Solution A
First I added the two 15s which gave me a total of 30 so I knew that the square number and its square root had to have a difference of 30. So I tried out 5 squared = 25 but there was a difference of only 20 so next I tried 6 squared which gave me 36 and that had a difference of 30. So I halved 30 which gave me 15 and I added that to 6 and the answer was 21. So he/she is 21 years old.

Solution B
If I find the difference between a square number and its root:
$3^2 = 9$, difference 6
$4^2 = 16$, difference 12
$5^2 = 25$, difference 20
The special age is equidistant from the square and its root, so I need to halve the difference
6/2 = 3
12/2 = 6
20/2 = 10
These are the triangle numbers. So I can say "In n years' time, my age will be the square of my age n years ago" if n is a triangle number.

Solution C

$x+15= (x-15)^2$
$0 = x^2 – 31x + 210$
$x = 21$ or 10

In general,

$x+n = (x-n)^2$
$0 = x^2-(2n+1)x +n^2-n$

If $ax^2+bx+c=0$, then for x to be an integer b^2-4ac must be a perfect square, t^2, say.

Therefore, $(2n+1)^2-4(n^2-n) = t^2$
$4n^2+4n+1-4n^2+4n = t^2$
$8n+1 = t^2$

$8n+1$ is odd, so t must be odd, so let $t=2r+1$

$8n = t^2-1 = (t-1)(t+1) = (2r+1-1)(2r+1+1) = 2r(2r+2) =4(r(r+1))$

Therefore, $n= ½r(r+1)$

This is the formula for a triangle number.

Solution D

$1^2=1$	0 years ago you were 1, in 0 years you will be 1, you are 1
$2^2=4$	1 year ago you were 2, in 1 year you will be 4, you are 3
$3^2=9$	3 years ago you were 3, in 3 years you will be 9, you are 6
$4^2=16$	6 years ago you were 4, in 6 years you will be 16, you are 10
$5^2=25$	10 years ago you were 5, in 10 years you will be 25, you are 15
$6^2=36$	15 years ago you were 6, in 15 years you are 36, you are 21
$7^2=49$	21 years ago you were 7, in 21 years you will be 49, you are 28
$8^2=64$	28 years ago you were 8, in 28 years you will be 64, you are 36
$9^2=81$	36 years ago you were 9, in 36 years you will be 81, you are 45
$10^2=100$	45 years ago you were 10, in 45 years you will be 100, you are 55

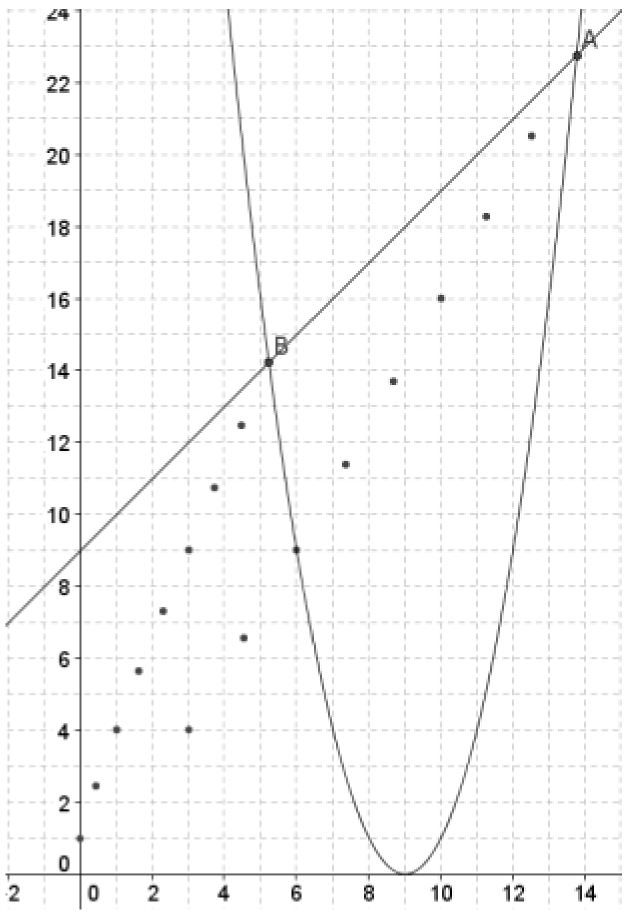
Solution E

I used GeoGebra to create a slider for a and plotted $y=(x-a)^2$ and $y=(x+a)$.

I used the 'Trace' function to mark the points of intersection.

Where they are on gridsquares, my age is a whole number.

It works for x=3 (n=1), x=6 (n=3) and x=10 (n=6).



Solution F

Let a = age now.
$(a-15)^2=a+15$
$a^2-30a+225=a+15$
$a^2-31a+210=0$
$(a-21)(a-10)=0$
age=21 (a=10 gives a-15=-5)

Try the other numbers:
$(a-3)^2=a+3$
$a^2-7a+6=0$
$(a-6)(a-1)=0$ so age=6

$(a-4)^2=a+4$
$a^2-9a+12=0$ no integer solution

$(a-5)^2=a+5$
$a^2-11a+20=0$ no integer solution

$(a-6)^2=a+6$
$a^2-13a+30=0$
$(a-10)(a-3)=0$ so age is 10

So 3,6 and 15 work. It looks as if there is a connection with triangular numbers, so I tried 10:
$(a-10)^2=a+10$
$a^2-21a+90=0$
$(a-15)(a-6)=0$ so age is 15
So all the triangular numbers seem to work and the age is the triangular number greater than the one in the question.

To prove it:
Does it work for k?
$(a-k)^2=a+k$
$a^2-(2k+1)a+k^2-k=0$

$$a=\frac{2k+1\pm\sqrt{(2k+1)^2-4k^2+4k}}{2}$$
$$=\frac{2k+1\pm\sqrt{8k+1}}{2}$$

This has integer solutions when 8k+1 is a square number.
This is true if k is a triangular number.
(see diagram when k=10)