1. I began by simplifying the expressing each value on the domino in their simplest form

For example the 1st domino simplified to a-b on the left and a+b/a-b on the right

Here are the rest:
a-b|a+b/a-b
b-a|b
b/a|a^2 - b^2
a-b|a^2 b
ab|a/b
a-b|ab^2
a^2+b^2|b
a+b|b-a
b/a|a+b
b|a+b/a-b
a|a^2+b^2
a^2 b|b
a b^2|a/b
b-a|a^2-b^2
ab|a-b
a|b-a

Overall you had, 4 a-b, b-a, b terms each
And the rest had 2 terms each

As a result, for 4 doughnuts you could utilise the expressions with lower frequencies because they narrow down the possible domino links and only 1 other domino will connect:

(let a domino like a|b be represented as a > b, so x > y > z is x|y and y|z)

1. a > b-a > b > a^2 + b^2 > a
2. a+b > b-a > a^2 - b^2 > b/a > a+b
3. ab^2 > a-b > ab > a/b > ab^2
4. a+b/a-b > a-b > a^2 b > b > a+b/a-b

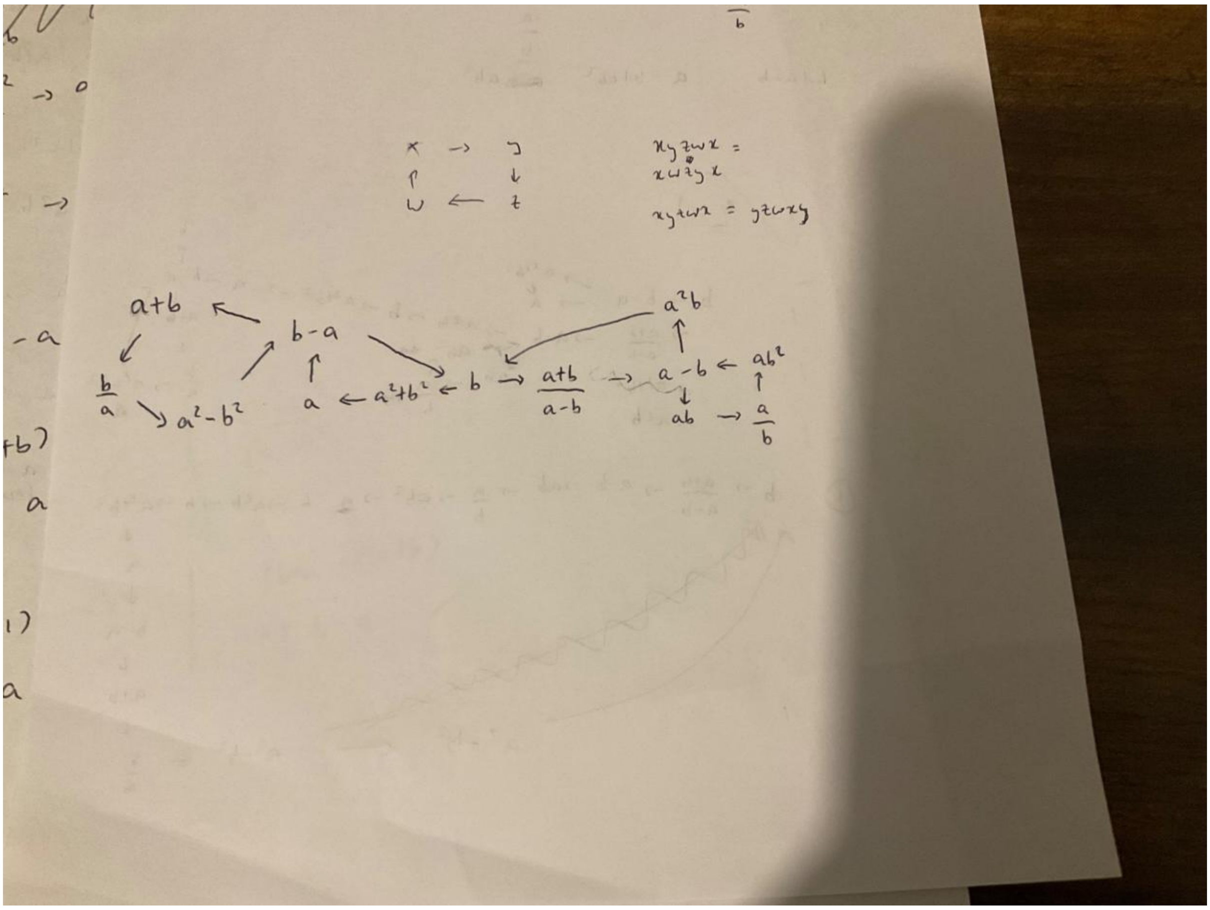
2. You could now use the terms with 4 possible links to for 8 doughnuts because of more possibilities , in this case I will use b

1. b > a+b/a-b > a-b > ab > a/b > ab^2 > a-b > a^2 b > b
2. b > b-a > a+b > b/a > a^2-b^2 > b-a > a > a^2+b^2 > b

Notice that in 8 links, you end up looping back to a 4 term twice but you don't have to choose the same option. As a result, you have exhausted 3 out of 4 possible ways to walk through that junction and we also haven't linked to 3 seperate 4 terms in those routes. In this way we can obtain 16 links .

1. **b** > a+b/a-b > **a-b** > ab > a/b > ab^2 > **a-b** > a^2 b > **b** > a^2+b^2 > a > **b-a** > a+b > b/a > a^2 - b^2 > **b-a** > **b**

3. Now the question is how many 16 links can we obtain , to solve this we can draw a map.



At the top we have the rules laid out for the equivalences.
We have rotational equivalence where we say
xyzwx = yzwxy, because they are the same map but done at different start points

Also we have directional equivalence
xyzwx = xwzyx because we are just traversing the same map but in a different direction.

In terms of the layout, notice that we have 3 nodes of degree 4 and the rest are degree 2. As a result, we will only come back to the nodes of degree 4 when making 16 links because there are more options. In addition , if we started from a node of degree 2 these will just rotationally be equivalent to another way. Also notice that b is a hub that links the left loop to the right loop, it’s then easier to find paths from b to b.

Let's say we start at b,
notice we have 2 choices , go left or right.
If we go right we have 2 more , go to a^2 b or to a+b/a-b
2 more when we approach the loop at a - b , we can traverse the ring CW or CCW
Due to the choices we made, we can only go to b through one route
We have 2 more choices to go to b - a or to a
We have 2 more choices to traverse the ring at b - a CW or CCW.

We now have 2^5 options for b to b which is 32 . We now also have 32 for the other degree 4 nodes. Despite this we only count one set as each completed route can be written from different starting points. Ultimately, you are still traversing the same path by rotational equivalence (xyzwx = yzwxy)

For example, take L1 to be any form of route through the left cyclic loop and R1 to be any route through the right cyclic loop.
In the 32 we will have L1+R1 and R1+L1 which are just equivalent as they are the same map but in a different order. Therefore, we divide by 2 to obtain 16 choices because of duplicates.

Now in the 16, we also have 8 perfectly opposite pairs. In the rules we laid out we said xyzwx = xwzyx, showing that CW and CCW traversals are just opposites of each other and are equal by directional equivalence.

Take L1+R1 to be our route , in the 16 we will also count reverse (L1+R1). These are just the same as we traverse the map in different directions. As a result we have 8 unique choices and 8 reversed forms.

Overall, we have 8 different 16 link doughnuts we can make.