass on, he engaged in many of the teaching strategies (for example, funneling and focusing) that (may) result in scaffolding the learning, found in the **ExThds** discussed in Chapter Nine earlier. Peter's strategies include: offering hints, using a different example to explain a technique and direct explanation. Examples of each are briefly reported below.

#### 10.5.1.1 Offering Hints

Just as Peter abided by the posting protocols when asking for help [see Section 10.3.2 earlier] he follows the protocol of offering some advice/hint on what to do next but not offering a solution. For example responding to a first time poster, Peter and one other offer help over one and a half hours. During the exchanges Peter engages in Socratic-Style Dialogue by posing a question back that implicitly includes the hints:

> *now that you know that the difference is 2, how do you write that in a formula involving n?* ***[Peter-Post306]***

… and later shows some partial working i.e. providing further but more explicit hints, ending with the remark:

> *i'll leave it to you from here* ***[Peter-Post308]***

Offering hints was normally Peter's initial strategy although he adapted this when appropriate.

#### 10.5.1.2 Offering an Alternative Example

Some five months in, Peter offers help to another first time poster on how to solve simultaneous equations. Near the beginning he posts:

> *multiply the equations by the number x is multiplied by and then subtract one equation from the other. since i havent explained very well i shall give another example not one of the questions you asked so you can still do the same question.* ***[Peter-Post374]***

and then does the example clearly and fully. He chose a different problem from the three the poster asked about, but ensured that, like the ones given, one equation had a negative coefficient rather than presenting the simplest type. The effective tactic of ensuring that the example offered maintained the same structure as the original is the same as that adopted by ***Help1*** in **ExThd1** in the previous chapter. Two of the three questions posted and Peter's example only required one of the equations to be multiplied throughout before addition of the two equations, but Peter's final, anticipatory line of advice [see also **ExThd1** and anticipate difficulties code **TRAD** Table 9.5 Chapter Nine p16/Thesis p199] made reference to at times needing both equations to be multiplied. This was a carefully thought through reply with the potential of being of great help to anyone embarking on this topic. Peter finished with the oft-used sign-off sentence …

> *Post back if you dont understand or get stuck.* ***[Peter-Post374]***

… in order to ensure that if this example was not successful then the exchange could continue[9].

#### 10.5.1.3 Direct Explanation

In the second of the three threads [**3Thd2]** forming the third Perspective [see Chapter Eleven], Peter was in a sustained exchange with a poster who could not solve a

[9] The Moderator, herself a teacher, did add a further response, beginning work on the first of the three questions posted by the originator. The reply, *'okay, thanks guys'* [**AskNRICHer-Post2**] came back, the plural implying help from more than one person had been useful.

part of the problem that Peter had earlier successfully solved unaided. Peter first attempted to help through offering a range of hints starting with the leading question:

> *what form do primes greater than 6 take* **[Peter-Post267]**

… which if known, would lead neatly towards the solution, or, as Peter added:

> *as soon as you see what to do this is very simple so I shall leave the hint at that.* **[Peter-Post267]**

However these hints proved insufficient and Peter continued trying to help. At one stage Peter mentioned modular arithmetic[10], which is essential to a solution, but it became clear that this would be a new topic for the poster. Peter then posted:

> *since I don't think that you understand modular arithmetic (don't worry about this) I shall write it in basic algebraic form.* **[Peter-Post270]**

… and after several further essentially didactic posts [see discussion of direct explanation in Section 9.4.3 p208], that nonetheless produced fruitful interactions, the poster arrived at the solution, leaving Peter to comment:

> *yes, well done this completes the proof. i remember fondly this question. this was my first bmo question i completed. arrr memories ... yes anyway. well done* **[Peter-Post271]**

The examples given in this section have been ones where Peter is entirely in command of the mathematics; the next section discusses episodes where he may not be.

### 10.5.2 Helping but Learning

Peter's enthusiasm for both the subject and AskNRICH sometimes led him to enter a thread in a helping role, but subsequent interactions provided him with the opportunity to also increase his own learning. van Lier [1996: 193] quotes the Latin dictum, *docendo discimus* and indeed many AskNRICHers openly subscribe to this dictum which translates as *we learn by teaching* [ibid], hence this section's title 'Helping but Learning'. In this respect Peter's interactions relate to the part of van Lier's [1996: 194] multiple ZPD: 'interactions with less capable peers'.

---

[10] see Houston [2009: 208] for the importance of this for number theorists.

This notion of 'helping but learning' is introduced using the first few posts of one particular thread [**H**], presented in Table 10.7. The thread occurred four-fifths of the way through Peter's contributions, thus at a stage when Peter had gained experience in helping others and was well settled into the helping role. The mathematics is unimportant, the thread is merely illustrating an exchange between Peter and someone he is trying to help.

| | Précis of Text | Critical Observations |
|---|---|---|
| H-P1 | ***O*** - Brand new poster. **Friday 8.20pm**<br>*I have read in several places that the algebraic numbers are closed under addition, subtraction and multiplication, and that this "could be easily proven", though I have not seen this done. .....*<br>*.....If x and y were algebraic numbers, what polynomial would x+y or xy be a root of? How can it be constructed? e.g. √2+√3.* | This is the very first post that ***O*** has made – or to be exact at least the first post made under the posting name. |
| H-P2 | ***Peter-Post390* 9.18pm**<br>*im afraid i dont know what closed under addition and subtraction means but the second question i can help you with.*<br>*let x=√2+√3now eliminating the square roots gives a polynomial with √2+√3 as roots.*<br>*i think this is what you wanted. somebody with more expertise shall be along soon though and give you more help than i can* | Peter will always admit when he does not know something but nevertheless selects the second part of the query and offers some help, again making clear that the query may be in need of more expertise than he is providing |
| H-P3 | ***Peter-Post391* 9.20pm**<br>*when i say eliminated i mean by squaring* | Shortly after posting instructions, Peter posts again to make instructions clearer – he has been thinking things through further or checking that his message makes sense. |
| H-P4 | ***O* 9.33pm**<br>*By closed I mean the sum or product of any two algebraic numbers is another algebraic number.* | The original poster turns helper in explaining what closed means in this context – Peter is thus learning something new too. |
| H-P5 | ***Peter-Post392* 9.59pm**<br>*if the two algerbraic numbers are expressable as the sum of roots of rationals then i think that it is quite easy to create a terminating algorithm to show there is a polynomial with that root. does this cover all algerbraic numbers? if not i'll leave it to some one else who knows there stuff.* | Peter continues to help even though the overall topic is beyond his experience. He then asks his own question about generality. He is still suggesting that someone more expert will help out … |
| H-P6 | ***Expert* 10.03pm**<br>*Yes .....* | … as they do here |

Table 10.7 Start of Thread [H] illustrating 'Helping but Learning'

Although in this thread, **H-P2** and **H-P5** exemplify Peter using existing knowledge to help as described in the previous section, analysis of the helping posts, finds examples in which Peter is:

- admitting if he is unsure of his help [**H-P2**, **H-P5**]
- making clear what he does not know [**H-P2**]
- gaining knowledge from originator or other helpers [**H-P4**, **H-P6**]
- posing his own question [**H-P5**]

The first three of these actions indicate that here Peter appears to be giving help in areas beyond his current knowledge and expertise, which he always acknowledges. The fourth, where he poses his own question, is a variant on other incidences in other threads where Peter picked up the problem posed and tried to find a solution not only for or with the person whose problem it originally was, but also for himself [see code **LRJ** Table 9.4 Chapter Nine p14/Thesis p197]. In all four Peter was essentially attempting to offer help, but the exchanges provided him with an opportunity to learn new work.

In some of his posts Peter appeared to be picking up a problem, trying it out and sharing his ideas, which were not necessarily always correct. In the extract below, Peter attempted to help with a problem posted[11] in **HD** (for university mathematics and thus well beyond the norm for his age) seemingly not to mind being told he was wrong:

> *yes, i realise ... sorry to anyone I mislead. ... sorry i seem to have led you down the wrong path. You are correct.* ***[Peter-Post****74]*

Shortly after, ostensibly offering help Peter posted his workings for a new example and feeling that final value was too small, asked for someone to check. When the person who posed the problem in the first place whom he was meant to be helping responded suggesting an error in the first line, Peter replied:

> *yeah, sorry i'm [worn out] and not thinking properly, that's what I meant by checking my answer.* ***[Peter-Post****77]*

Between them, they never get it correct. Eventually an 'expert' comes in: '*Right well, I think its just that you* [suggests the mistake] *…… or I have [got it wrong] and you guys are right'* [**AskNRICHer-Post1251**], a kindly let down perhaps. The thread was started from someone who is *'practising some questions for my interviews at Cambridge on Tuesday and thought it best to ask someone who is very good at maths!*' [**AskNRICHer-Post17**]. The impression gained from the friendly exchange was that it was potentially a valuable experience for the prospective interviewee. Working through things together, errors, misleads and all, as Peter was doing here in an attempt to help someone else, could actually help the originator clarify his/her own learning and understanding.

[11] Peter is simultaneously asking for help on one of his own questions.

In another thread some seven weeks later, after others had offered suggestions, Peter, even though as the final sentence reveals he had actually never met the topic(!), joined in with his own idea posed as a question:

> *would*[12] *the best strategy if A started be for A to bid £9.99 then it is not worth while B bidding so B gives up and loses nothing and A wins 1p? i have no idea of game theory though.* ***[Peter-Post155]***

In the following example Peter provided a method for solving the problem and was thus more fully in a teaching role. The problem was a relatively basic question, could the specific quadratic equation be found given two roots (solutions). Peter's post appeared some ten minutes after another AskNRICHer had easily addressed the problem two minutes after the question was posted. Peter's method was correct but tortuous, maybe a signal of not yet having full mastery of quadratics. Peter's post was greeted by one word: *'Um...'* [**AskNRICHer-Post973**] made by the person who had quickly solved the problem. Peter appeared content to accept this criticism with good grace:

> *Yes my method is not particularly elegant but i didn't see your solution when i posted mine. O well ☺* ***[Peter-Post379]***

… and in the process had been made aware of an alternative more elegant (efficient) solution[13].

This section has focused on Peter's posts where he has entered a thread in some form of helping role but the interactions provided him with the opportunity to increase his own learning. Peter undoubtedly has a mathematical attainment well in excess of his chronological age. For those occasions where a lack of experience appeared to show through, Peter was at the very least an enthusiastic 'amateur', with an apparent keenness to fully participate in AskNRICH. This extended yet further to Peter acting as teacher (or moderator) if standards slipped as demonstrated in the next section. Peter was helping to uphold the ethos and etiquette of AskNRICH as elaborated below.

---

[12] Reading all of the thread, the word 'would' in this context is being used in the sense of 'I feel that the best strategy would be'.

[13] This example is incidentally also illustrative of the asynchronous aspect of different helpers finding a solution and pressing the send button later than others.

### 10.5.3 Reaching out to other AskNRICHers: Perpetuating Ethos and Etiquette

Two posts selected in Section 10.5.1.3 above to illustrate direct explanation also demonstrate Peter showing care and consideration [***Peter-Post270]*** about any lack of experience on the other person's part and making social comments [***Peter-Post271***] to be friendly. Such posts highlight the ethos that makes AskNRICH 'a nice place to be' [see Section 8.4.2 Chapter Eight pp9-12/Thesis page pp173-176].

Peter's posts clearly show him being polite in welcoming newcomers to AskNRICH. For example a first time poster posting a question at three minutes past midnight and then three hours later making a further plea for help, is likely to be in a different time zone. Whilst the Moderator responds at 8.37am with a message suggesting patience, at 9.02am Peter provides help prefaced by a welcome:

> *first of all welcome to nrich. i am going to assume that you have done all of your other working* ***[Peter-Post333]***

A further examination of **H-P3** [Table 10.7 earlier] reveals Peter returning quickly to clarify meaning, appearing to write, post and then re-read, checking the help he has provided. Although this could be interpreted to imply some lack of confidence, it could equally imply conscientiousness on Peter's part to offer the most accurate help and advice he could. Peter, in still thinking about what he had written after he had posted, is perpetuating the ethos by example.

Peter appeared equally keen to ensure that other users of AskNRICH adhere to the protocols too. When a first time poster incorrectly started their thread in **PE**, Peter promptly 'reprimanded' them in a supportive manner:

> *can you post the question please. also bmo questions for future reference should be in onwards and upwards.* ***[Peter-Post288]***

The person responded by posting the question.

A further illustration can be seen in the following episode where Peter is 'defending' AskNRICH. A regular poster was trying to re-ignite the debate about the role of zero and was suggesting some fairly outlandish definitions that six other hardworking AskNRICHers were trying politely and using rigorous mathematics to refute. Eventually Peter joins in:

> *why do you insist in asking the same question in a different way when you have the AskNRICH team and other people have categorically told you that division by zero is undefined* ***[Peter-Post179]***

This did not exactly stop the debate immediately but it probably encapsulated what many were thinking.

The examples in this section are typical of the AskNRICHers' normal 'self-moderation' and their expectations of how AskNRICH should be used.

## 10.6 Features Summary 3

The Features Catalogue [a concept explained in Section 8.6 Chapter Eight pp16-17/Thesis pp179-180] for this chapter, relating to People Characteristics, is presented in Figure 10.1.

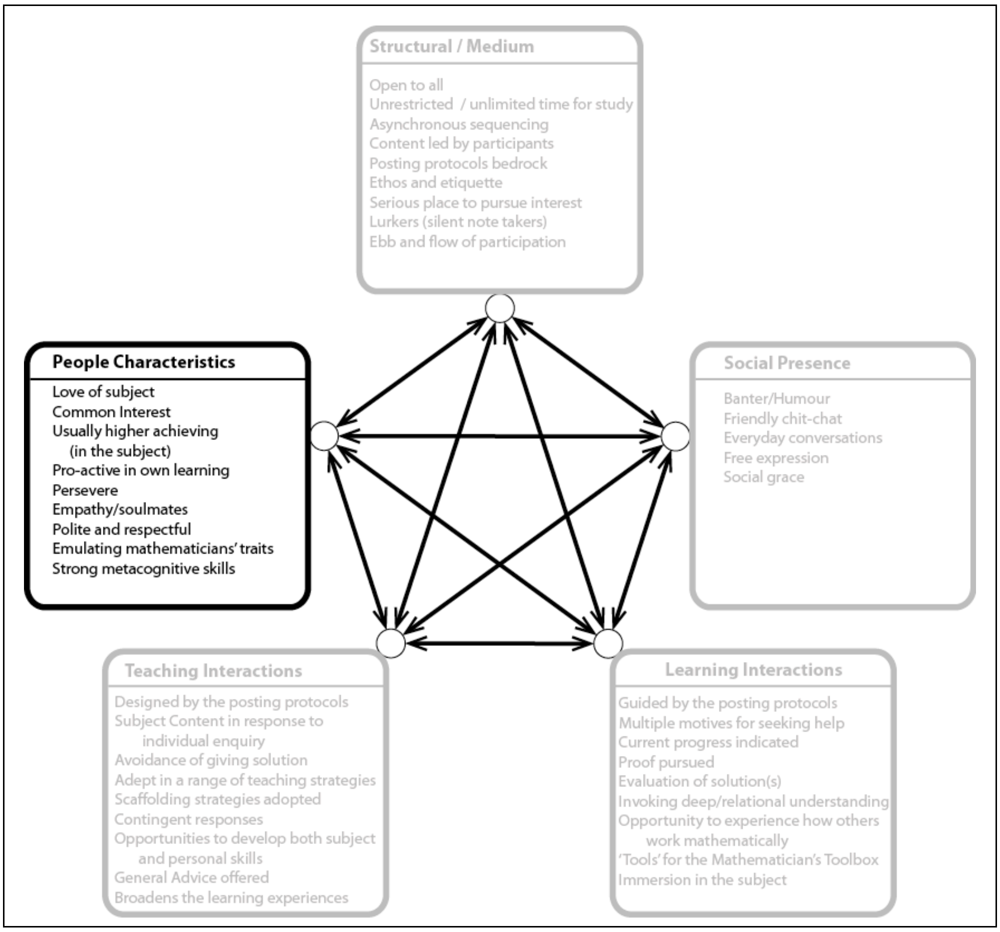

Figure 10.1 Features Catalogue: People Characteristics

## 10.7 Conclusions

This chapter has reported the second Perspective, an in-depth case study of one young mathematician, Peter, through his participation in AskNRICH and interaction with other AskNRICHers, analysing around 1900 posts in all. Peter used AskNRICH over an eighteen month period, at a time when he was much further advanced in his mathematics studies than other members of his school class and needed to work at challenging topics alongside others of comparable ability; a keen and enthusiastic mathematician pursuing independent study at a level above his current chronological age and beyond the school curriculum. AskNRICH provided the means for people working on their own, at home and alone, to (remotely) connect with like-minded others within this virtual environment [Sawyer 2006: 569] an opportunity rarely available anywhere else in the physical or virtual worlds.

From the threads used in the analysis throughout the case study a set of people characteristics are apparent that reinforce the picture of (school-aged) AskNRICHers engaged in mathematical study portrayed in the previous chapter. Peter perseveres to understand deeply the mathematics, seeking connections and relationships, pursuing proof and discussing aesthetic solutions. Peter is able to be open about his own achievements, thoughts and limitations. Peter is imaginative in his working, participating with good fun in his banter and display of humour. Peter is well-behaved in adhering to and maintaining the posting protocols. Peter shows and is shown politeness, respect, empathy, care and consideration to and by others.

Analysis of Peter's posting patterns confirmed that his participation was predominantly out of school hours and his posts were equally divided between asking for and offering help. In reporting the findings of this case study, the varying roles that Peter takes on at different times have been tested against van Lier's [1996: 194] conceptualisation of an individual's four-part ZPD. The analysis starts with studying Peter in a learning role through two sample threads, and includes a discussion of learning opportunities through: the mathematics involved, experiencing habits of more proficient mathematicians and how his self-determined thinking and understanding allowed other AskNRICHers to exploit these qualities. Threads resulting from Peter's persistent interest in mathematical induction initiated by a 'have you heard of' remark are then used to track his transition from a learning role to a helping role. The interest led Peter over a period of three months or so to gain familiarity and mastery [Wenger 1998] of the topic that he could later share with others. An extensive analysis of threads with Peter in a helping role brought out Peter's engagement with other people's problems, involving him in: offering expert help on topics he had already mastered; at times offering help when he was himself unsure of the answer but could work with the person requesting help to find the solution, and joining in another's thread asking his own questions to further his own development and interest. These varied ways of *'teaching but learning'* allowed Peter to work with, and gain knowledge, from more experienced, equal experienced and less experienced others, whilst at the same time using his own internal processes.

In presenting the results of the various analyses of Peter in a Learning and/or Teaching role, instances of each of the four parts of the individual's ZPD, as presented by van Lier [1996: 194] were exemplified. That is, van Lier's four-part ZPD can be adopted to model Peter's interactions in AskNRICH and hence those of AskNRICHers in general. Furthermore, given that this case study is based in a virtual environment, these findings also show that van Lier's model originally derived within a classroom context has the potential to be appropriated for a web-board context.

The focus of the next chapter, which concludes the three-way exploration of AskNRICH, is three distinct threads, all on the same mathematical question but posted at different times, incidentally all involving Peter. The exchanges in the threads are used to illustrate two subjects already touched on in this chapter: AskNRICHers' contingent conversations [van Lier 1996] and behaviours demonstrating traits attributable to professional mathematicians' ways of working [Cuoco et al. 1996].

## Postscript

Peter's use of AskNRICH is now only spasmodic. Even though after many months of regular posting the need to use AskNRICH decreased, it had helped him to become even more independent:

> *... From then* [the day I ventured in AskNRICH] *I began to use the site regularly to use askNRICH when I got stuck. ... I ask questions in askNRICH much less now as I do not use it so much anymore because I now have more of a determination to finish a problem than I used to and so spend more time on a single problem.*
>
> *[Peter email communication]*

## References


Anthony, G. (1996). Active Learning in a Constructive Framework. *Educational Studies in Mathematics*, 31(4), 349-369.

Byers, V., & Herscovics, N. (1977). Understanding school mathematics. *Mathematics Teaching*, 81, 24-27.

Cuoco, A., Goldenberg, E.P., & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curricula. *Journal of Mathematical Behavior*, 15(4), 375-402.

Department of Education and Science (DES). (1982). *Mathematics Counts*. London: DES/HMSO.

Houston, K. (2009). *How to think like a mathematician: a companion to undergraduate mathematics*. Cambridge: CUP.

Jones, K. (2002). Issues in the Teaching and Learning of Geometry. In L. Haggarty (Ed.), *Aspects of Teaching Secondary Mathematics* (pp. 121-139). London: RoutledgeFalmer.

Office for Standards in Education (Ofsted). (2008). *Mathematics: Understanding the Score.* London: Ofsted.

Sawyer, R.K. (2006). The New Science of Learning. In R.K. Sawyer (Ed.), *The Cambridge Handbook of The Learning Sciences* (pp. 1-16). New York: CUP.

Skemp, R. (1987). *The Psychology of Learning Mathematics* (American Expanded Edition). Mahwah, NJ: Lawrence Erlbaum Associates.

Swan, M. (2001). Dealing with misconceptions. In P.Gates (Ed.), *Issues in Mathematics Teaching* (pp. 147-165). London: RoutledgeFalmer.

Upitis, R., Phillips, E., & Higginson, W. (1997). *Creative mathematics exploring children's understanding*. London: Routledge.

van Lier L. (1996). *Interaction in the Language Curriculum: awareness, Autonomy & Authenticity*. Harlow: Pearson Education.

Wenger, E. (1998). *Communities of Practice – Learning, Meaning and Identity*. Cambridge: CUP.

| Post | Post text |
|---|---|
| Peter<br>Mon<br>8.12pm | I do appologise for keeping on asking all these questions you but I would be grateful for any help.<br>I am looking at mathematical induction and have been shown this proof<br>"we show that f(n)=4^(3n-2)+2(3n-2)+1 is divisible by 7 for all positive intergers n. call this proposition p(n). Now f(1)=4+2+1+7 so p(1) is true.<br>If p(k) is true then there exists an interger m such that f(k)=4^(3k-2)+2^(3k-2)+1=7m.<br>multiplying by 64 we get 4^(3k+1)+2^(3k+4)+64=448m and so f(k+1)=4^(3k+1)+2^(3k+1)+1=448m+2^(3k+1)-2^(3k+4)-63=448m-(2^(3k+1)(8-1))-63=7(64m-2^3k+1)"<br>----------------------------------------------------------<br>i understand all before and after the induction of f(k+1).<br>i cannot see why<br>f(k+1)=4^(3k+1)+2^(3k+1)+1 when f(k)=4^(3k-2)+2^(3k-2)+1<br>thanks in advance. |
| Help 1<br>8.16pm | substitute in k+1 for k in the formula f(k)=4^(3k-2)+2^(3k-2)+1 |
| Peter<br>8.22pm | GRRRRRR I've just spent about 1 1/2 hours staring at it in this book and couldnt see what I was missing and all it took was to see that it was 3k not k. just goes to show how easy it is to miss the silliest things. thanks anyway, I'll sleep tonight now. |
| Peter<br>8.50pm | Could I please ask two further questions.<br>1. Can anybody reccomend an indtroductory website to induction with some basic questions to "break me in to it".<br>2. I'm having some trouble with applying it in a proof im constructing myself though I can follow the proof when I see it. Is this normal when learning induction as a new subject? |
| Nick<br>(Help 2)<br>8.58pm | If you're at a college or a sixth form, try to get hold of an FP3/P6 book (it's FP3/P6 in Edexcel, not sure about other exam boards). I imagine you'll be able to get the hang of it by doing some of the questions in there ... |
| Peter<br>9.12pm | im currently doing my gcse's. im reading a book on number theory and am trying to get the hang of induction.<br>would you recommend that book any way?<br>if so would it be available readily from libaries or good book shops?<br>also is it good for covering other simlair topics? |
| Nick<br>9.19pm | Hmm. I guess a good idea in that case then would be to get your maths teacher to spend a little time aside with you to explain the general principle, and to give you some questions to have a go at. If he or she explains it to you then it could 'click', and you'll be fine from then on.<br>Here's a pdf I found after a quick Google search:<br>http://www.maths.uwa.edu.au/~gregg/Academy/1995/inductionprobs.pdf<br>Questions 1 and 2 look approachable, you should start with those ...<br>Also, there are solutions with the questions ^_^ ... |
| Peter<br>9.29pm | thanks for that.<br>its not that i dont understand the principle of how it works it just i struggle to construct any proves myself.<br>thanks for the questions.<br>they should help me understand how to construct them.<br>has anybody else on here had simlar problems?<br>thanks once again. |

| Post | Post text |
|---|---|
| Nick<br>9.38pm | Once you get the hang of it I'm sure you'll be fine. I remember my first induction proof hehe.<br>"Prove that the sum of the first 'n' natural numbers is equal to n(n + 1)/2."<br>Try that ^_^ , post back if you need help with it. |
| Peter<br>9.55pm | can you tell me if my method is correct please.<br>1+2+3...+n=f(n)=(n(n+1))/2<br>so f(n+1)=(n(n+1)/2)+n+1<br>=(n^2+3n+2)/2<br>if n=even,  f(n+1)=even+even+even=even<br>and if n=odd<br>f(n+1)=odd+odd+even=even<br>so f(n+1)/2<br>is this correct? thanks |
| Help 3<br>10.04pm | You should make sure you have a base case. Just put f(1)=1=1*2/2 so it is true for n=1. Once you have your $(n^2+3n+2)/2$ you wnat to show this is the same as f(n+1) (i.e what you get by substituting n+1 into f(n)) which isn't too hard if you factorise it. |
| Peter<br>10.13pm | thanks, i forgot f(1).<br>actually i sort of went off the point here i just realised, i ve been trying to prove that sequences equal interger values tonight so i was in that mind set. lol. |
| Peter<br>10.21pm | i've ran myself into knots. i appricieate i have gone down totally the wrong path. could i have some hints please? just so that i know the way in which i need to approach the question. thanks. |
| Peter<br>10:32pm | i can prove this by pairing of the 1st and last and 2nd and 1 from last numbers but cannot do it through induction. i think that i may need to fill in gaps in my knowledge. thanks for all the help you have offered. |
| Help 4<br>10.33pm | 1. Show it is true for some integer (usually 1).<br>2. Assume it is true for n=k.<br>3. Show that it is true for n=k+1. In this case, just see what you get when you factorise (n^2 + 3n + 2)/2. |
| Moderator<br>11.16pm | There's an NRICH article (given as a hyperlink) with an introduction and some questions which may be of interest. |
| Next day | |
| Peter<br>8.37am | when factorised it equals (n+1)(n+2) and this is the same as n(n+1))/2 with n=n+1, is this correct?<br>thanks to everyone for your help. |
| Help 5<br>4.25pm | Yes, that is correct. You pretty much had it first time until you went off on a tangent about odd and even numbers! ;)<br>Now try the $n^2$ one, that $1^2 + 2^2 + 3^2 + .... + n^2 = (2n^3 + 3n^2 + n)/6$, that is n(n + 1)(2n + 1)/6. |
| Peter<br>4.42pm | when n=1 (2n^3+3n^2+n)/6=1<br>now assume it is true for n<br>now induce (is induce the correct word) n+1<br>(2n^3+3n^2+n)/6+(n+1)^2=1^2 + 2^2 + 3^2 + .... + n^2+(n+1)<br>=(2n^3+3n^2+n)/6+n^2+1+2n<br>=(2n^3+3n^2+n)+6n^2+12n+6/6<br>=2n^3+9n^2+13n+6/6<br>=(2n^3+3n^2+n)/6 when n=n+1<br>so by induction this is true<br>yay!!!<br>can you check this and tell me whether it is correct. Thanks |

| Post | Post text |
|---|---|
| Help 6<br>4.48pm | quote:<br>it's FP3/P6 in Edexcel, not sure about other exam boards<br>It's fp1 on OCR MEI (and one of my favourite sections from the module) and I had the impression that it's fp1 on most other exam boards too, though I don't actually know for certain for anything other than OCR.<br>quote:<br>2n^3+9n^2+13n+6/6 =(2n^3+3n^2+n)/6<br>This stage doesn't seem to make any sense. Plus you appear to have ended up with what you started with instead of what you started with subed in. I suggest working with everything factorised - it allows you to see what you are going to end up with when you've proved it much easier.<br>The statement n=n+1 doesn't make much sense either.<br>Edit: Having reconsidered I think you may actually be correct (though I still can't really tell). It's just the use of n=n and n=n+1 (which is why you are supposed to stick to n=k and n=k+1) and the means that I have trouble seeing what you are saying. |
| Help 7<br>4.59pm | =2n^3+9n^2+13n+6/6<br>=(2n^3+3n^2+n)/6 when n=n+1<br>I see what you're trying to say, but you need to be much clearer than this! |
| Help 8<br>5.08pm | In particular, you need to explicitly write it out in terms of (n+1), so you get<br>...<br>$= (2n^3 + 9n^2 + 13n + 6)/6$<br>$= (2(n+1)^3 + 3(n+1)^2 + (n+1))/6$ |
| Peter<br>7.16pm | yeh i see that it would be helpful to use another symbol, it does look confusing. thanks to everyone who has helped me with this, you have realy improved my understanding of this topic. |
| Peter<br>7.40pm | would anybody mind explaining the meaning of xxxxx. thanks because i have encountered it in the induction article for the first time and can not decifer its meaning. Thanks. |
| Nick<br>7.47pm | That means the sum of all the integral values of 'n', from n = 1 to n = infinity. This is known as an 'infinite sum' I think. |
| Deputy Moderator (DM)<br>7.53pm | Nick has explained what you've written, and he's quite right, it's an infinite sum. I just wanted to point out that none of the sums in the article is infinite. They're all things like xxx which is the sum from *i*=1 to *i*=*n* of *i*, i.e., the sum of the integers from 1 to *n* inclusive. |
| Help 9<br>7.53pm | 1+2+3+4+5+... |
| Peter<br>8.11pm | yes i used the infinite sum above because i had trouble formatting. how would one go about reading such an expression? is the top number the upper limit, the bottom number the lower limit and the middle number the way in which it adds, so if it is xxx then it would increase by cubes? i appolagise for all these questions but am trying to learn a totally new subject.<br>thanks to all. |
| Help 6<br>8.20pm | $=1^3+2^3...+n^3$ |
| DM<br>8.36pm | I'd read what you've written as ``the sum from i=1 to n of i cubed", which is what raoulh has written out in symbols. Please don't apologise for asking questions: we're here to try to answer them! (And asking how to read maths is always a good idea, because books and articles very seldom tell you.) |
| Peter<br>8.39pm | thanks, your all really helpful. |

| Post | Post text |
|---|---|
| Next day | |
| Peter<br>8.23pm | xxxx<br>= (r^n)-1 when r =/= 1<br>--------<br>r-1<br>btw r(i-1) is r^(i-1)<br>this question was in the article on nrich on induction im reading and i guess that it means for all values of r. am i correct, im new to this notation as yoou may have guessed. Thanks |
| DM<br>8.28pm | You mean xxx, I think. (I'm putting that there so that you can click on it to find out how to get it in LaTeX. The important thing is that if you want more than one thing in a superscript (or subscript) then you have to include it in curly brackets.)<br>Yes, this is for any *r* (except 1, of course, because then we'd be dividing by 0, which isn't allowed). You might like to write out what this means without a big sigma sign, to get some practice at decoding. This thing is called a geometric series, by the way; I think they come up in A level maths.<br>I hope that the article is starting to make sense! |
| Peter<br>8.41pm | yes thank you i have found the article enjoyable and informative even though the questions after the first are hard for me to understand but at least its a challenge<br>am i correct that = 1 + r^1 + r^2 .. + r^n? |
| DM<br>8.42pm | Very, very close. But you might want to check exactly where the sum stops. |
| Peter<br>10.13pm | would it be r^(n-1) because of the i-1 rather than r? thanks |
| DM<br>10.19pm | Spot on well done! |