Tower of Hanoi

http://nrich.maths.org/6690

In this problem, you will be working on a famous mathematical puzzle called The Tower of Hanoi. There are three pegs, and on the first peg is a stack of discs of different sizes, arranged in order of descending size. The object of the game is to move all of the discs to another peg. However, only one disc can be moved at a time, and a disc cannot be placed on top of a smaller disc.

1

Look at the sequence: 1, 2, 4, 8, 16... Can you describe how to get from one term to the next? Can you describe the n^{th} term of the sequence?

Now try adding together terms from the sequence: 1+2, 1+2+4, 1+2+4+8. Do you notice anything interesting?

Can you predict what 1+2+4+...+64+128 would be? Check to see if you are right.

How could you write the answer to 1+2+4+...+2n? Justify why your formula works.

2

What is the smallest number of moves needed to complete the Tower of Hanoi game with: one, two, three or four discs?

Do you notice anything interesting about the way the number of moves increases? Can you explain any patterns you find?

3

The Tower of Hanoi puzzle can be completed in 3 moves with two discs. Can you use this to work out how many moves would be needed with three discs?

The Tower of Hanoi puzzle can be completed in 15 moves with four discs. Can you use this to work out how many moves would be needed with five discs?

In general, can you describe a way of working out how many moves are needed when one extra disc is added?

Final Challenge

Explain how you could work out the number of moves needed for the Tower of Hanoi puzzle with n discs.

You Will Need:

- A Tower of Hanoi set, or a set of nesting cups

Teachers' Notes

Why do this problem?

The Tower of Hanoi is a well-known mathematical problem which yields some very interesting number patterns. This version of the problem involves a significant 'final challenge' which can either be tackled on its own or after working on a set of related 'building blocks' designed to lead students to helpful insights.

Initially working on the building blocks gives students the opportunity to then work on harder mathematical challenges than they might otherwise attempt.

The problem is structured in a way that makes it ideal for students to work on in small groups.

Possible approach

Start by explaining how the Tower of Hanoi game works, making clear the rules that only one disc can be moved at a time, and that a disc can never be placed on top of a smaller disc.

Hand out a set of building block cards to groups of three or four students. Within groups, there are several ways of structuring the task, depending on how experienced the students are at working together.

Each student, or pair of students, could be given their own building block to work on. After they have had an opportunity to make progress on their question, encourage them to share their findings with each other and work together on each other's tasks.

Alternatively, the whole group could work together on all the building blocks, ensuring that the group doesn't move on until everyone understands.

When everyone in the group is satisfied that they have explored in detail the challenges in the building blocks, hand out the final challenge.

The teacher's role is to challenge groups to explain and justify their mathematical thinking, so that all members of the group are in a position to contribute to the solution of the challenge.

Set aside some time at the end for students to share and compare their findings and explanations, whether through discussion or by providing a written record of what they did.

Key questions

What important mathematical insights does my building block give me?
How can these insights help the group tackle the final challenge?

Possible extension

Of course, students could be offered the Final Challenge without seeing any of the building blocks.

Possible support

Encourage groups not to move on until everyone in the group understands. The building blocks could be distributed within groups in a way that plays to the strengths of particular students.