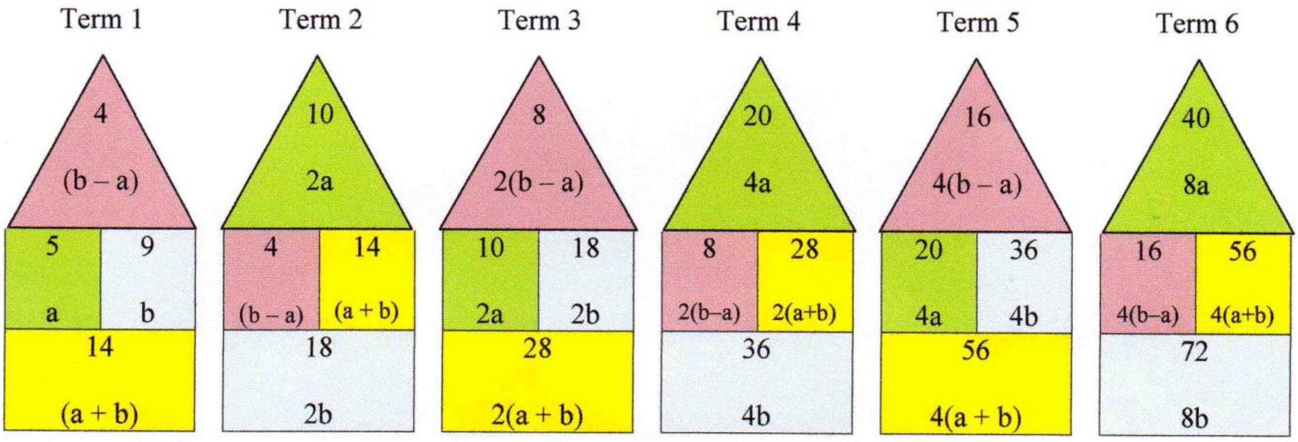
I thought of each shack as a term in a sequence. To complete the first term, I added the two middle numbers to make the floor number (5 + 9 = 14), and subtracted (9 – 5 = 4) to make the roof number. The floor and roof numbers of Term 1 then became the two middle numbers in Term 2, where I repeated the process.

This repeated action causes the numbers to flow in a particular way through the sequence. The numbers show up several patterns so I decided to use colour to try to illustrate this. I also used algebra to explain the patterns and labelled the two middle numbers in the first term *a* and *b*. This is what I found:



<u>Follow the colours, numbers or algebra to see the patterns:</u>

(1): roof numbers and floor numbers become the middle numbers of the next term

(2): middle numbers double to become the roof number and floor number of the next term

(3): alternate terms double, so $T_1 \times 2 = T_3 \times 2 = T_5$ and $T_2 \times 2 = T_4 \times 2 = T_6$ etc ...

I did explore many, many other sequences of number shacks by inputting different values for *a* and *b*. I worked forwards and backwards, used negative numbers and decimals, but because I followed the same repeated action to generate the first term and all subsequent terms, the patterns in algebra were the same, only the numbers varied. But I was able make some other observations:

(1) If *a* and *b* are odd, then all subsequent numbers will be even. If *a* is odd and *b* is even then both (b – a) and (a + b) are odd, but all other numbers will be even. The same thing happens when *a* is even and *b* is odd.

(2) If *b* = *2a*, then you will get repeating roof numbers:
e.g. *a* = 2 and *b* = 4

2	4	4	8	8	16
2 | 4	2 | 6	4 | 8	4 | 12	8 | 16	8 | 24
6	8	12	16	24	32

I substituted *b* = *2a* into the roof expressions to see why this works. Second roof term = *2a*, but *2a* = *b*, so second roof term = *b*. Third roof term = *2(b – a)* or *(2b – 2a)* but *2a* = *b*, so third roof term = *(2b – b)* = *b*. This pattern continues (*4a* = *2b* and *4(b – a)* = *2b* where *b* = *2a*, etc...).

2(b – a) = (a + b) also works:
e.g. (b – a) = 2 and *(a + b)* = 4.

2	2	4	4	8	8
1 | 3	2 | 4	2 | 6	4 | 8	4 | 12	8 | 16
4	6	8	12	16	24

This time I substituted in a different way:
2(b – a) = *(a + b)* expand the brackets to get *2b – 2a* = *(a + b)* and rearrange: *2b* – *(a + b)* =*2a*, then expand again: *2b – a – b* = *2a* or *(b – a)* = *2a*, so first roof term *(b – a)* = second roof term *2a*. The pattern continues.

Term nr	Roof nr	Algebra
1	4	(b – a)
2	10	2a
3	8	2(b – a)
4	20	4a
5	16	4(b – a)
6	40	8a
7	32	8(b – a)
8	80	16a

(3) If $a = 5$ and $b = 9$ (our original question), I noticed a deeper pattern:

20 – 8 = 12 or 3(4) ... 40 – 16 = 24 or 3(8).... 80 – 32 = 48 or 3(16)

Just looking at the numbers it's obvious why the pattern will hold (each number doubles so the relationship is the same). Algebra helps explain why:

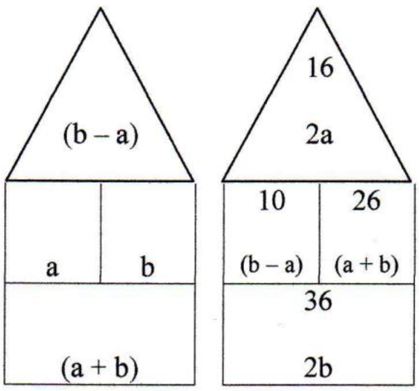
4a – 2(b – a) = 3(b – a) simplifies to 4a = 5(b – a), and 20 = 5(4)

8a – 4(b – a) = 3(2(b – a)) simplifies to 8a = 10(b – a), and 40 = 10(4)

(5) Similar patterns exist for some other numbers, e.g. If $a = 4$ and $b = 6$ then:
16 – 4 = 12 or 6(2) because 4a – 2(b – a) = 6(b – a) simplifies to 4a = 8(b – a), and 16 = 8(2),
32 – 8 = 24 or 6(4) because 8a – 4(b – a) = 6(2(b – a)) simplifies to 8a = 16(b – a), and 32 = 16(2),
64 – 16 = 48 or 6(8), because 16a – 8(b – a) = 6(4(b – a)) simplifies to 16a = 32(b – a), and 64 = 32(2),

But not for every set, e.g. if $a = 4$ and $b = 7$, there wasn't a roof number that could act as a common factor.

When I was working, I used a sheet with 10 blank number shacks, which were easy to fill. I just worked out the first (or last or middle) shack from any pair of numbers using the three rules I'd discovered at the beginning then 'leapfrogged' forwards or backwards ($T_1 \times 2 = T_3 \times 2 = T_5$ ... $T_{10} \div 2 = T_8 \div 2 = T_6$), and used reverse logic to find the shack to the left if I had to. But this can also be explained using algebra:



The number shack could be any term, but I just called it T_2 because T_1 and T_2 form a pair, and applied the algebra I'd discovered. Next I used simultaneous equations:

$$\begin{array}{rrrrrl} & b & + & a & = & 26 \\ + & b & - & a & = & 10 \\ \hline & 2b & & & = & 36 \\ & & & & & \\ & b & & & = & 18 \end{array} \qquad \begin{array}{rrrrrl} & b & + & a & = & 26 \\ - & b & - & a & = & 10 \\ \hline & & & 2a & = & 16 \\ & & & & & \\ & & & a & = & 8 \end{array}$$

Now that I had a value for a and b, I could easily work out the roof number $(b - a) = 10$, and floor number $(a + b) = 26$

Number shacks are an infinite sequence. Because there are lots of different patterns running through the sequence, I thought I would start by separating out patterns with common features, so I began with odd termed shacks (O_1, O_2, O_3...) and even termed shacks (E_1, E_2, E_3...)

If $a = 5$ and $b = 9$, we know that the roof numbers are: 4,10,8,20,16,40,32,80,64,160...Odd Term roof numbers are: 4,8,16,32,64...Even Term roof numbers are: 10,20,40,80,160...

E_1: $2a = 10$, E_2: $4a = 20$, E_3: $8a = 40$, etc. Because the maths was just 10 x 2 x 2 x 2 etc, the pattern must be using exponents. I realised that I was actually calculating $E_1 = (10 \times 2^0) = 10$, $E_2 = (10 \times 2^1) = 20$, $E_3 = (10 \times 2^2) = 40$, $E_4 = (10 \times 2^3) = 80$, etc.

I noticed that the exponent is the term number less 1, and the number 2 represented the common factor. Now I could write a rule which works $2a \times 2^{(n-1)}$ (I did lots of calculations to check).

I then repeated this for the odd numbered terms:
$O_1 = (4 \times 2^0) = 4$, $O_2 = (4 \times 2^1) = 8$, $O_3 = (4 \times 2^2) = 16$, etc. Again, the exponent was the term number less 1, with 2 the common factor. Again, I could write a rule: $(b - a) \times 2^{(n-1)}$. Again this held.

I went on to calculate a rule and test this for each the variables in the first two number shacks. Each rule was (the first term) $\times\ 2^{(n-1)}$. Because I'd used a lot of letters, I thought it easiest to think of the complete sequence as Z_1, Z_2, Z_3, Z_4, Z_5 etc. I could now write a general rule for all of the variables: $Z_1 \times 2^{(n-1)}$.

Example: $a = 4$, $b = 6$, $(b - a) = 2$, $(a + b) = 10$

	Z_1	O_1	O_2	O_3	O_4	O_5
Odd Terms	$(b - a)$	$2 \times 2^0 = 2$	$2 \times 2^1 = 4$	$2 \times 2^2 = 8$	$2 \times 2^3 = 16$	$2 \times 2^4 = 32$
	a	$4 \times 2^0 = 4$	$4 \times 2^1 = 8$	$4 \times 2^2 = 16$	$4 \times 2^3 = 32$	$4 \times 2^4 = 64$
	b	$6 \times 2^0 = 6$	$6 \times 2^1 = 12$	$6 \times 2^2 = 24$	$6 \times 2^3 = 48$	$6 \times 2^4 = 96$
	$(a + b)$	$10 \times 2^0 = 10$	$10 \times 2^1 = 20$	$10 \times 2^2 = 40$	$10 \times 2^3 = 80$	$10 \times 2^4 = 160$

Example: $2a = 8$, $2b = 12$, $(b - a) = 2$, $(a + b) = 10$

	Z_1	E_1	E_2	E_3	E_4	E_5
Even Terms	$2a$	$8 \times 2^0 = 8$	$8 \times 2^1 = 16$	$8 \times 2^2 = 32$	$8 \times 2^3 = 64$	$8 \times 2^4 = 128$
	$(b - a)$	$2 \times 2^0 = 2$	$2 \times 2^1 = 4$	$2 \times 2^2 = 8$	$2 \times 2^3 = 16$	$2 \times 2^4 = 32$
	$(a + b)$	$10 \times 2^0 = 10$	$10 \times 2^1 = 20$	$10 \times 2^2 = 40$	$10 \times 2^3 = 80$	$10 \times 2^4 = 160$
	$2b$	$12 \times 2^0 = 12$	$12 \times 2^1 = 24$	$12 \times 2^2 = 48$	$12 \times 2^3 = 96$	$12 \times 2^4 = 192$

The expressions work, but they doesn't describe the complete sequence from Term 1 to Term *n*, which is what I wanted, so I thought about the actual position of terms and how they translate:

Number Shack Term (n)	Odd Term	Even Term
1	1	
2		1
3	2	
4		2
5	3	
6		3
7	4	
8		4
9	5	
10		5

To convert an even Number Shack Term into an Even Term, divide by 2:

Even numbers: $\frac{n}{2}$

To convert an odd Number Shack Term into an Odd Term, add 1 and divide by 2:

Odd numbers: $\frac{n+1}{2}$

Now I could unify my expressions.

To find any Odd Number Shack to the right:

$$Z_1 \times 2^{\left(\frac{(n+1)}{2} - 1\right)}$$

To find any Even Number Shack to the right:

$$Z_1 \times 2^{\left(\frac{n}{2} - 1\right)}$$

You can also work the figures backwards by using a negative exponent to divide by multiples of 2, so:

To find any Odd Number Shack to the left:

$$Z_1 \times 2^{-\left(\frac{(n+1)}{2} - 1\right)}$$

To find any Even Number Shack to the left:

$$Z_1 \times 2^{-\left(\frac{n}{2}\right)}$$

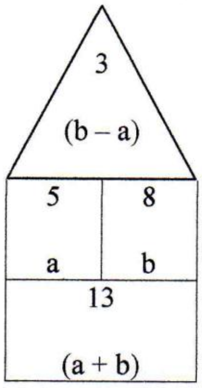
Even without the rule, it is very easy to generate the next few terms, left or right, because you just follow the pattern, dividing or multiplying. But it is harder to visualise say, 20 to the left, just in your head. The last part of the question was asking for this, so I used my expressions:

1) (i) Calculate the 4th shack to the right of :
(ii) Calculate the 10th shack to the right of:
(iii) Calculate the 100th shack to the right of:
(iv) Calculate the 7th shack to the right of:
(v) Calculate the 99th shack to the right of:

I started with two numbers: a = 5 and b = 8.

Next I calculated the roof and floor numbers.

Then labelled this Term 1 (T_1), an odd shack.



The 4th shack to the right of T_1 is T_5 as 1 + 4 = 5. The target term is odd, so I used my odd formula.

$a \times 2^{\left(\frac{(5+1)}{2} -1\right)} = 5 \times 2^2 = 20$ left middle

$b \times 2^{\left(\frac{(5+1)}{2} -1\right)} = 8 \times 2^2 = 32$ right middle

$(b-a) \times 2^{\left(\frac{(5+1)}{2} -1\right)} = 3 \times 2^2 = 12$ roof

$(a+b) \times 2^{\left(\frac{(5+1)}{2} -1\right)} = 13 \times 2^2 = 52$ floor

The 10th shack to the right of T_1 is T_{11} as 1 + 10 = 11.

$a \times 2^{\left(\frac{(11+1)}{2} -1\right)} = 5 \times 2^5 = 160$ left middle

$b \times 2^{\left(\frac{(11+1)}{2} -1\right)} = 8 \times 2^5 = 256$ right middle

$(b-a) \times 2^{\left(\frac{(11+1)}{2} -1\right)} = 3 \times 2^5 = 96$ roof

$(a+b) \times 2^{\left(\frac{(11+1)}{2} -1\right)} = 13 \times 2^5 = 416$ floor

The 100th shack to the right of T_1 is T_{101} as 1+100=101.

$a \times 2^{\left(\frac{(101+1)}{2} -1\right)} = 5 \times 2^{50}$ left middle

$b \times 2^{\left(\frac{(101+1)}{2} -1\right)} = 8 \times 2^{50}$ right middle

$(b-a) \times 2^{\left(\frac{(101+1)}{2} -1\right)} = 3 \times 2^{50}$ roof

$(a+b) \times 2^{\left(\frac{(101+1)}{2} -1\right)} = 13 \times 2^{50}$ floor

The 7th shack to the right of T_1 is T_8 as 1 + 7 = 8. The target term is even so I used my even formula.

$(b-a) \times 2^{\left(\frac{8}{2} -1\right)} = 3 \times 2^3 = 24$ left middle

$(a+b) \times 2^{\left(\frac{8}{2} -1\right)} = 13 \times 2^3 = 104$ right middle

$2a \times 2^{\left(\frac{8}{2} -1\right)} = 10 \times 2^3 = 80$ roof

$2b \times 2^{\left(\frac{8}{2} -1\right)} = 16 \times 2^3 = 128$ floor

The 99th shack to the right of T_1 is T_{100} as 1+99=100.

$(b-a) \times 2^{\left(\frac{100}{2} -1\right)} = 3 \times 2^{49}$ left middle

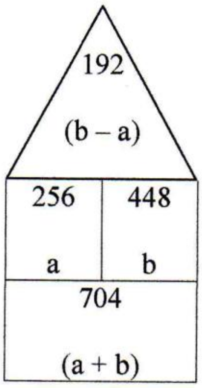
$(a+b) \times 2^{\left(\frac{100}{2} -1\right)} = 13 \times 2^{49}$ right middle

$2a \times 2^{\left(\frac{100}{2} -1\right)} = 10 \times 2^{49}$ roof

$2b \times 2^{\left(\frac{100}{2} -1\right)} = 16 \times 2^{49}$ floor

Now to use negative exponents and work backwards:

2) (i) Calculate the 6th shack to the left of :
(ii) Calculate the 12th shack to the left of:
(iii) Calculate the 100th shack to the left of:
(iv) Calculate the 5th shack to the left of:
(v) Calculate the 99th shack to the left of:



I started with two numbers: a = 256 and b = 448

As before I calculated the roof and floor numbers.

Then labelled this Term 1 (T_1), an odd shack.

Working to the left I started with T_1, but ignored zero. This is because I was continually dividing by 2, so needed to find the correct term and exponent. It makes sense once you start working through examples:

The 6th shack to the left of T_1 is an odd shack as 1 + 6 = 7, so I used my odd formula.

$a \times 2^{-\left(\frac{(7+1)}{2}-1\right)} = 256 \times 2^{-3} = 32$ left middle

$(b-a) \times 2^{-\left(\frac{(7+1)}{2}-1\right)} = 192 \times 2^{-3} = 24$ roof

$b \times 2^{-\left(\frac{(7+1)}{2}-1\right)} = 448 \times 2^{-3} = 56$ right middle

$(a+b) \times 2^{-\left(\frac{(7+1)}{2}-1\right)} = 704 \times 2^{-3} = 88$ floor

The 12th shack to the left of T_1 is an odd shack as 1 + 12 = 13

$a \times 2^{-\left(\frac{(13+1)}{2}-1\right)} = 256 \times 2^{-6} = 4$ left middle

$(b-a) \times 2^{-\left(\frac{(13+1)}{2}-1\right)} = 192 \times 2^{-6} = 3$ roof

$b \times 2^{-\left(\frac{(13+1)}{2}-1\right)} = 448 \times 2^{-6} = 7$ right middle

$(a+b) \times 2^{-\left(\frac{(13+1)}{2}-1\right)} = 704 \times 2^{-6} = 11$ floor

The 100th shack to the left of T_1 is an odd shack as 1 + 100 = 101

$a \times 2^{-\left(\frac{(101+1)}{2}-1\right)} = 256 \times 2^{-50}$ left middle

$(b-a) \times 2^{-\left(\frac{(101+1)}{2}-1\right)} = 192 \times 2^{-50}$ roof

$b \times 2^{-\left(\frac{(101+1)}{2}-1\right)} = 448 \times 2^{-50}$ right middle

$(a+b) \times 2^{-\left(\frac{(101+1)}{2}-1\right)} = 704 \times 2^{-50}$ floor

The 5th shack to the left of T_1 is an even shack as 1 + 5 = 6, so I used my even formula.

$(b-a) \times 2^{-\left(\frac{6}{2}\right)} = 192 \times 2^{-3} = 24$ left middle

$2a \times 2^{-\left(\frac{6}{2}\right)} = 512 \times 2^{-3} = 64$ roof

$(a+b) \times 2^{-\left(\frac{6}{2}\right)} = 704 \times 2^{-3} = 88$ right middle

$2b \times 2^{-\left(\frac{6}{2}\right)} = 896 \times 2^{-3} = 112$ floor

The 99th shack to the left of T_1 is an even term as 1 + 99 = 100

$(b-a) \times 2^{-\left(\frac{100}{2}\right)} = 192 \times 2^{-50}$ left middle

$2a \times 2^{-\left(\frac{100}{2}\right)} = 512 \times 2^{-50}$ roof

$(a+b) \times 2^{-\left(\frac{100}{2}\right)} = 704 \times 2^{-50}$ right middle

$2b \times 2^{-\left(\frac{100}{2}\right)} = 896 \times 2^{-50}$ floor