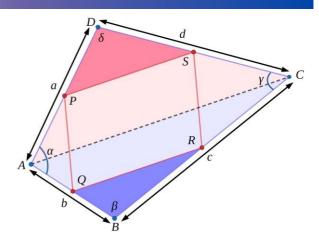


Quad in Quad – Area of PQRS

Draw a convex quadrilateral and then join the adjacent midpoints of the four edges. You should find that the area of the new quadrilateral is half the area of the original quadrilateral.

Here is a diagram and a proof that has been scrambled up. Can you rearrange it into its original order?



Therefore $X = \frac{1}{2}ad\sin\delta + \frac{1}{2}bc\sin\beta$	A
Area of $\triangle ADC = \frac{1}{2}ad \sin \delta$ and area of $\triangle ABC = \frac{1}{2}bc \sin \beta$	В
Therefore $PQRS = X - \frac{1}{4} \times 2X = \frac{1}{2}X$, and so the area of <i>PQRS</i> is equal to half the area of <i>ABCD</i>	С
Area of $\triangle SDP = \frac{1}{2} \times \frac{1}{2} a \times \frac{1}{2} d \sin \delta = \frac{1}{8} a d \sin \delta$	D
Area of $\triangle BAD = \frac{1}{2}ab \sin \alpha$ and area of $\triangle BCD = \frac{1}{2}cd \sin \gamma$	E
Let the area of $ABCD = X$	F
Rearranging gives $PQRS = X - \frac{1}{4} \left[\left(\frac{1}{2}ab\sin\alpha + \frac{1}{2}cd\sin\gamma \right) + \left(\frac{1}{2}ad\sin\delta + \frac{1}{2}bc\sin\beta \right) \right]$	G
Using previous results for X this gives $PQRS = X - \frac{1}{4}[X + X]$	н
The area of <i>PQRS</i> is given by <i>PQRS</i> = <i>ABCD</i> – (\triangle <i>PAQ</i> + \triangle <i>QBR</i> + \triangle <i>RCS</i> + \triangle <i>SDP</i>)	Ι
Therefore $X = \frac{1}{2}ab\sin\alpha + \frac{1}{2}cd\sin\gamma$	J
Similarly we have areas $\triangle PAQ = \frac{1}{8}ab\sin\alpha$, $\triangle QBR = \frac{1}{8}bc\sin\beta$ and $\triangle RCS = \frac{1}{8}cd\sin\gamma$	к
We have $PQRS = X - \left(\frac{1}{8}ab\sin\alpha + \frac{1}{8}bc\sin\beta + \frac{1}{8}cd\sin\gamma + \frac{1}{8}ad\sin\delta\right)$	L
Let $\angle DAB = \alpha$, $\angle ABC = \beta$, $\angle BCD = \gamma$ and $\angle CDA = \delta$	М