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Becoming confident and competent as a problem solver is a complex process that requires a range of skills and experience. As teachers we can support this process in three principal ways:

- Through our choice of task
- Through structuring the stages of the problem-solving process
- Through explicitly and repeatedly providing children with opportunities to develop key problem-solving skills.

Choice of task

NRICH offers a wide range of rich tasks to engage learners in the problem-solving process. We want all our tasks to be used in such a way that they enable learners to explore and work from their own level of understanding, and then build on this towards new understandings. We believe that this approach offers the opportunity for rich, embedded learning.

Some of our NRICH tasks are simple games that can be played time and time again to develop numerical fluency as well as problem solving and reasoning, such as Spiralling Decimals <https://nrich.maths.org/10326>, and Totality <https://nrich.maths.org/1216> (see also Developing Number Fluency – What, Why and How <https://nrich.maths.org/10624>), whilst others may be non-routine problems that have a specific solution (such as Shape Times Shape <https://nrich.maths.org/5714>). Some may take a short time, like Shut the Box <https://nrich.maths.org/6074>, whilst others may intrigue and challenge over more than one lesson, like Dice in a Corner <https://nrich.maths.org/8586>.

The key is to be clear how you want to use a particular problem to support the development of the children's skills to problem solve. It is tempting to choose a problem that relates to the mathematical content that you have been working on in class. However, whilst children need to be fluent with the mathematical content demands of any problem they tackle, it may be more productive to choose a problem that builds on a specific element of problem solving that you are working on as a class, and uses content that they are very familiar, and more confident, with. The Primary National Strategy (May 2004) suggested that there are five different types of problem:

- Finding All Possibilities (Half Time <https://nrich.maths.org/7408>)
- Logic (Teddy Town <https://nrich.maths.org/108>)

- Visual (Baravelle <https://nrich.maths.org/6522>)
- Rules and Patterns (Ip Dip Sky Blue <https://nrich.maths.org/7185>)
- Word Problems

This is one way of trying to divide up problems into categories. However, it is clear that not all problems fit neatly into just one category and we may debate the categories. What this idea of different types does offer, however, is a way of giving children the experience of a similar type of problem over a number of weeks so that they can gain some proficiency. For example, you might like to explore a number of problems that encourage the children to Find All Possibilities – see our collections: Lower Primary <https://nrich.maths.org/9779>
Upper Primary <https://nrich.maths.org/9784>

The stages of the problem-solving process

The problem-solving process can usually be thought of as having four stages:

- Stage 1: Getting started
- Stage 2: Working on the problem
- Stage 3: Digging deeper
- Stage 4: Concluding

We can helpfully spend time with children concentrating on one of these stages explicitly, in turn, as they learn to become confident problem solvers.

Stage 1: Getting started will mean offering them strategies to help them engage with the problem. These could be prompts such as:

- Tell me/a partner what you think the problem is about.
- What would help you understand the problem? You might like to draw a diagram, act it out or represent it with a model.
- What other problems have you seen that are ‘a bit like’ this one?
- What mathematical skills have you got that could be helpful here?
- Try making a simpler case to get an idea of how the problem works.

Stage 2: Working on the problem will usually involve using one or several problem-solving skills such as:

- Trial and improvement

- Working systematically (and remember there will be more than one way of doing this: not just the one that is obvious to you!)
- Pattern spotting
- Working backwards
- Reasoning logically
- Visualising
- Conjecturing.

These problem-solving skills are in a random order, although the first two, trial and improvement and working systematically, are key skills that will support children to become competent as problem solvers.

The children will benefit from becoming proficient in each of these skills and working on one of them as a key focus in a lesson or series of lessons could be a useful strategy.

Stage 3: Digging deeper usually happens when the problem has been explored and then it is possible to look for generalisations and proof. Here's an example from our NRICH problem Make 37: <https://nrich.maths.org/1885>

This problem is not possible because with an even number of odd numbers you cannot make an odd number. You can make 36 and 38 using 10 numbers but not 37. You can make 37, but by using 9 numbers. Here are some examples:

36 (10 numbers): $5 + 5 + 5 + 5 + 5 + 3 + 3 + 3 + 1 + 1$

38 (10 numbers): $1 + 1 + 1 + 3 + 3 + 5 + 5 + 5 + 7 + 7$

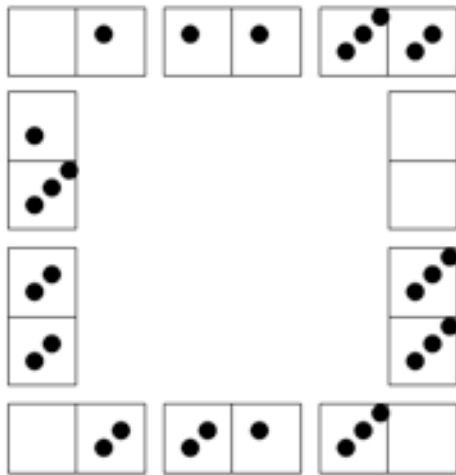
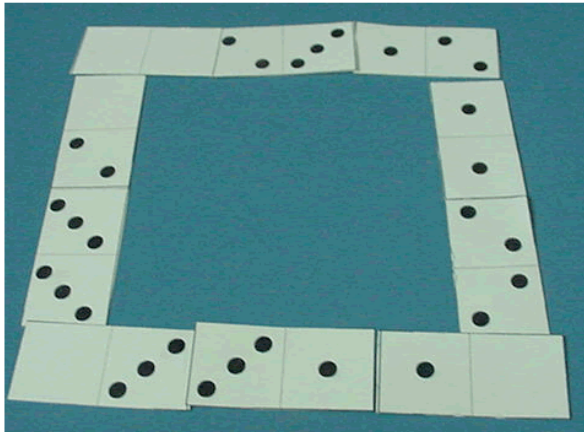
37 (9 numbers): $5 + 5 + 5 + 5 + 5 + 5 + 5 + 1 + 1$

Stage 4: Concluding is the part of the problem-solving process where we support the children to learn to explain their findings both verbally and in writing. They may need some support to explain succinctly, use words such as 'because' and to use the appropriate mathematical vocabulary correctly. This all takes time, attention and practice.

Written recording could be in the form of a photograph, diagram or written explanation. Here are some children's solutions to our NRICH problem Domino Square <https://nrich.maths.org/146> that illustrate how recording may develop:

<http://nrich.maths.org/10865>

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0	3 1	1 1	2
0			2
3			2
3			1
2			1
0	3 2	3 0	0

Children will need support to develop their proficiency with written recording. Here's an example of a child explaining their thinking for Eggs in Baskets <https://nrich.maths.org/2002>. She has used a trial and improvement approach.

If there was 1 egg in the brown basket, no eggs in the red basket and 3 eggs in the pink basket, that would only make 4 eggs.
If there were 2 eggs in the brown basket, 1 egg in the red basket and 4 eggs in the pink basket, this only makes 7 eggs.
So finally, if there were 3 eggs in the brown basket, 2 eggs in the red basket and 5 eggs in the pink basket, there would be 10 eggs altogether.

You can read more about types of recording in our article at <https://nrich.maths.org/9871>.

Also in the concluding part of the problem-solving adventure children will need to be supported to compare different strategies that were used to solve the problem in order to consider the efficiency of the method and the elegance of the solution. This will enable them to see how they might refine their own methods or adopt a different one next time they encounter a similar problem.

The skills needed for a problem-solving task

By this we mean the problem-solving skills listed above in Stage 2: working on the problem. It will help the children become fluent in these if you take every opportunity to explicitly talk about them and use the appropriate language when they occur in games or larger problem-solving activities. You may like to focus on developing one or two at a time.

Our youngest learners can start thinking about 'working systematically' in contexts such as choosing two toppings out of sprinkles, sugar stars or flakes to go on top of iced biscuits they are making. The key question is – 'how do you know that you have got them all?'. This comes after, 'I can find some solutions' and 'I can recognise ones that are the same'. For example, is having sprinkles and sugar stars the same as having sugar stars and sprinkles on top of my iced biscuit?

Children are often quite good at having a random guess as to how to solve a problem. To become fluent at trial and improvement they need to be able to think about how to adapt their first guess so that it is more likely to become a solution rather than throwing the first one out and starting again. When starting to explore Dice in a Corner <https://nrich.maths.org/8586> children may well put the dice together at random and be surprised when they get the magic total of 18. Those children who

are becoming fluent at trial and improvement will then want to adjust the dice to see if they can make 18 in another way, rather than trying another random arrangement.

There are lots of NRICH problems that will help you develop these skills with children. You will find some of our collections at <https://nrich.maths.org/8935>

Being a competent and confident problem solver is central to the mathematical development of all our learners. It is also the major aim of our new national curriculum. This article has detailed the individual elements that teachers can focus on to support children to gain this level of proficiency. We trust you will find it useful and we are always interested in your feedback and experiences as you explore problem solving together with the children in your class.

References

Primary National Strategy (2004) Problem solving A CPD pack to support the learning and teaching of mathematical problem solving. DfES Publications