Nrich Solution for Two and Two (Sat Batch)

This task of Two and Two was taken for 2 sessions with a group of 15 students in Ganit Kreeda, Vicharvatika, India by **Shubhangee(facilitator).**

The names of the students are:
Ahana, Ananthajith, Dhruv, Sehar, Saanvi, Nikhil, Insiya, Inaaya, Ishaan, Ishanvi, Sanat, Abhay, Karthik, Vishnuvardhan, Rudra

Kids together found out that:
ONE + TWO = THREE
Not possible, as two 3-digit numbers' sum cannot be 5-digit.
ONE + THREE = FOUR
Not possible as one 3-digit number and one 5-digit number cannot be added to get 4-digit sum.

Karthik created one problem as a solution to:
Can you create other similar cryptarithms?

FOUR	**8532**
+FOUR	**+8532**
EIGHT	**17064**

R=2, T=4, U=3, H=6, O=5, G=0, E=1, F=8, I=7

While experimenting with numbers, Vishnuvardhan realized that any addition problem can be converted as a subtraction problem in 2-ways.
TWO – ONE = ONE,
NINE - FOUR = FIVE,
NINE - FIVE = FOUR

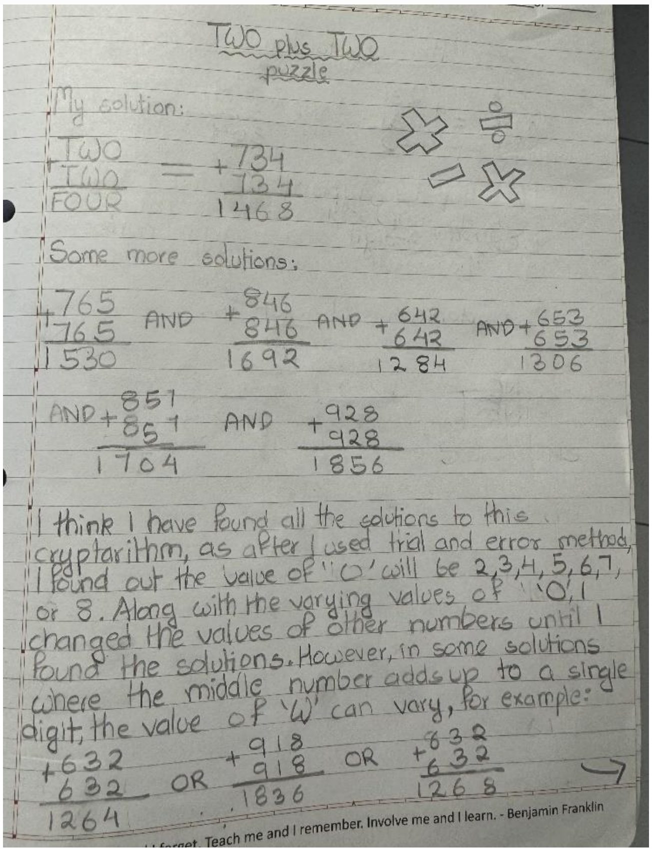
Kids shared different observations for

TWO
+TWO
FOUR

- **R is always even, as R= O+O.**
- **O cannot be equal to 0/1/9.**
- **F is always equal to 1.**

Ahana, Karthik and Inayaa worked on this in a breakout room to find all possible solutions along with the reason. Here is Ahana's work:

Ahana's work:



TWO plus TWO puzzle

My solution:

TWO + TWO = FOUR = 734 + 734 = 1468

Some more solutions:

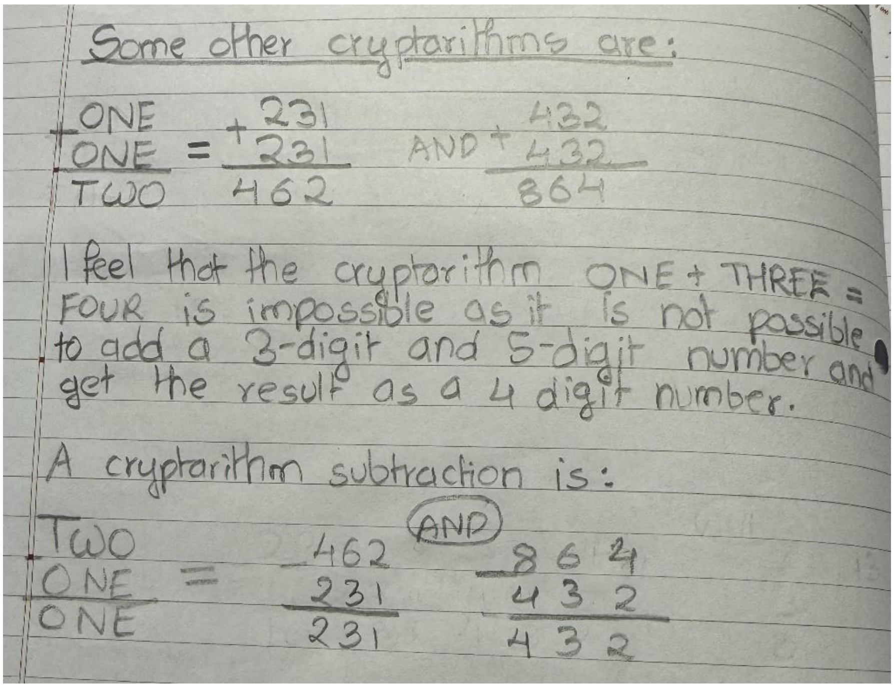
765 + 765 = 1530 AND 846 + 846 = 1692 AND 642 + 642 = 1284 AND 653 + 653 = 1306

AND 851 + 851 = 1704 AND 928 + 928 = 1856

I think I have found all the solutions to this cryptarithm, as after I used trial and error method, I found out the value of 'O' will be 2, 3, 4, 5, 6, 7, or 8. Along with the varying values of 'O', I changed the values of other numbers until I found the solutions. However, in some solutions where the middle number adds up to a single digit, the value of 'W' can vary, for example:

632 + 632 = 1264 OR 918 + 918 = 1836 OR 632 + 632 = 1268

Teach me and I remember. Involve me and I learn. - Benjamin Franklin



Some other cryptarithms are:

ONE + ONE = TWO = 231 + 231 = 462 AND 432 + 432 = 864

I feel that the cryptarithm ONE + THREE = FOUR is impossible as it is not possible to add a 3-digit and 5-digit number and get the result as a 4 digit number.

A cryptarithm subtraction is:

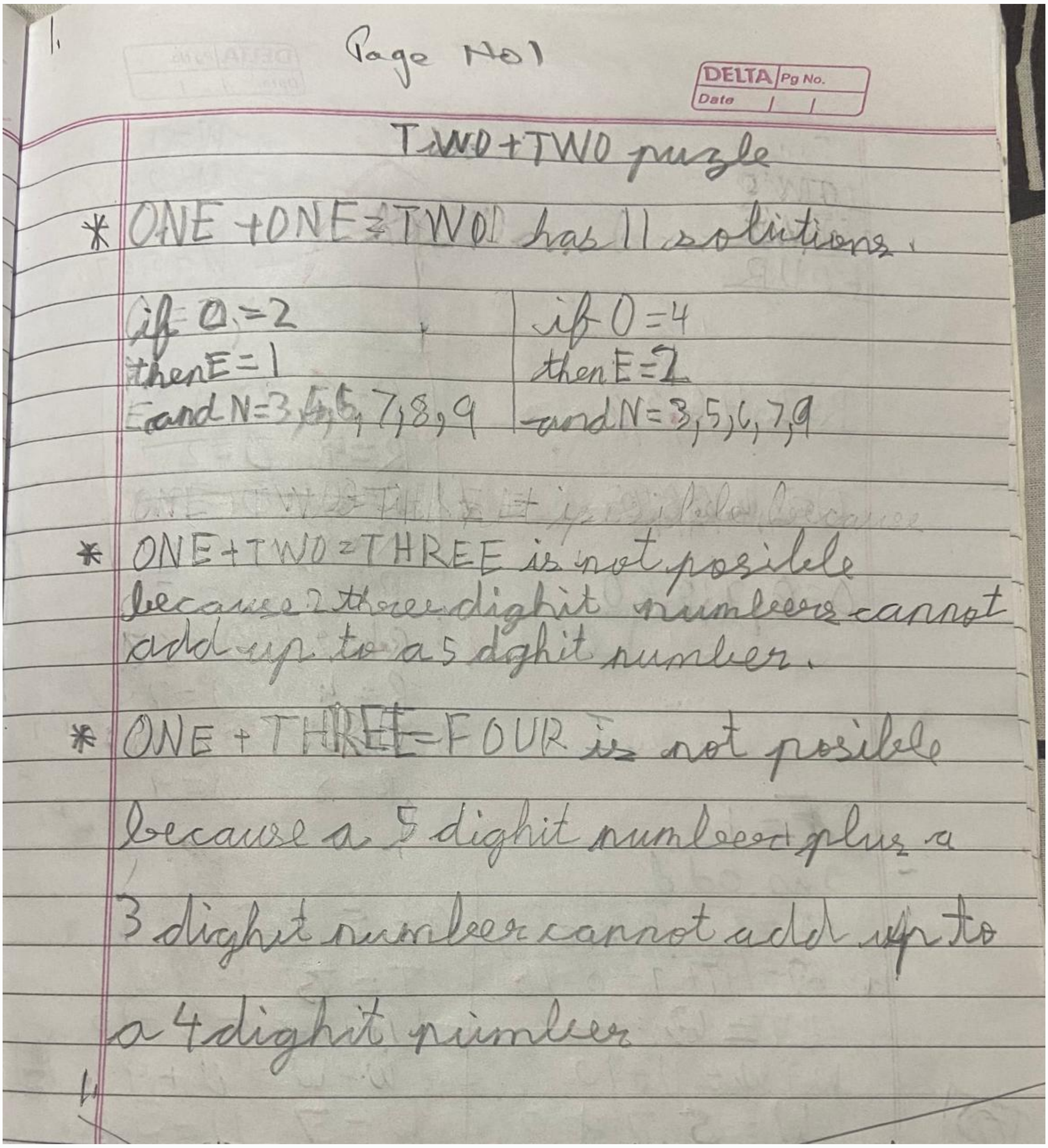
TWO − ONE = ONE = 462 − 231 = 231 AND 864 − 432 = 432

Abhay says that for TWO + TWO = FOUR….

The problem suggests that the addends have to be the same and above 500 and the ones digit of the addend and the hundredth digit of the sum have to be the same so I tried a trial and error method.

Anathajith, Sehar, Dhruv and Saanvi worked together and their work is summarized by Sehar here.

Here is Sehar's Work for ONE + ONE =TWO


1.

Page No 1

DELTA Pg No.
Date / /

TWO+TWO puzzle

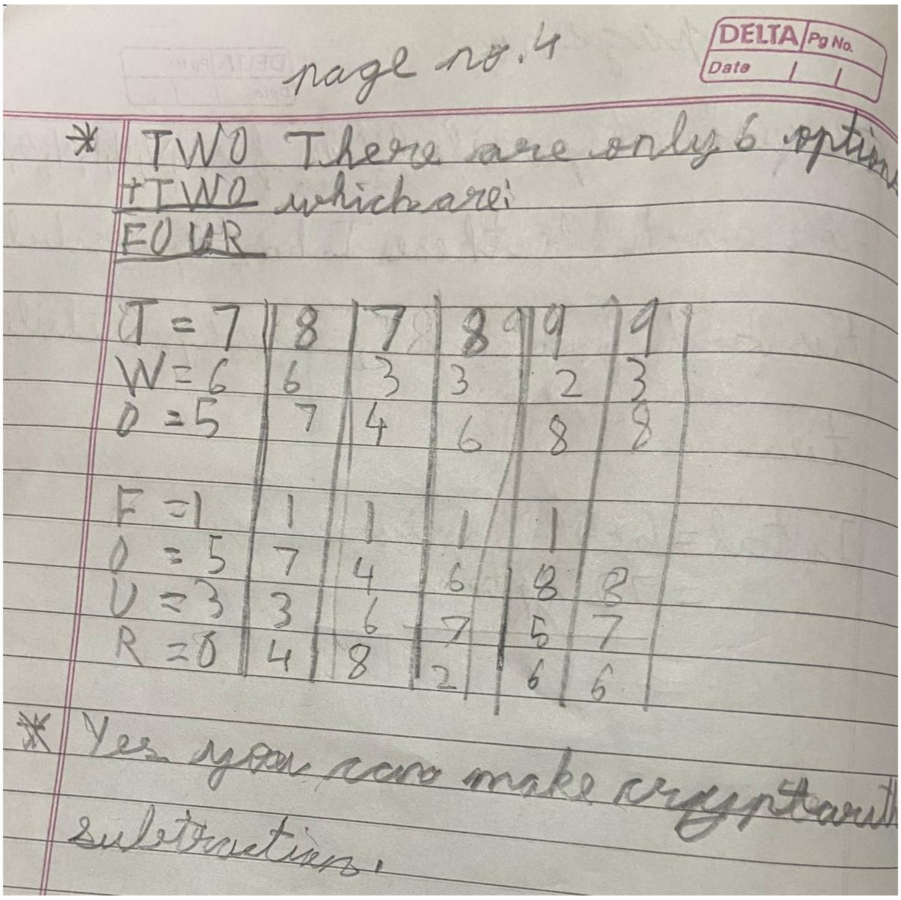
* ONE+ONE=TWO has 11 solutions.

if O=2	if O=4
then E=1	then E=2
and N=3,4,6,7,8,9	and N=3,5,6,7,9

* ONE+TWO=THREE is not posible because 2 three dighit numbers cannot add up to a 5 dighit number.

* ONE + THREE=FOUR is not posible because a 5 dighit number plus a 3 dighit number cannot add up to a 4 dighit number

Here is Sehar's Work for TWO + TWO = FOUR



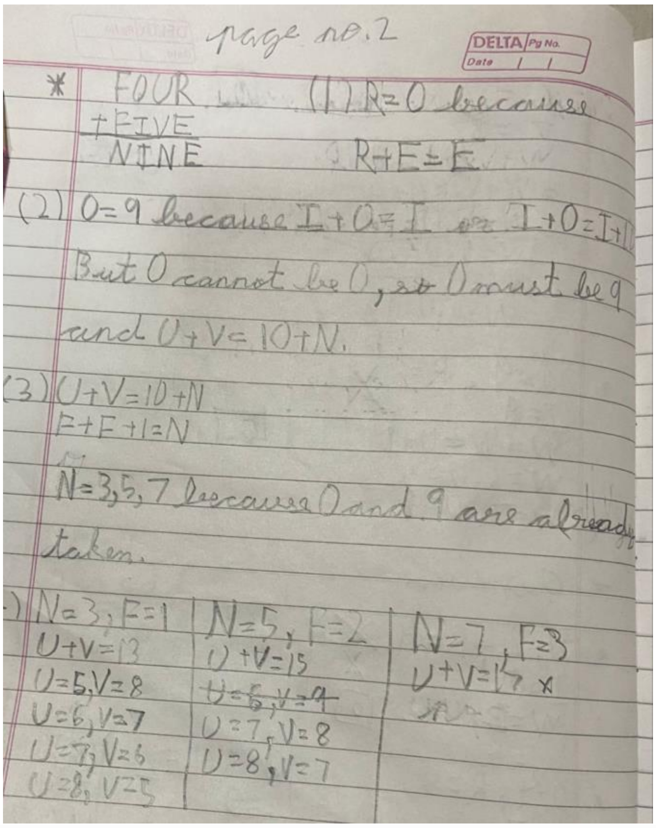
page no.4

DELTA Pg No. Date / /

* TWO There are only 6 options
+TWO which are:
FOUR

T = 7	8	7	8	9	9
W = 6	6	3	3	2	3
O = 5	7	4	6	8	8
F = 1	1	1	1	1	
O = 5	7	4	6	8	8
U = 3	3	6	7	5	7
R = 0	4	8	2	6	6

* Yes you can make cryptarith subtraction.



page no.2

DELTA Pg No. Date / /

* FOUR (1) R=0 because
+FIVE R+E=E
NINE

(2) O=9 because I+O=I or I+O=I+10
But O cannot be 0, so O must be 9
and O+V=10+N.

(3) U+V=10+N
F+F+1=N

N=3,5,7 because 0 and 9 are already taken.

(4) N=3, F=1	N=5, F=2	N=7, F=3
U+V=13	U+V=15	U+V=17 x
U=5, V=8	~~U=6, V=9~~	
U=6, V=7	U=7, V=8	
U=7, V=6	U=8, V=7	
U=8, V=5		

This is Sehar's reasoning for FOUR + FIVE = NINE

page no. 3

(5) There are 6 possibilities for N, U, F, V, O, R
For each of these I has 4 possibilities and for each I, E has 3 possibilities.

Total = $6 \times 4 \times 3$ options
= 72 options

Kids used Sehar's reasoning and found out all possible solutions for:

$$\begin{array}{rcccc} & F & O & U & R \\ + & F & I & V & E \\ \hline & N & I & N & E \end{array}
\qquad
\begin{array}{rcccc} & 1 & 9 & 5 & 0 \\ + & 1 & 6 & 8 & 2/4/7 \\ & 1 & & & \\ \hline & 3 & 6 & 3 & 2/4/7 \end{array}$$

I & E can be exchanged among themselves.
So, 12 diff^n solutions.

R=0 and O=9, U and V can be exchanged.
F < 5 and N is odd as N=F+F+1

N=1 F=0	N=3 F=1	N=5 F=2	N=7 F=3	N=9 F=4
Not possible. As starting digit can't be zero.	U=5,V=8	U=7,V=8	U=8,V=9	**Not possible** asU+V=19 is impossible to get using 2 digits.
	U=6,V=7	U=8,V=7	U=9,V=8	
	U=7,V=6			
	U=8,V=5			
	4 solutions	2solutions	2solutions	

$\therefore$ Total sums $= (4+2+2)\times 12 = 8 \times 12 = 96$ solutions.

Shubhangee found one new Cryptarithm problem as:

$$\begin{array}{r} FIVE \\ -\ ONE \\ \hline FOUR \end{array}$$

$\Rightarrow$ R=0
E can be anything other than 0.
F can also be anything other than 0.
I cannot be smaller than 0.

$$\begin{array}{r} 2879 \\ -\ 469 \\ \hline 2410 \end{array} \qquad \begin{array}{r} 2754 \\ -\ 364 \\ \hline 2390 \end{array}$$

$\leftarrow$ These 2 are sample solutions.