Problem Sheet 4

Choose some of these statements to prove. You may wish to start with some numerical examples to get some insight into the problem, then sketch out a proof, then try to write it up using formal mathematical notation.

1.

$$x_1 = 2^2 + 3^2 + 6^2$$
$$x_2 = 3^2 + 4^2 + 12^2$$
$$x_3 = 4^2 + 5^2 + 20^2$$

Show that x_n is always a perfect square.

2. Prove that if you add 1 to the product of four consecutive whole numbers the answer is ALWAYS a perfect square.

3. Prove that if you double the sum of two squares you get the sum of two squares. That is,

$$2(a^2 + b^2) = m^2 + n^2$$

for some m, n

4. Prove that for any prime $p > 3$, $p^2 - 1$ is divisible by 24.

5. Prove that if the integer n is divisible by 4 then it can be written as the difference of two squares.

6. Prove that 8T+1 is a perfect square if and only if T is a triangular number.

7. Prove that if $a^2 + b^2$ is a multiple of 3 then both a and b are multiples of 3.