



Rearrange these statements to form a proof about triangular and square numbers.

Hence,  $8k + 1 = 4n^2 + 4n + 1$

Suppose  $8k + 1$  is a square number

Factorising the right hand side,  $8k = 4n(n + 1)$

This is the general form for a triangular number.  
Therefore, if  $8k + 1$  is square,  $k$  is triangular.

We wish to prove that if  $8k + 1$  is a square number  
then  $k$  is a triangular number.

Simplifying,  $k = \frac{1}{2}n(n + 1)$

All odd squares are of the form  $(2n + 1)^2 = 4n^2 + 4n + 1$

Dividing both sides by 8,  $k = \frac{4n(n+1)}{8}$

Subtracting 1 from both sides,  $8k = 4n^2 + 4n$

Since  $8k$  is even,  $8k + 1$  is odd