

Draw any convex quadrilateral and find the midpoints of the four edges. Join the midpoints to form a new quadrilateral.

A convex quadrilateral is one where all of the angles are less than 180°. Alternatively you can use the definition that both diagonals lie inside the shape.

Try it a few times starting with different convex quadrilaterals.

What do you notice about the new quadrilaterals formed by joining the midpoints? Does this always happen?

Can you find a counter-example? Can you make a conjecture?

Can you prove your conjecture?

Is the area of PQRS always the same fraction of the area of ABCD? Can you explain why?

Do these results still hold if ABCD is a concave quadrilateral?

A concave quadrilateral is one where one angle is greater than 180°, for example you could draw an "arrowhead" shape.