

These are the 4 possible solutions, made up of 2 sets of numbers, repeated in a different order.

2	3	2	1	1	2	1	4
1	8	3	8	4	7	2	7
23		21		12		14	
21		23		14		12	
38		18		27		47	
<u>18</u>		<u>38</u>		<u>47</u>		<u>27</u>	
100		100		100		100	

We discovered a digit pattern that happened with the placement of the digits in the boxes

tens (a)	tens and ones (b)
tens and ones (c)	ones (d)

After a lot of trial and error, we discovered that the sum of the units had to be 20 and the sum of the tens equalled 80. So we know that the units column will have two of the same number. An important thing to consider is that there has to be a carry over from the ones to the tens because the ones digit ends in zero. If there is a large number that repeats though, we know that the answer will definitely be higher than ten and if there are three more different numbers, so the answer of the ones column will be 20. There will be a carryover of 2. So the total of the tens column will be 80. So if the tens column is meant to equal 10 (the value is 100) and there is a carryover of two then the other numbers will equal 8 ($10 - 2 = 8$).

We knew that larger digits couldn't be used in the tens because two of the same tens digit in the top right hand corner would make the value greater than 100. The tens have to add up to 8 tens in the formula $2a + b + c = 8$. If $a = \{3, \dots, 9\}$ $2a + b + c > 8$ even with $b = 1, c = 2$ (the smallest totals; b and c must be different). For $a = 3$ to be possible, the units can only add up to 10. The formula for the units is $2d + b + c$. This is not possible because a can't be greater than 3.

Here is a table to explain our results

a	$2a + b + c$	Total	d	$2d + b + c$	Total
3	$6 + 3$	9 tens (possible?)	1	$2 + 8$	10 but not possible b and c can't be more than 4 or both 4 (see a table)
4	$8 + 3$	11 tens (too high)	2	$4 + 6$	10 but not possible b and c can't be both 3 or d and c both 4
5	$10 + 3$	13 tens	3	$6 + 4$	10 but not possible b and c can't be both 2 and a and d can't both be 3
6	$12 + 3$	15 tens (too high)	4	Every result over 10. To get next ten (20) need b and c to =12 but b and c ≤ 4	
7	$14 + 3$	17 tens (too high)	5	Every result over 10. To get next ten (20) need b and c to =10 but b and c ≤ 4	
8	$16 + 3$	19 tens (too high)	6	Every result over 10. To get next ten (20) need b and c to =8 but b and c ≤ 4	
9	$18 + 3$	21 tens (too high)	7	$14 + (4 + 2)$	20 ones. b and c are 4 and 2 (can't both be 3)
			8	$16 + (3 + 1)$	20 ones b and c are 3 and 1 (can't both be 2)
			9	Every result over 20. because b and c can't both be 1	

From these results, we proved that

sum of tens	sum of the units
80	20

this table shows the way it works

Tens	Ones
a	b
a	c
b	d
c	d
80	20
$80 + 20 = 100$	

We then created formulas for the results

(a) $2 \times 10a$	(b) $10b + b$
(c) $10c + c$	(d) $2d$

For the result

2	3
1	8

$$(2 \times 20) + (30 + 3) + (10 + 1) + (2 \times 8)$$
$$40 + 33 + 11 + 16 = 100$$

The 3 and 1 can change place for same result

For the result

1	4
2	7

$$(2 \times 10) + (40 + 4) + (20 + 2) + (2 \times 7)$$
$$20 + 44 + 22 + 14 = 100$$

The 4 and 2 can change place for same result

Another way of describing correct numbers in the 2 x 2 grid is

(a) a	(b) $8 - 2a = b + c$
(c) $8 - 2a = b + c$	(d) $(20 - b - c)/2$

Therefore if $a = 1$, then $b+c = 8 - 2$ or 6 and $d = (20 - 6)/2$, meaning $a = 1$, $b = 2$ or 4, $c = 4$ or 2 and $d = 7$

Or

If $a = 2$, $b + c = 8 - 4$ or 4 and $d = (20 - 4)/2$, meaning $a = 2$, $b = 1$ or 3, $c = 1$ or 3 and $d = 8$

These are the only possible results.