Association of Teachers of Mathematics
Micro CPD Event

Habits of Mind
&
Habits of Interaction

Using oracy to promote structural thinking in the maths classroom

My background

- KS2 classroom teacher from 2008
- MaST 2010-12 (Mathematical Thinking; Proportionality; Pattern; Generality; Representation)
- Introduced to the ideas of ATM
- Mathematics Mastery from 2015
- MAT primary maths lead from 2019
- Maths Hub Work Group Lead from 2020
- Currently working for NCETM since September 2023

Session aims

- Enjoy doing some maths (choosing your own level)
- Share an interpretation of “Mathematical Habits of Mind and Habits of Interaction”
- Share a HoM model focused on developing structural thinking (and what has happened with it so far)
- Taking time to reflect (and so what?)
- Seeking critical responses and feedback

What is 'learning mathematics'?

Do many secondary teachers struggle to describe this too?

…matics is certainly a discipline whose principal component is thinking."

Goldenberg, E.P. et al. (2002)

How to use mathematics to solve unfamiliar, non-routine problems

"Learning to think mathematically means (a) **developing a mathematical point of view** — valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) **developing competence** with the tools of the trade, and using those tools **in the service of the goal of understanding structure** — **mathematical sense-making**."

Schoenfeld, A. H., (1992)

Mathematical content

Number	Ratio & proportion	Algebra	Geometry & Measures	Statistics
• Number sense • Place value • Calculations • Fractions	• Relations between quantities (Y6)	• Patterns • Early algebraic thinking • Variables & expressions (Y6)	• Lines, shapes & solids • Position & direction • Measurement	• Data collection & organisation • Data visualisation • Data analysis

"Know, **apply** and **understand**"

"the matters, skills and processes"

Time to do or observe some maths

"Any possibility of intimidating with mathematical expertise is to be avoided."

"Encouraging a questioning approach and giving due attention to the ideas of others are attitudes to be encouraged."

Treat everything that is said as a conjecture; be open to others' and to modifying your own.

- 5 minutes of thinking time – choose a question, and join a breakout room with cameras off/mics off until the signal is given
- 5 or 6 per breakout room maximum
- Agree who will focus on observing and recording (max 2), who will focus on solving the chosen problem and justifying their thinking
- 10-15 minutes of doing some maths together
- 5 minutes of reflecting in your breakout on what was drawn upon
- Spoiler Alert – there will be no answers to the problems!

https://tinyurl.com/microcpd2

What was drawn upon?
(Jamboards)

Social emotional learning skills?

- Collaborating & communicating
- Holding your thought / Suspending your disbelief
- Turn-taking
- (Mathematical) resilience
- Thinking creatively & critically

Mathematical attitudes?

- Curiosity
- Willingness to play (with numbers, shapes, ideas)
- Willingness to take risks
- Open to the ideas of others
- Perseverance, tenacity, determination
- Self-belief
- Sense of wonder
- Inclined to experiment/try out

Birth to 5 Matters (2021)


“Mathe d be like roller-
like being
g and
om.”
Richard Winter (1991)

https://rb.gy/cr9zy7

Mathematical strategies?

Which of these aspects of 'being a mathematician' did colleagues use and demonstrate?

Imagining
Specialising
Characterising
Conjecturing
Generalising
Extending

Expressing
Classifying
Organising
Convincing
Reflecting

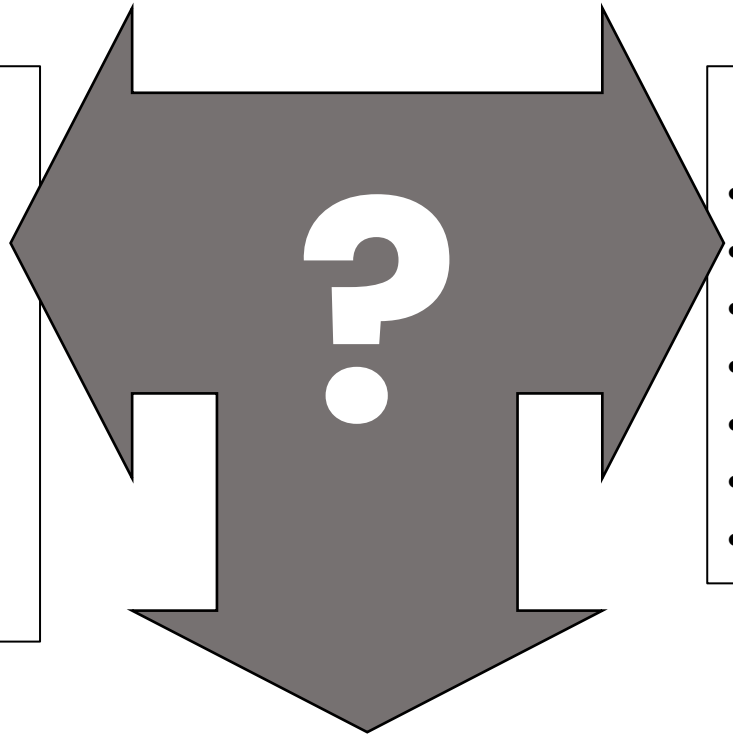
Those familiar with the terms, what others did you use?

Learning mathematics

Being a mathematician

Social-emotional learning skills

- Mathematical resilience & 'other' dispositions
- Collaboration
- Developing a sense of self and sense of place
- Thinking creatively & critically



Mathematical processes

- Problem solving
- Reasoning & proving
- Reflecting
- Connecting
- Communicating
- Representing
- Selecting tools & strategies

Developing conceptual understanding and procedural fluency with Mathematical content

Number	Ratio & proportion	Algebra	Geometry & Measures	Statistics
• Number sense • Place value • Calculations • Fractions	• Relations between quantities (Y6)	• Patterns • Early algebraic thinking • Variables & expressions (Y6)	• Lines, shapes & solids • Position & direction • Measurement	• Data collection & organisation • Data visualisation • Data analysis

A social-constructivist model for learning mathematics

Being a mathematician

Social-emotional learning skills

- Mathematical resilience & 'other' dispositions
- Collaboration
- Developing a sense of self and sense of place
- Thinking creatively & critically

Habits of mind & Habits of interaction

Mathematical processes

- Problem solving
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California Mathematics Framework (2023)

"...the **habits of mind and habits of interaction** that the field increasingly recognizes are essential for the kind of deep learning of mathematics that students require for their lives and careers and to better interpret and understand their world.

...habits of mind include making or using mathematical representations, attending to mathematical structure, persevering in solving problems, and reasoning, with the latter including the processes of inferencing, conjecturing, generalizing, exemplifying, proving, arguing, and convincing (Jeannotte and Kieran, 2017).

Habits of interaction are linguistic processes and include such things as explaining one's thinking, justifying a solution, listening to and making sense of the thinking of others, and raising worthy questions for discussion.

Both kinds of habits are fundamentally tied to language development and linguistic processes. To support reasoning processes and habits of interactions, teachers need to support language development as students engage in these disciplinary practices."

Chapter 4, Exploring, Discovering, and Reasoning With and About Mathematics

Habits of interaction a.k.a. “oracy”

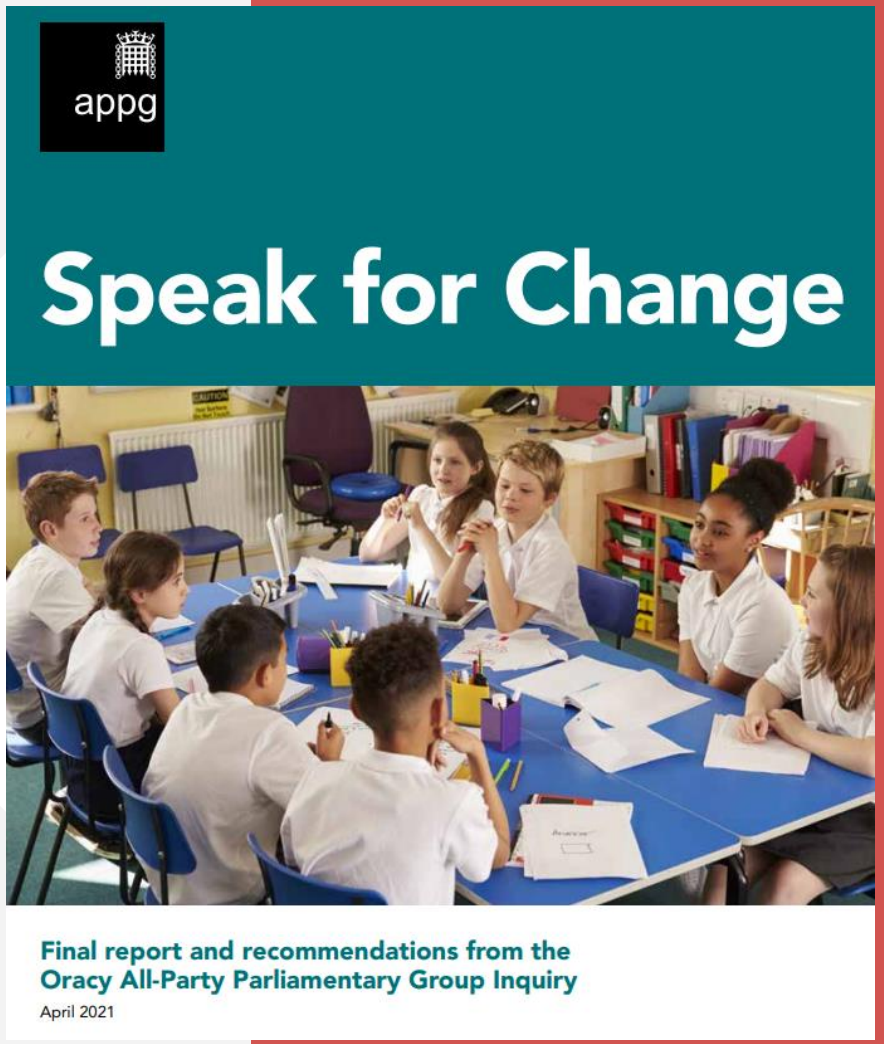
“Habits of interaction are linguistic processes and include such things as explaining one’s thinking, justifying a solution, listening to and making sense of the thinking of others, and raising worthy questions for discussion.

…teachers need to support language development as students engage in…disciplinary practices.”

- Learning **through** talking and listening
- Learning **to** talk and **to** listen
- Learning to think about others’ ideas
- Learning to respond thoughtfully to others’ ideas
- Learning to talk and think **collaboratively**

Benefits of oracy

- Improves academic outcomes
- Underpins literacy and vocabulary acquisition
- Supports wellbeing and confidence
- Enables young people to access employment and thrive in life beyond school
- Develops citizenship and agency



Oracy creates conditions for mathematical thinking

“We learn to think through oral language, [so oracy is about] advancing individual capacity for productive, rational and reflective thinking.”

Professor Frank Hardman in ‘The State of Speaking in Our Schools’ (2016, Voice 21)

“Mathematical thinking is central to how pupils learn mathematics … Pupils should actively engage in mathematical thinking in all lessons…”

Five Big Ideas in Teaching for Mastery (2022, NCETM)

“Mathematical thinking can only take place in what I call **a conjecturing atmosphere**."

John Mason, Conjecturing Atmosphere (2015, OUP Blog)

Conjecturing atmospheres

“This means that **everything that is said is treated as a conjecture** that may require modifying. If you disagree with someone you “invite them to modify their conjecture” or you ask how their conjecture applies to some example (which perhaps you think might be a counter example)."

“**The aim is to reduce the pressure on ‘being correct’…**when a learner offers a response to a question, try to catch yourself before you declare whether it is right or wrong; praise it as a conjecture, and invite others to consider whether they agree with it, or whether they would like to suggest a modification or a counter-example. In this way you can be responsible for the process of thinking, while the learners work together to decide correctness.

A conjecturing atmosphere requires trust: learners trusting the teacher that they can make progress on the tasks and questions posed to them, because effort is worthwhile; teachers trusting learners to make an effort because they will be rewarded by a more comprehensive appreciation of the topic…”

John Mason, Conjecturing Atmosphere (2015, OUP Blog)

Oracy supporting conjecturing atmospheres

Equity & engagement

- Talk (and listening) expectations
- Structures for talk
- Everyone listens ‘to’ (not ‘for’)
- Avoid Initiation-Response-Feedback (IRFs)
- Learners do the heavy lifting

Privileging exploratory talk*

- Exploring ideas
- Making sense of
- Reasoning & explaining
- Willing to change mind
- Building to a consensus
- Develops understanding

*Mercer & Littleton (2007)

Oracy as a pedagogical tool

- Supports formative assessment by making learners' thinking available
- Requires putting structures in place to facilitate all learners to contribute to and learn from classroom talk
- Requires the belief (from teachers and learners) that all learners can contribute to and learn from classroom talk (in every lesson)
- Subject content & pedagogic knowledge…and preparation (and professional development!)

  (e.g. Stein et al. Five Practices for Orchestrating Productive Mathematical Discussion)

Why ‘habits of mind’?

“mathematics is both a body of facts accumulated over the millennia, and **a body of ways of thinking that has allowed people to discover or invent these facts and ideas**… teaching these ways of thinking, which we refer to as *mathematical habits of mind*, is a vital part of mathematical instruction at every level.”

Goldenberg, Shteingold and Feurzeig (2002)

“Using these ways of thinking as the organizer of a curriculum helps to emphasize them and bring them to the fore…”

Cuoco, Goldenberg & Mark (1996)

When mathematical thinking is prioritised:

- More exploration, more creativity; more enjoyment
- Develops ways of thinking that develop understanding of content
- Develops ways of thinking that are crucial to solve unfamiliar problems
- Unlikely that a curriculum organised in this way will ever become obsolete (see above!)

Conceptualising habits of mind

"We demonstrate MHoM when we habitually choose **actions** and **strategies**, **pose questions** and **display attitudes** that are PRODUCTIVE in a mathematical context.

They help us understand the mathematics, solve problems and ~~maybe~~ even help us create mathematics."

Gourdeau, Oesterle & Stordy (2014)

"Only awareness is educable."

Caleb Gattegno (1987)

"Only behaviour is trainable."

John Mason (1994)

A collection of mathematical habits of mind

Attitudes	Actions
• Curiosity • Willingness to play (e.g. with numbers, shapes, ideas) • Willingness to take risks • Perseverance, tenacity, determination • Self-belief • Sense of wonder • Inclined to experiment/try out	• Noticing features/properties • Seeking patterns • Making comparisons—noticing variance & invariance • Describing what is seen ("Say what you see") • Visualising • Organising information efficiently • Tinkering deliberately • Taking care/checking/verifying • Thinking about thinking (metacognition)
Strategies	**Questions**
• Specialising • Generalising • Conjecturing • Convincing • Imagining • Expressing (e.g. explaining, recording, manipulating, representing) • Classifying • Characterising • Organising • Reflecting • Extending	• What if...? I wonder what would happen if... • What am I given? What do I know? • What's the same? What's different? • What changes? What stays the same? • What do I need to find out? • How can I show this? • Have I seen something like this before? • Why is this true? What is really going on here? • Is this the best way to start? • How am I approaching this? What else could I try? • Could I have done this differently? • What related questions could I explore?

What to prioritise?

Why attitudes are so vital

"It is worth mentioning that what enables training of behaviour and educating of (one's own) awareness is the harnessing of emotion, the source of energy.

Hence, "only emotion is harnessable."

John Mason (2021)

Attitudes

Curiosity

Willingness to play

Willingness to take risks

Perseverance, tenacity, determination

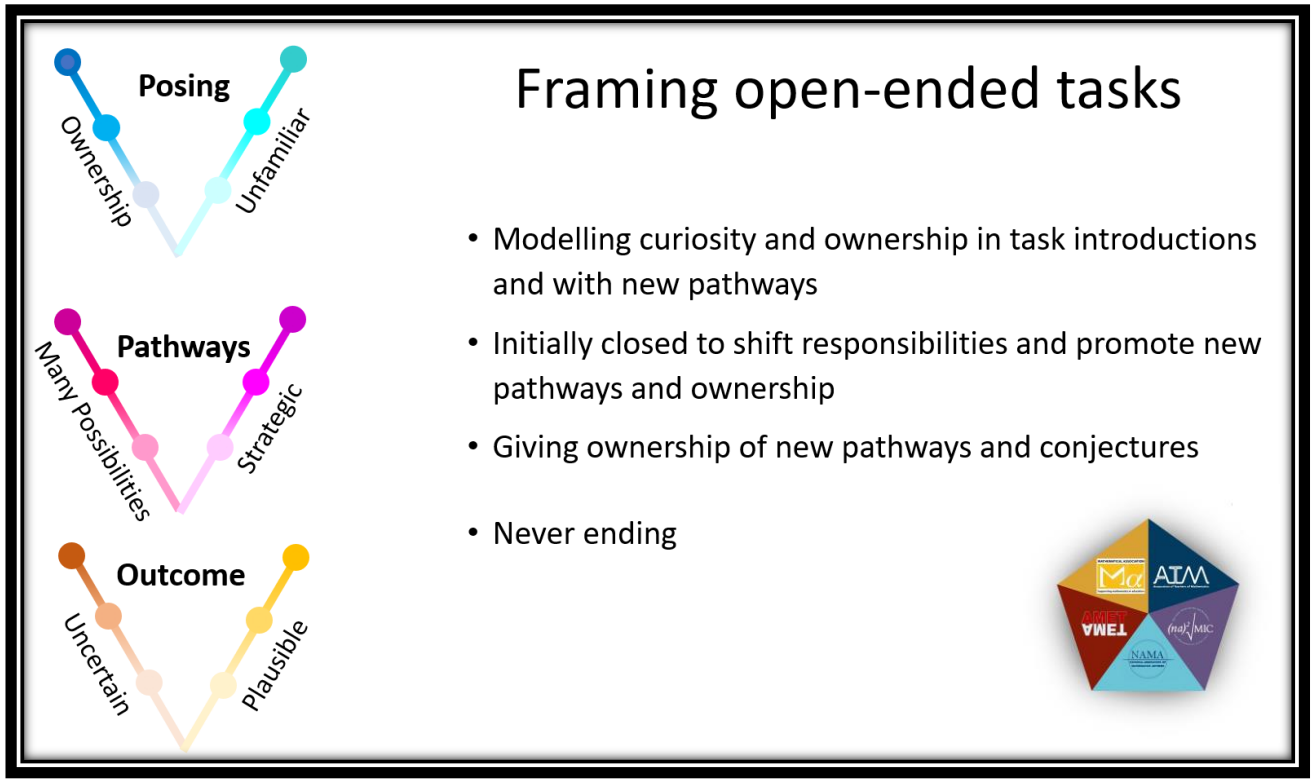
Self-belief

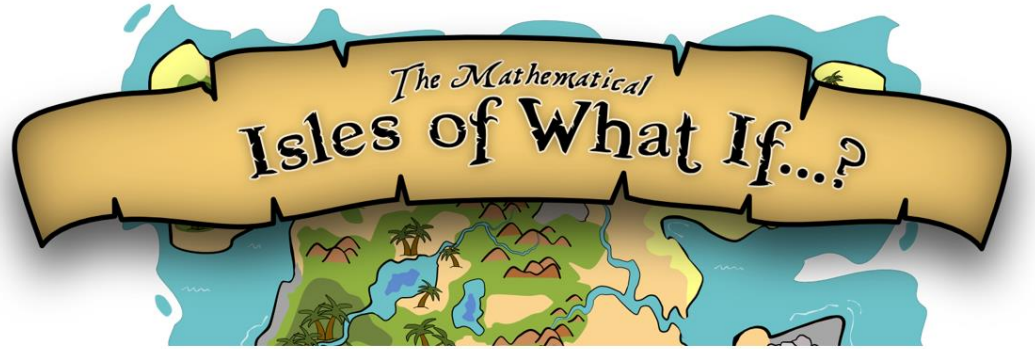
Sense of wonder

Inclined to experiment/try out

Stimulating & sustaining mathematical curiosity

Cultivating a mathematically curious classroom





islesofwhatif.com

Jacob Merrill

Twitter: @islesofwhatif

'Mathematical creations?' MT 290

Sustaining harnessable emotion

"…enjoyment of mathematics cannot be characterised solely by the existence of the positive emotions that occur in moments of success, because such moments cannot and do not exist in isolation."

"When positive emotions do occur, they are often connected to what I have called **emotions-of cognition**, the sensations involved with **knowing**, which I have argued are distinct from emotions of success."

Teaching, of whatever style or philosophical standpoint **that genuinely supports learners in the making-sense of journey**…will lend itself to the occurrence of moments of cognition…Such a learner experiences successes that are decreasingly dependent on outer affirmation and increasingly rooted in inner confidence."

Jay Timotheus (2023) 'Should pupils enjoy mathematics?'
Mathematics Teaching 286

Pedagogical attitudes?

Pedagogical strategies?

What pedagogical habits are needed to teach in this way?

‘Innate learning powers’ for Structural Thinking

The ways of thinking we should prioritise are the ones that help learners to develop their own awareness to make sense of and understand mathematical concepts.

If we can, as much as possible, build these from natural ways of thinking that all learners possess, surely that promises to be an efficient strategy.

Thinking Mathematically

Mason et al. (2010, 2nd Edition)

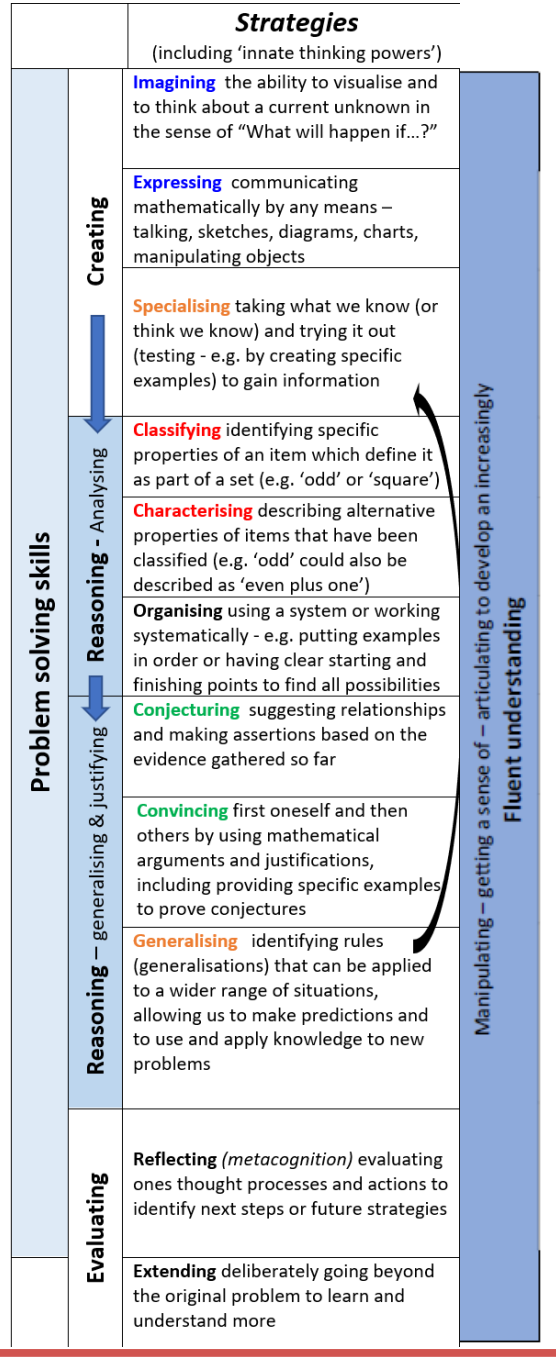
"Our claim is that there are **natural powers** possessed by every child, and that thinking mathematically is really about learning to use these powers in mathematical ways and in the exploration of mathematical problems."

Offer opportunities for learners to think and **create** their own examples.

Encourage them to **analyse** and, over time, to **organise** their work…

…in order to sniff out patterns, to make **conjectures** and **generalise.** And choose or create examples to **convince**, **justify** and **prove**.

As learners develop their proficiency as thinkers we should also assist them in **evaluating**.



Structural thinking

"We take mathematical structure to mean the identification of general properties which are instantiated [*represented by an instance*] in particular situations as relationships between elements."

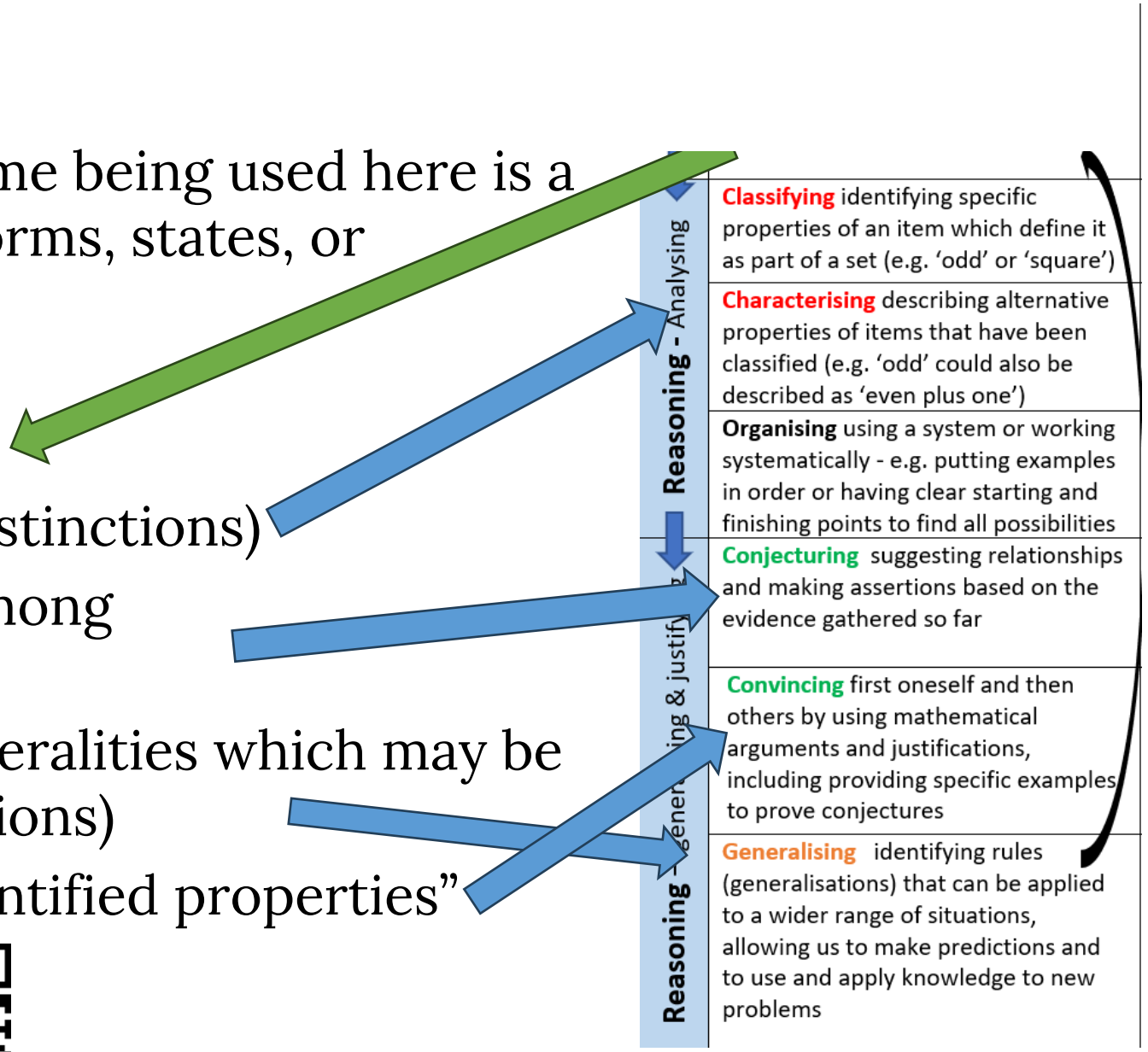
"Awareness of the use of properties lies at the core of structural thinking. We define structural thinking as a disposition to use, explicate [*analyse and develop in detail*] and connect these properties in one's mathematical thinking."

Mason, J., Stephens, M. and Watson, A., (2009)

Structural thinking

"The underlying theoretical frame being used here is a distinction between different forms, states, or structures of attention:

- Holding wholes (gazing)
- Discerning details (making distinctions)
- Recognising relationships (among specific discerned elements)
- Perceiving properties (as generalities which may be instantiated in specific situations)
- Reasoning on the basis of identified properties"



https://tinyurl.com/2maak57s

The purpose of developing structural thinking

To support the learner in altering “the structure of their attention by becoming aware of not just an idea, but details and distinctions, properties and relationships.” (p.38)

A previously abstract idea becomes secure or manipulable enough to be a starting point for more complex, abstract ideas. “The concept is reified.” (p.38)

“This is how learners learn anything: forming a view, becoming familiar with it, using it, adapting it and developing it to take account of new challenges, and exchanging ideas about it with others through language and other interactions so that what was initially hazy and ill-formed becomes an element used to express further hazy ideas built upon it.” (p.39)

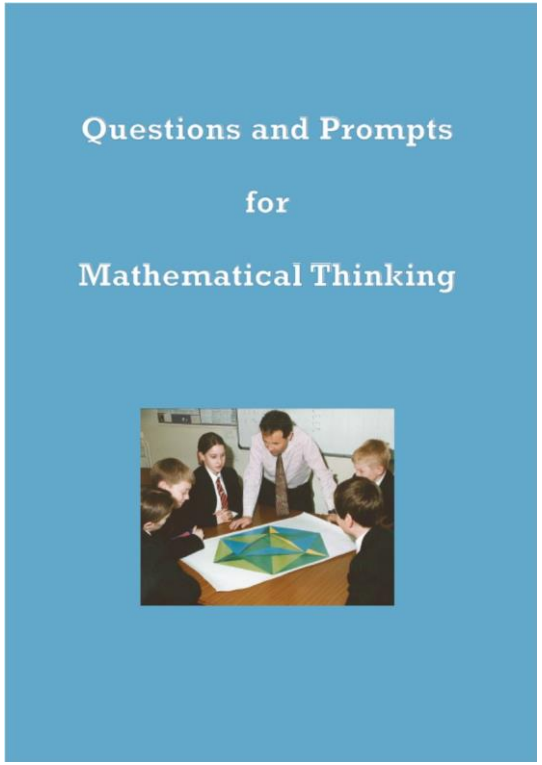
Anne Watson & John Mason (1998) Questions & Prompts for Mathematical Thinking: ATM

How can we prompt structural thinking?

"...a net to catch the practice of effective mathematicians..."

"...higher order mathematical thinking can be promoted as an integral part of teaching and learning core school mathematics." because "...as important as a topic is, even more important is inducting students into a mathematical way of thinking..."

Questions & Prompts for Mathematical Thinking



How do we develop and improve structural thinking?

Some suggestions that have worked for me over time:

- Using learners' powers as a metacognitive frame for subject leader and teacher CPD (since 2020)
- Using learners' powers as a metacognitive/reflective frame during whole-class teaching and learning and for problem-solving (since 2010)

A more recent strategy that I have tried (since 2021):

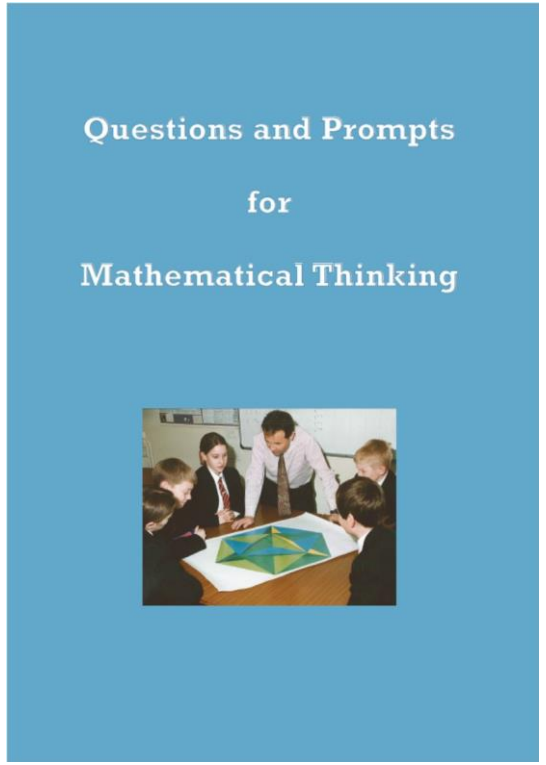
- Using a 'reasoning' rubric* Trust-wide, to improve formative assessment and develop teaching of 'reasoning' (definition corresponds to the definition of 'structural thinking' used here)

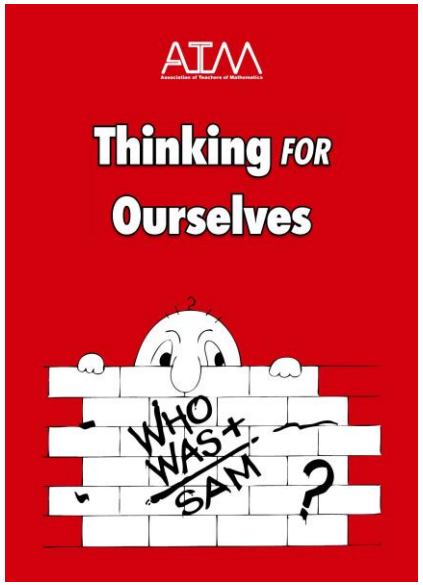
*Loong, E. et al (2018) Developing a Rubric for Assessing Mathematical Reasoning

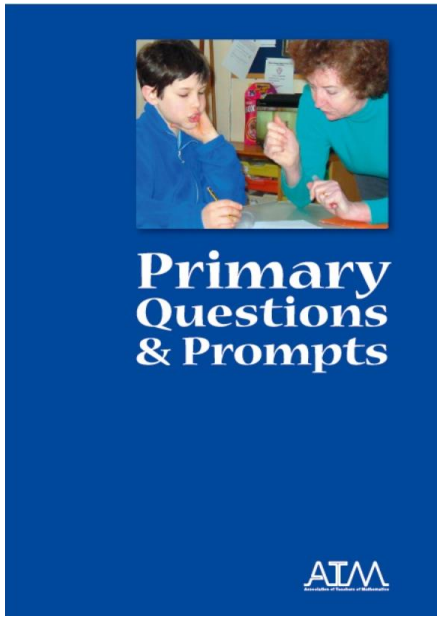
How do we habituate structural thinking?

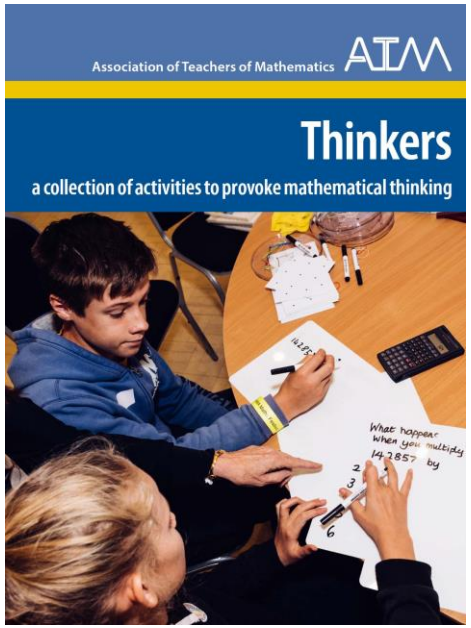
The eventual aim of these resources is to encourage learners to think structurally and, by doing so, to understand and learn mathematics as a rich, connected web of concepts and ideas.

They intend that learners adopt the mathematical habit of mind of asking themselves questions that provoke their own mathematical thinking and that of each other.









A case study – Oracy & Structural Thinking

Summerhill Academy

Trusting in learners of maths.

Trusting in teachers of maths.

Trusting in leaders of maths.

Voice 21 & OUP: Benchmarks Report

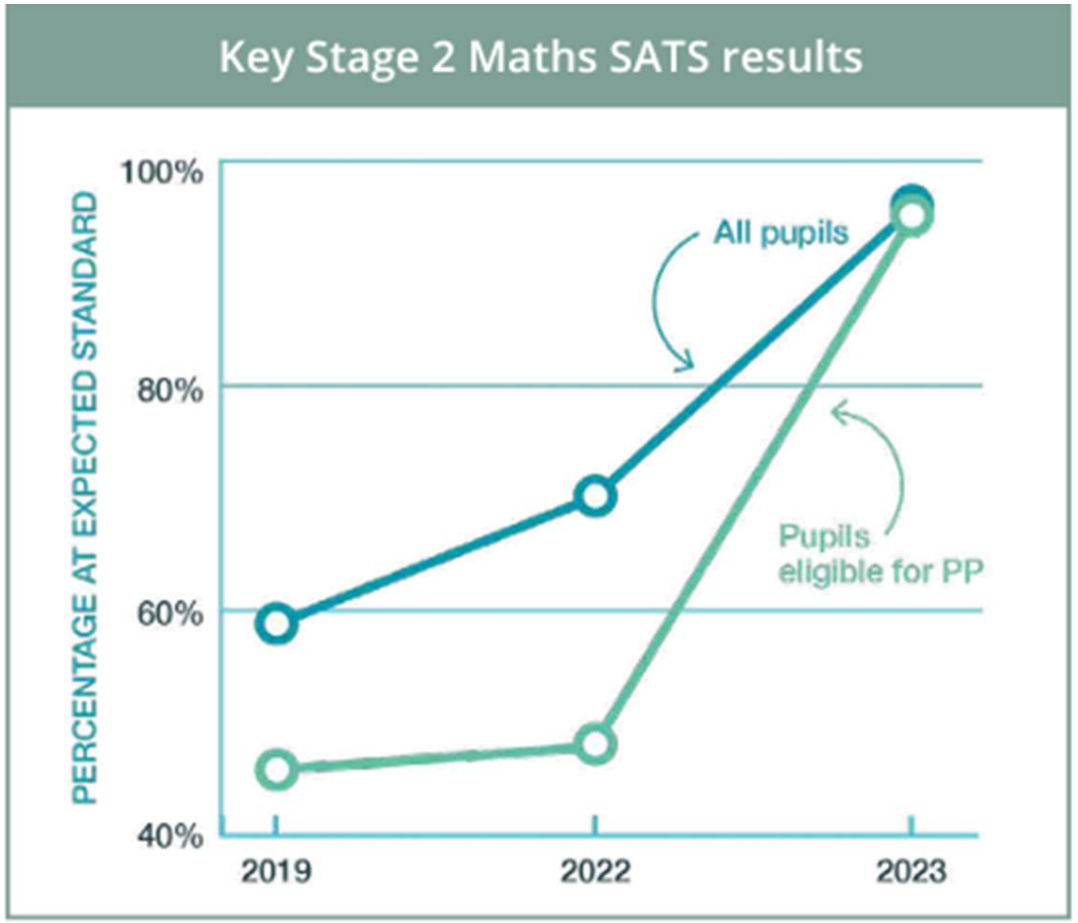
Summerhill Academy, Bristol

In my previous role, from Autumn 2019.

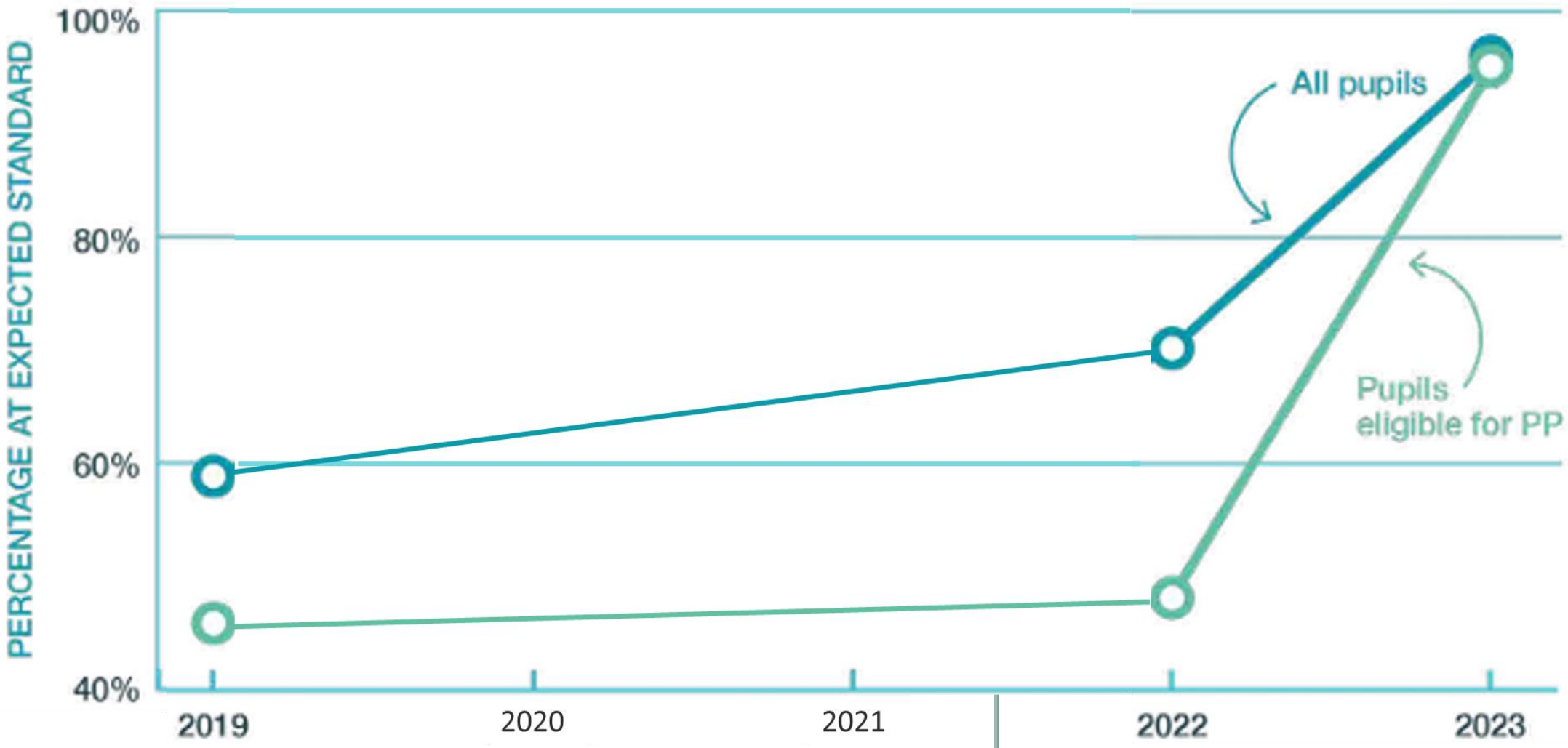
Significant deprivation, low community expectations, low esteem endemic.

Significant developments during COVID:

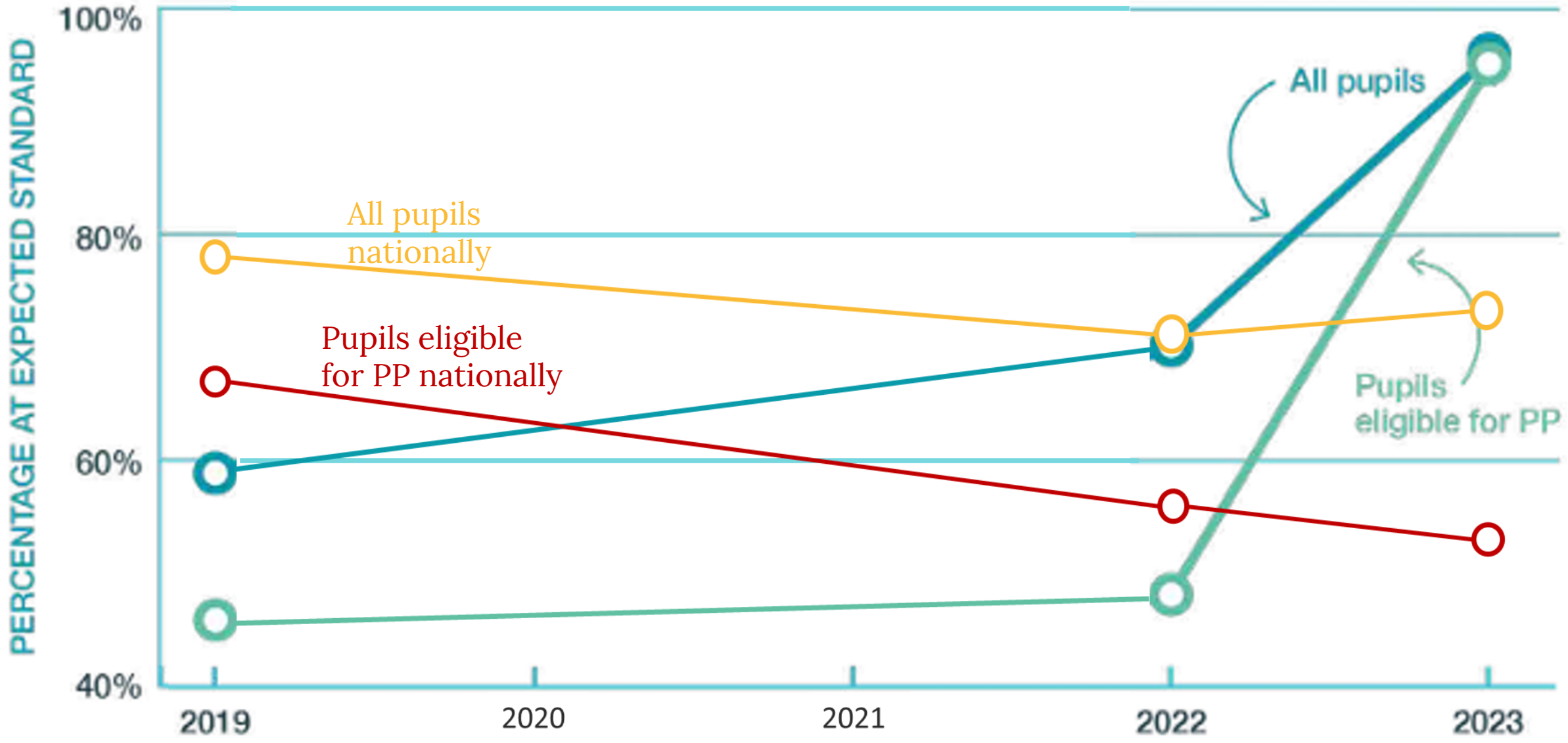
- Deputy Head maternity cover - former specialist from the Maths Hub
- New Headteacher (and interim Exec Head)
- School accessing Maths Hub workgroups
- Developed conjecturing atmospheres and structural thinking approach - with a curriculum based on DfE/NCETM 2020 Guidance (Trust-wide)



Key Stage 2 Maths SATS results



Some additional context



The slow grind of (sustainable) school improvement

In 2019 KS2 Maths SATS outcomes were as follows:

All pupils at expected standard: 59% (20% below National 79%)
Pupil premium learners at expected standard: 46% (21% below National 67%)
All pupils at the higher standard: 14% Pupil premium at the higher standard: 10 %
All pupils progress: -4.9 Pupil premium progress: -5.9

In 2022 KS2 Maths SATS outcomes were as follows:

All pupils at expected standard: 70% +11% vs 2019 (1% below National 71%)
Pupil premium learners at expected standard: 48% +2% vs 2019 (9% below National 57%)
All pupils at the higher standard: 17% Pupil premium at the higher standard: 4 %
All pupils progress: -0.1 Pupil premium progress: +2.0

In 2023 KS2 Maths SATS outcomes were as follows:

All pupils at expected standard: 95% +25% vs 2022 (+22% above National 73%)
Pupil premium learners at expected standard: 94% +46% vs 2022 (+41% above National 53%)
All pupils at the higher standard: 41% Pupil premium at the higher standard: 22%
All pupils progress: +2.9 Pupil premium progress: +5.3 (4.8 above national = +0.5)

Time to reflect (5 minutes)

What has resonated with you?

From this session, what is the one thing that you want to do something about?

Spend some time considering what you will do differently this time next month, as a result of your own experience today.

Think carefully about your first step towards making this happen. Write it down along with a note of when (realistically!) you will do it.

If you want to share any of those reflections in the chat as a means of committing to them, please do.

Micro CPD Number 3

What's the problem with problem solving?
Leader: Richard Perring

Tuesday 9th July 2024
17:00 - 18:15

In the meantime, do not forget to spend your ATM vouchers £5 for individual attendees, £20 for schools!

Questions?

Critical feedback, please!

For example:

- Is this relevant in Secondary as much as Primary?
- Is there a context in which this does not apply?
- What should I be taking better account of?

References

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