Konigsberg

Pre-requisite knowledge

No significant pre-requisite content knowledge is required for this problem.

Why do this problem?

Konigsberg demonstrates the power of an exhaustive approach in a spatial context. In addition, the underpinning theory is of interest in its own right. This problem offers a good opportunity to introduce a little history of mathematics into the lesson. An article about the origin of this problem can be found on the NRICH website.

Resources

"Tourism" problem (published on NRICH July 2004), sheet of Konigsberg diagrams.

Skipping ropes and hoops (optional)

Time

One to two lessons

Lesson 1

Introducing the problem

Layout skipping ropes and hoops to recreate some of the networks shown in the Tourism problem (for example numbers 1, 2, 4, 5, 7, 8). It might be helpful if these were arranged in a circuit for pupils to move around.

Explain to the pupils that if you can walk around a network without going over any line twice, then it is traversable (you can visit a hoop/node more than once but not a rope and you do not have to end up where you started).

Groups could move around the networks using copies of the networks used to record which are traversable (and how) and indicate which ones are not. For those that are traversable they should record where they started and where they finished.

Main activity

Feedback about anything they notice about traversable networks.

- How many lines or paths "come out" of each blob (node) in the different cases?
- Why might this be important?

Each group can make a traversable or non traversable network of their own. Groups swap and try out circuits and feed back.

Plenary

What makes a network traversable? You can only have two odd nodes (a node with an odd number of paths entering, or leaving, it) or no odd nodes.

In other words:

A traversable path must have all even nodes or all but two of the nodes even.

Note: If there is time it is worth discussing why there is a limitation on the number of odd nodes, which is to do with where you start and where you finish. This will lead into the next lesson.

Lesson 2

In the last lesson pupils were asked to identify traversable networks. In this introduction they are asked to distinguish traversable circuits from traversable paths as part of the recapitulation. Traversable circuits are ones where you must end up back where you started.

Introduction

Pupils will need copies of their network sheet from the previous lesson.

Recap what a traversable network is.

Ask the pupils to identify which of the remaining networks on the sheet are traversable (remembering to show where they started and where they finished). This could be done by trying to draw the circuits without taking their pencil off the paper and only traversing each path once.

Pull the findings of the class together, checking results and asking them to think about where all the pupils started and finished each network, and what differences they noticed. The reasons for the differences will be explored in the main part of the lesson.

Main part of the lesson

Describe the distinction between traversing paths and traversing circuits (easy to remember because circuit implies you end up back where you started).

- Of the networks that are traversable, in which ones did you end up at the same place that you started? (traversing circuit)
- In which of the networks that are traversable did you end up at a different point to the one at which you started? (traversing paths)

At this point you are looking for the pupils to make a connection between traversable/non-traversable networks and the number of odd and even nodes. If this does not arise from discussions of findings, it might help to look at some examples of

networks and ask pupils how they might describe them. They might have comments such as:
"it has five nodes and 10 lines – four of the nodes have an even number of lines – we need to start at the 'odd' node but we do not have anywhere to finish"
"we started and finished at the odd nodes – it is not a circuit"
"we started and finished at an even node – there were no odd nodes – it was a circuit"
"this had three odd nodes – it means we cannot get into one of them without needing to leave the node again, so unless the odd nodes are at the beginning or end they have to be even!"

From the discussions can the pupils suggest what the differences are between a circuit and a path?
Look at a couple of further examples to check hypotheses.

Present the Konigsberg problem as stated on the website. To help with the representation of the problem the Konigsberg diagram sheet includes a plan of Konigsberg and how this relates to the diagram on the problem sheet. In addition there is also a diagram of nodes without lines and a plan of the islands without bridges that pupils might use for their solutions.

Using what they have concluded in the main part of the lesson, students should be able to predict and explain their solutions to the problem.

Plenary

Using the teacher resources sheet with no bridges as an OHT, challenge pupils to add bridges to convert it from a non traversable to a traversable network.

Is their solution a circuit or a path?