A Pack of Little Fibs

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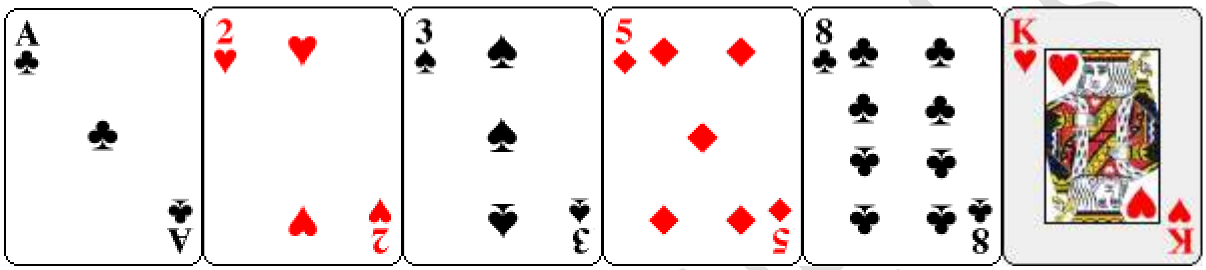
Make up a pack of little Fibs (any Ace, 2, 3, 5, 8 and King), such as these:



Turn them face down and mix them up a lot. Ask two people to each select one card and peek at it. Say, "I couldn't possibly know what cards you now have."

Have each card memorised, and shown to somebody else, just in case. Next, the two helpers compare their cards and tell you, the mathemagician, the total of those two values. Always stress, "An Ace is worth 1, a Jack 11, a Queen 12 and a King 13" (this is subtle: you seem to be saying that any cards could turn up!).

Suppose you are told that the total is 16. Think of the largest Fib you can less than the total—which would be 13 in this case. Subtract it from the total, to get 3. The cards must be the 3 and the King. Announce this to thunderous applause.

That idea works for any total! Try it. (*Why* does it work….? Think it *over*…)

If you also memorise the suits, and keep these six key cards at the top of a whole packet throughout some fair-seeming shuffles, before dealing them out as allegedly random cards, and then have any two of those picked as above, the overall effect is even more impressive.

Details on that option, and why the whole thing works, are on the other side.

Adapted from MAA Online's *Card Colm* (February 2008) – colm@spelman,edu 10.10. 2011

Additional Certainties (why the **A Pack of Little Fibs** card trick works)

http://www.maa.org/columns/colm/cardcolm200803.html

Here's the full performance version *Additional Certainties* of the trick on the other side of this page. Pick up a deck of cards and "shuffle it" a few times. Have two selections made by different spectators. They share the results and announce the sum of the chosen card values. The cards are plunged back in the deck and lots more shuffling is done. You then rummage through the deck and pull out the cards. How is this possible? The secret is the following:

When several numbers are added up, each may be determined with certainty from the sum.

There are of course restrictions as to exactly which cards may be selected: while free choices are offered, they are from a carefully controlled small subset of the deck. The possibilities in question may be narrowed down by using your favourite technique; the simplest method is just to have a stack of key cards (about half a dozen) at the top of the deck, and keep those there throughout some appropriately managed and seemingly fair shuffles.

Despite the apparent difficulty of decomposing the sum into its summands, that's exactly what you do, also deducing the card suits, hence being able to find the cards later, no matter how shuffled the deck really ends up.

We need a wonderful fact gleaned from Burger & Starbird's generally excellent *Heart of Mathematics* text book:

Every natural number has a unique representation as a sum of (distinct)
non-consecutive Fibonacci numbers (1, 1, 2, 3, 5, 8, 13, 21, 34, ...).

This so-called Zeckendorf representation was seemingly first established in 1939, by Edouard Zeckendorf from Belgium, an amateur mathematician---but a real doctor. By the time he published it, in retirement, in 1972, the word had got out: Cornelis Gerrit Lekkerkerker beat him to print by 20 years. Can you see why this result must hold?

For instance, 6 = 5 + 1 (we don't allow 3 + 2 + 1 or 3 + 3), and 20 = 13 + 5 + 2. Given any natural number, it's easy to find the corresponding decomposition, by first peeling off the largest possible Fibonacci number, and repeating that process for the difference, until we are done. For instance, 50 = 34 + 16 = 34 + 13 + 3. (Burger & Starbird focus on a strategic application of this greedy algorithm to the game of Nim; it's worth looking that up.)

Now, consider any Ace, 2, 3, 5, 8 and King (value 13). For instance, for ease of memorization—you'll need to think fast further down the road—we suggest the usual suit rotation CHaSeD. (Turn over this page to see those six cards.)

There are (6 x 5)/2 = 15 possible sums of two values, and by the Zeckendorf result, most of these are distinct, except possibly 1 + 2 = 3, 2 + 3 = 5, 3 + 5 = 8, 5 + 8 = 13, and 8 + 13 = 21. But those are themselves Fibonacci numbers, and hence elements of an increasing sequence, so none can be expressed in another way as a sum of two Fibonacci numbers. Hence, all such 15 sums are distinct; they turn out to be the numbers: 3–11, 13–16, 18 and 21. Thus,

If two cards are selected from any Ace, 2, 3, 5, 8 and King, then
the cards selected can be determined from the sum of their values.

Each total can only arise in one way, and it's easy to decompose a given sum into its constituent summands. <u>The easy to perform version is on the other side of this page.</u> (The February 2008 *Card Colm* has info on other such sequences. The June 2009 *Card Colm* discusses the possibility of doing such a trick with a really shuffled deck.)

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