

Frogs

First of all I tried what is the quickest amount of moves to find 1 red frog and 1 blue frog. Then 2 red frogs and 2 blue frogs. After that I tried 3 red frogs and 3 blue frogs, then I continued the same thing until 5. I used red cubes and blue cubes as frogs instead because it makes it easier and help imagine the situation. The table below shows my findings.

<i>Red Frogs</i>						<i>Blue Frogs</i>
	1	2	3	4	5	
1	3					
2		8				
3			15			
4				24		
5					35	

Now I try to find the formulae so I find the second difference.

1	2	3	4	5
3	8	15	24	35
	+5	+7	+9	+11
	(+2)	(+2)	(+2)	

Quadratic sequences have a common second difference. They are of the form $UN = an^2 + bn + c$. Where the 2nd difference is $2a$. So if $2a = 2$. So $a = 1$ because $2/2 = 1$.

So we start with ***n squared***. We square all the bold numbers above. So it will be . . .

1	2	3	4	5
1	4	9	16	25

Now we subtract the two answers. And find the difference. So it will be . . .

	3	8	15	24	35
-	1	4	9	16	25
	2	4	6	8	10
	+2	+2	+2	+2	

Because it is 2 the formulae is $n^2 + nb$. I checked it, and it worked for all the cases. But it does not work for cases such as two red frogs and 3 blue frogs. So with further investigating with cubes, I found the least amount of moves without going backwards in the table below.

<i>Red Frogs</i>						<i>Blue Frogs</i>
	1	2	3	4	5	
1	3	5	7	9	11	
2	5	8	11	14	17	
3	7	11	15	19	24	
4	9	14	19	24	29	
5	11	17	24	29	35	

So for 1 blue frog the difference is two between them. The diagram below shows it.

3 5 7 9 11

+2 +2 +2 +2

Therefore it would be $2n + \text{something}$. I tried adding 1 to each one and it works.

Then I tried 2 blue frogs and I got the difference of 3. The diagram below shows it.

5 8 11 14 17

+3 +3 +3 +3

I repeated the same thing over and over until 5 blue frogs. My results are the following bellow. By the way, M is the amount of moves. B is the number of blue frogs and R is number of red frogs.

$$B = 1 \quad M = 2R + 1$$

$$B = 2 \quad M = 3R + 2$$

$$B = 3 \quad M = 4R + 3$$

$$B = 4 \quad M = 5R + 4$$

$$B = 5 \quad M = 6R + 5$$

I found out that the + something is the same amount of blue frogs and the red frogs is the 1 more than blue frogs. So with this information, I came up with the formulae $B(R+1)+R$. If I expand the brackets the answer would be . . .

$$\underline{\underline{BR + B + R}}$$

After checking it thoroughly with others, I concluded that the formula is correct.