LUMEN CHRISTI POINT COOK AUSTRALIA MEP (Maths Extension Program) Grade 4-6 teams

When we first started playing the game we tried to find target numbers to reach from where you couldn't lose. Our first aim was to find a way to always win the game.
The first number we discovered was 29 since it was 8 away from the target of 37. the numbers 1,3,5 and 7 in pairs can always make 8 so whatever number your opponent plays, you just say the other number in the pair (1,7/7,1, 3,5/5/3).

This made us think about working with 8. 8 x 4 = 32. If you start with 5, you get to 37 (5 + 32) just using combinations of 8. If you start on 3, you get to 37 (3 + 32 = 35 then +1+1) with 8s then your opponent can only use 1 then you win with 1. If you start on 7, 3 more 8s make 24 that gets you to 31, then 3 and 3 or 5 and 1 gets you to 37 and the win. If you start on 1, 4 x 8 gets you to 33, then a 3 and 1 gets you the win.

Start number	Combinations of 8	Running total	Add on to reach target	Target reached
1	4 x 8 = 32	1 + 32 = 33	1 + 3 or 3 + 1	33 + 4 = 37
3	4 x 8 = 32	3 + 32 = 35	1 + 1	35 + 2 = 37
5	4 x 8 = 32	5 + 32 = 37		37
7	3 x 8 = 24	7 + 24 = 31	3 + 3 or 5 + 1	31 + 6 = 37

We spent a lot of time looking for numbers to get to so we could always win. In all our trials, we found 3,7,9,15,17,19,23,29 or 31 are the "special" numbers because no matter what number the other player uses, you can get to the next "special" number.

But then we realised all the numbers had something in common - they were all odd numbers. When we drew up a table of the running totals, after the first and second players' turns, we noticed that the first player's total was ALWAYS odd and the second player's was ALWAYS even. Of course, we are adding odd numbers. Odd + Odd = Even. Even + Odd = Even.
The second player is always adding a odd number to an odd number, so will always end on an even number. The first player is always adding an odd number to an even number to end with an odd number. So we realised it didn't need special numbers to win.
If you start first you ALWAYS WIN because you always get an ODD number and the target 37 is ODD. If you start second you ALWAYS LOSE because you always get an EVEN number and the target 37 is ODD. We then had a discussion about it being an unfair game tried to make it fairer by changing the numbers.

Others in the group looked for the quickest ways to reach the number.Using mostly 7s we acheived the target in 7 moves and we thought this would be the quickest. (5 x 7) + (2 x 1) = 37

But we also found ways to do it in 7 moves starting with the other numbers as well.

Start no.	Combinations	No. of moves
1	(**1** x 1) + (**3** x 7) + (**3** x 5) = 37	7
3	(3 x 3) + (4 x 7) = 37	7
5	(**3** x 5) + (**1** x 1) + (**3** x 7) = 37	7
7	(5 x 7) + (2 x 1) = 37	7

Then we started looking for possible results for every number of moves between 7 (the least number of moves) and 37 (the most number of moves - 1 x 37) but remembered that you can't win on a even number of moves because the even numbered move results in an even number so we just looked at odd numbered moves

Combination example	No. of moves
(5 x 7) + (2 x 1) = 37	7
(4 x 7) + (4 x 1) + 5 = 37	9
(3 x 7) + (6 x 1) + (2 x 5) = 37	11
(2 x 7) + (8 x 1) + (3 x 5) = 37	13
(1 x 7) + (10 x 1) + (4 x 5) = 37	15
(5 x 3) + (2 x 5) + (12 x 1) = 37	17
(6 x 3) + (1 x 7) + (12 x 1) = 37	19
(4 x 3) + (2 x 5) + (15 x 1) = 37	21
(3 x 3) + (2 x 5) + (18 x 1) = 37	23
(2 x 3) + (2 x 5) + (21 x 1) = 37	25
(1 x 3) + (2 x 5) + (24 x 1) = 37	27
(2 x 5) + (27 x 1) = 37	29
(1 x 7) + (30 x 1) = 37	31
(1 x 5) + (32 x 1) = 37	33
(1 x 3) + (34 x 1) = 37	35
37 x 1 = 37	37

With the combinations of 3,5,1, taking one 3 out and adding three 1s, kept the total at 37 but increased the number of moves by 2. As you increase the number of 1s needed to make more moves, you have to take away a value to maintain 37 as a total. Take away a 5, add five more 1s, take away a 7, add 7 more 1s. This makes all odd number of moves up to 37 possible.