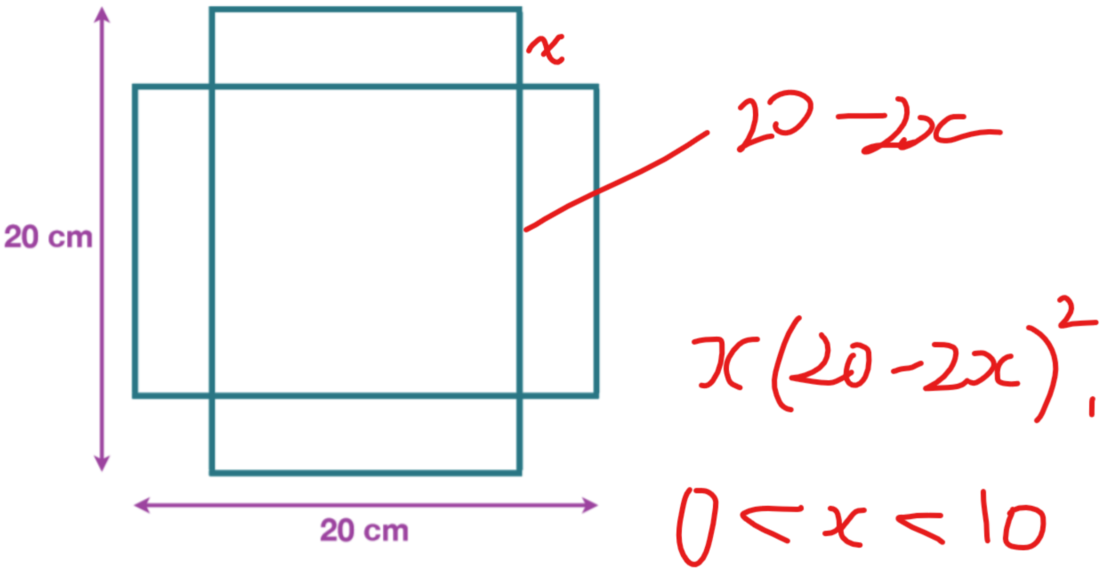

For a box generated this way, the height is x, and the side lengths of the square are $20-2x$. The volume is $x(20-2x)^2$.

$$=x(400-80x+4x^2)$$
$$=4x^3-80x^2+400x$$
$$=4x(x^2-20x+100)$$

Also, the domain of the function is $0<x<10$.

1. Using AM-GM

The AM-GM equality states that $\frac{a+b+c}{3}\geq\sqrt[3]{abc}$ for real numbers a, b, and c. We can set them to cancel out like this:

$$a=x$$
$$b=\frac{10-x}{2}$$
$$c=\frac{10-x}{2}$$
$$\frac{a+b+c}{3}\geq\sqrt[3]{abc}$$
$$abc=\frac{4x(10-x)\text{^}2}{3}$$
$$a+b+c=10$$
$$\sqrt[3]{\frac{x(10-x)^2}{4}}\leq\frac{10}{3}$$
$$\frac{x(10-x)^2}{4}\leq\frac{10^3}{3^3}$$
$$\frac{100x-20x^2+x^3}{4}\leq\frac{1000}{27}$$

$$4(100x - 20x^2 + x^3) \leq \frac{16000}{27}$$

2. Using Graphical

We can simply plot the graph of $y = x(20-x)^2$ between 0 and 10 to find the maximum of the value, using a GDC or any graphing calculator. We find this value is $x = 3\frac{1}{3}$ or $\frac{10}{3}$, plugging it back in we get $\frac{16000}{27}$ for the volume.