Proof (Triangular Number x 8) + 1 Is Equal to Perfect Square Number

- Before equate of 8(△ num) + 1 = □ num we must find a formula for △ numbers
  - Instead of writing △ numbers like this:

1
2 3
4 5 6
7 8 9 10

Write them like this:

1
2 3
4 5 6
7 8 9 10

which is a right angled △ => ◺

If we flip this and set it next to each other, you get a rectangle

So in this example, we've used 10 which is the forth triangle number and our rectangle has area 4×5, so we can deduce that for

$$\text{Triangular number}_n = \frac{n(n+1)}{2}$$

- So now we know that $T_n = \frac{n(n+1)}{2}$

If we sub this into $8n+1$ where $n = T_n$, we get

$$8 \cdot \frac{n(n+1)}{2} + 1$$

$$= 4(n^2+n) + 1$$

$$= 4n^2 + 4n + 1$$

$$= (2n+1)^2$$ which is always square number; odd.

So a quick way to check is to take the number, and multiply by 2.

eg

$$6214 \times 2 = 12428$$

Then if we take a square root

$$\sqrt{12428} = 111.48\ldots$$

But if we take the integer value i.e. 111 and sub it back into $T_n = \frac{n(n+1)}{2}$

if the number is triangular, it should return us to the original number.

eg $\frac{(111)(112)}{2} = 6216 \neq 6214$

$\therefore$ 6214 not $\triangle$ ✗

$3655 \times 2 = 7310$

$\sqrt{7310} = 85.49\ldots$ $\frac{1}{2}(85)(86) = 3655$ $\therefore \triangle$ ✓

$7626 \times 2 = 15252$

$\sqrt{15252} = 123.49\ldots$ $\frac{1}{2}(123)(124) = 7626$ $\therefore \triangle$ ✓

$8656 \times 2 = 17312$

$\sqrt{17312} = 131.57\ldots$ $\frac{1}{2}(131)(132) = 8646$ $\therefore$ not $\triangle$ ✗