Answer Sheet 1

1. $x + 15 = (x - 15)^2$ $x + 15 = x^2 - 30x + 225$ $x^2 - 31x + 210 = 0$

   Using the quadratic equation formula,

   $x = \frac{31 \pm \sqrt{31^2 - 4 \times 1 \times 210}}{2 \times 1}$

   $x = \frac{(31+11)}{2} = 21$ or $x = \frac{(31-11)}{2} = 10$

   As he had a birthday 15 years ago, he cannot be 10 years old so his age is 21.

2. The product simplifies to $\frac{n+1}{2}$, so it is an integer when n is odd.

3. Let the areas of the dark blue and light blue circles be$R$ and r respectively. Calculating the yellow area by subtraction: $A = \pi(R + r)^2 - \pi R^2 - \pi r^2 = 2\pi Rr$. Given that the yellow area is equal to the dark blue area, $2\pi Rr = \pi R^2$. So $R = 2r$. The radius of the yellow circle is $R + r$, so the three radii are in the ratio $1 : 2 : 3$

4. To end with a 9, the last digit must be a 3 or a 7. If the units digit is a 3, $(10a + 3)^2 = 100a^2 + 60a + 9$ so in order for the final digits to be 09, 6a has to have a 0 in the units, so a ends with 0 or 5. If the units digit is a 7, $(10a + 7)^2 = 100a^2 + 140a + 49$ so in order for the final digits to be 09, 14a has to have 6 in the units, so a ends with 4 or 9. So the last two digits are 03, 47, 53 or 97. Trying different cases, it turns out that 1503 is the smallest.

5. $81 + 36 + 4 = 121; 9 + 6 + 2 = 17$

6. There are a variety of different solutions that can be verified quite easily!