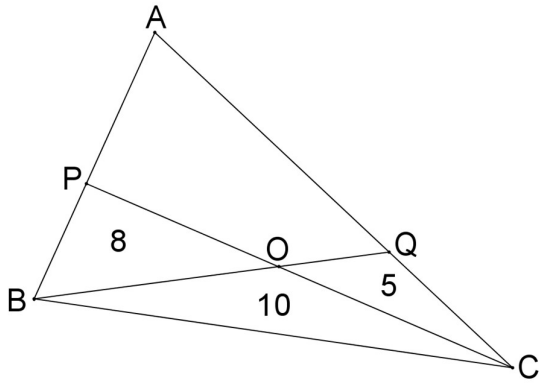
Areas and Ratios

Can you rearrange the statements below to find the area of the quadrilateral AQOP?



A	Substituting this back into equation (1) gives $E = 12$.
B	Triangles QAO and OAB have the same height above the line BQ, so $\frac{F}{OQ} = \frac{E+8}{BO}$.
C	Triangles PAO and OAC have the same height above the line PC, so $\frac{E}{PO} = \frac{F+5}{CO}$.
D	Combining these two equations gives $\frac{E}{8} = \frac{F+5}{10}$.
E	The area of quadrilateral $AQOP$ is $E + F = 10 + 12 = 22$.
F	Triangles PBO and OBC also have the same height above the line PC, so $\frac{8}{PO} = \frac{10}{CO}$.
G	Solving for F gives $F = 10$.
H	Doubling equation (2) and rearranging gives $10E = 20F - 80$.
I	Draw in the line AO. Label the area of triangle AOP as E, and that of AOQ as F.
J	Clearing denominators gives $10E = 8F + 40$. Call this equation (1).
K	Clearing denominators gives $10F = 5E + 40$. Call this equation (2).
L	Combining these two equations gives $\frac{F}{5} = \frac{E+8}{10}$.
M	Triangles QCO and OBC also have the same height above BQ, so $\frac{5}{OQ} = \frac{10}{BO}$.
N	Substituting this into equation (1) gives $20F - 80 = 8F + 40$.

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