

My problem: I've chosen has the problem code 1,6,0,4,7,16,0,2

I will be answering the extension problems on the question:

1. The first 3 digit positive number that turns on all lights (ANSWER: 196)
2. The first 3 digit negative number that turns on all lights (ANSWER: -164)
3. The first 4 digit positive number that turns on all lights (ANSWER: 1096)
4. The first 4 digit negative number that turns on all lights (ANSWER: -1064)

Approach: I will attempt to solve this problem by generating the nth number linear sequence that turns on all four lights, which can be used to solve all four problems. To do this, I'll create a linear sequence for each of the lights, and combine the sequences to form the equation that turns on all lights. For each of the four lights, I've noted down the first three numbers that turn on the respective light, to create the first linear sequences.

The lights turn on in a linear sequence. I will find each of the four linear sequences as my first step:

1. Green: Lights turn on at 0, 2, 4... The nth term is  $2n-2$ , or all multiples of two
2. Red: Lights turn on at 0, 4, 8... The nth term is  $4n-4$ , or all multiples of four
3. Yellow: Lights turn on at 1, 6, 11... The nth term is  $5n+1$
4. Blue: Lights turn on at 7, 16, 25... The nth term is  $9n-2$

To make this easier to solve, I've decided to combine the sequences to solve the problem more easily:

1. Green + Red: simply becomes multiples of 4, as multiples of 4 are multiples of 2.
2. Yellow + blue: To find numbers that appear in both sequences, I'll form the equation  $5n+1=9m-2$   
 $\rightarrow n=(9m-3)/5$ . m must be an integer after dividing by 5, the first two numbers will be 2, 7... Once m is replaced with these numbers, the nth term of the number sequence ( $5n+1$ ) is equal to 16, 61, 106.... By observation, the nth term of this new sequence (which turns on yellow and blue lights) is  $45n-29$ .

I only need the number 45, and I need the LCM of 45 (turns on yellow and blue lights) and 4 (turns on green and red lights), which is 180. This means all numbers will light up once every 180 integers. We know 16 is the first positive integer to meet this criteria, since it's a multiple of 4 and follows the sequence  $45n-29$  ( $n=1$ ). This means the nth term sequence is  $180n-164$ , and the first positive 3 digit number where all lights turn on is 196 ( $n=2$ ).

The next problem to solve is the first negative 3-digit number. Since this number is relatively small, I can reduce n by 1 until I reach the first number below -99. Zero works because  $180 \times 0 - 164 = -164$ , thus the answer is -164.

To find the first positive 4 digit number, I can use the nth term sequence I created before, while forcing the nth term of the sequence to be larger than or equal to 1000. That is,  $180n-164 \geq 1000$ . n is therefore equal or greater than 6.46. After rounding n up to next integer 7 and plugging it into the sequence,  $180 \times 7 - 164$  is equal to 1096. Thus the answer is 1096.

To find the first negative 4 digit number, I can simply use the same equation but instead change 1000 to -1000, the equation would be  $180n-164 \leq -1000$ , n is equal to approximately -4.64, thus the answer will be  $-5 \times 180 - 164 = -1064$ .