Climbing Powers

We can define $\sqrt{2}^{\sqrt{2}^{\sqrt{2}}}$ as either $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$ or $\sqrt{2}^{\left(\sqrt{2}^{\sqrt{2}}\right)}$. Does it make any difference?

Which is bigger:

$$\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdots}}}$$

where the powers of $\sqrt{2}$ go on forever,

or

$$\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$$

?

Thousands more problems can be found on the NRICH maths website:

http://nrich.maths.org